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Forum for CSIR-UGC JRF/NET, GATE, IIT-JAM/IISc, JEST, TIFR and GRE in PHYSICS & PHYSICAL SCIENCES

ELECTRONICS

(IIT-JAM/JEST/TIFR/M.Sc Entrance)

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1. Semiconductor Physics

1.1 Metals, Semiconductors, and Insulators

Every solid has its own characteristic energy band structure. This variation in band structure is responsible for the wide range of electrical characteristics observed in various materials. The diamond band structure for example, can give a good picture of why carbon in the diamond lattice is a good insulator. To reach such a conclusion, we must consider the properties of completely filled and completely empty energy bands in the current conduction process.

Before discussing the mechanisms of current flow in solids further, we can observe here that for electrons to experience acceleration in an applied electric field, they must be able to move into new energy states. This implies there must be empty states (allowed energy states which are not already occupied by electrons) available to the electrons. For example, if relatively few electrons reside in an otherwise empty band, ample unoccupied states are available into which the electrons can move. On the other hand, the diamond structure is such that the valence band is completely filled with electrons at $0\,K$ and the conduction band is empty. There can be no charge transport within the valence band, since no empty states are available into which electrons can move. There are no electrons in the conduction band, so no charge transport can take place there either. Thus carbon in the diamond structure has a high resistivity typical of insulators.

Semiconductor materials at $0\,K$ have basically the same structure as insulators-a filled valence band separated from an empty conduction band by a band gap containing no allowed energy states (Figure 1.1). The difference lies in the size of the band gap E_g which is much smaller in semiconductors than in insulators. For example, the semiconductor Si has a band gap of about $1.1\ eV$ compared with $5\ eV$ for diamond. The relatively small band gaps of semiconductors allow for excitation of electrons from the lower (valence) band to the upper (conduction) band by reasonable amounts of thermal or optical energy.

For example, at room temperature a semiconductor with a $1\,eV$ band gap will have a significant number of electrons excited thermally across the energy gap into the conduction band whereas an insulator with $E_g = 10\ eV$ will have a negligible number of such excitations. Thus an important difference between semiconductors and insulators is that the number of electrons available for conduction can be increased greatly in semiconductors by thermal or optical energy.

In metals the bands either overlap or are only partially filled. Thus electrons and empty energy states are intermixed within the bands so that electrons can move freely under the influence of an electric field. As expected from the metallic band structures, metals have a high electrical conductivity.

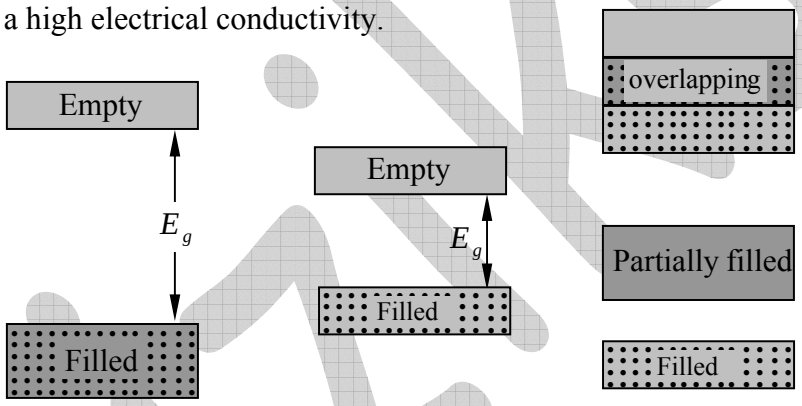


Figure 1.1:

1.2 Direct and Indirect Semiconductors

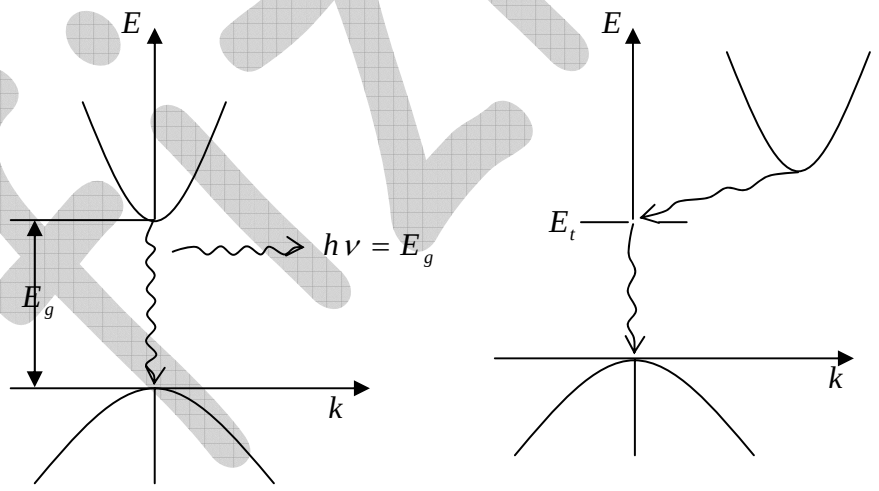
When quantitative calculations are made of band structures, a single electron is assumed to travel through a perfectly periodic lattice. The wave function of the electron is assumed to be in the form of a plane wave moving, for example, in the x-direction with propagation constant k, also called a wave vector. The space-dependent wave function for the electron is

$$\psi_k(x) = U(k_x, x)\, e^{ik_x x}$$

where the function $U(k_x, x)$ modulates the wave function according to the periodicity of the lattice.

In such a calculation, allowed values of energy can be plotted vs. the propagation constant k. Since the periodicity of most lattices is different in various directions, the (E,k) diagram must be plotted for the various crystal directions and the full relationship between E and k is a complex surface which should be visualized in three dimensions.

The band structure of $GaAs$ has a minimum in the conduction band and a maximum in the valence band for the same k value $(k=0)$. On the other hand, Si has its valence band maximum at a different value of k than its conduction band minimum. Thus an electron making a smallest-energy transition from the conduction band to the valence band in $GaAs$ can do so without a change in k value; on the other hand a transition from the minimum point in the Si conduction band to the maximum point of the valence band requires some change in k. Thus there are two classes of semiconductor energy bands ***direct*** and ***indirect*** (Figure 1.2). We can show that an indirect transition involving a change in k requires a change of momentum for the electron.



(a) Direct (b) Indirect

Figure 1.2: Direct and indirect electron transitions in semiconductors: (a) direct transition with accompanying photon emission; (b) indirect transition via a defect level.

In a direct semiconductor such as $GaAs$, an electron in the conduction band can fall to an empty state in the valence band, giving off the energy difference E_g as a photon of light. On the other hand, an electron in the conduction band minimum of an indirect semiconductor such as Si cannot fall directly to the valence band maximum but must undergo a momentum change as well as changing its energy. For example, it may go through some defect state (E_t) within the band gap. In an indirect transition which involves a change in k, the energy is generally given up as heat to the lattice rather than as an emitted photon. This difference between direct and indirect band structures is very important for deciding which semiconductors can be used in devices requiring light output. For example, semiconductor light emitters and lasers generally must be made of materials capable of direct band-to-band transitions or of indirect materials with vertical transitions between defect states.

1.3 Electrons and Holes

As the temperature of a semiconductor is raised from $0\,K$, some electrons in the valence band receive enough thermal energy to be excited across the band gap to the conduction band. The result is a material with some electrons in an otherwise empty conduction band and some unoccupied states in an otherwise filled valence band (Figure 1.3). For convenience, an empty state in the valence band is referred to as a *hole.* If the conduction band electron and the hole are created by the excitation of a valence band electron to the conduction band, they are called an ***electron-hole pair*** (abbreviated EHP).

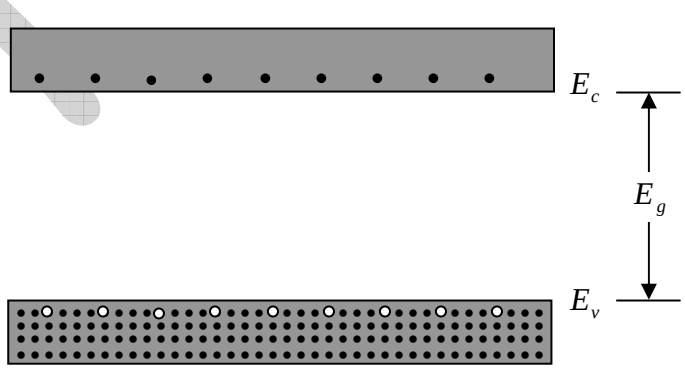


Figure 1.3: Electron-hole pairs in a semiconductor.

After excitation to the conduction band, an electron is surrounded by a large number of unoccupied energy states. For example, the equilibrium number of electron-hole pairs in pure Si at room temperature is only about $10^{10}\ EHP/cm^3$, compared to the Si atom density of more than $10^{22}\ atoms/cm^3$. Thus the few electrons in the conduction band are free to move about via the many available empty states.

1.3.1 Effective Mass

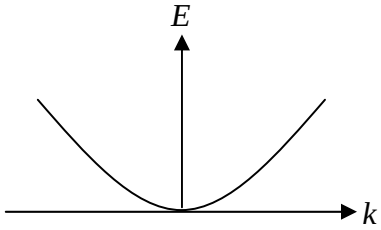
The electrons in a crystal are not completely free, but instead interact with the periodic potential of the lattice. As a result, their "wave-particle" motion can-not be expected to be the same as for electrons in free space. Thus, in applying the usual equations of electrodynamics to charge carriers in a solid, we must use altered values of particle mass. In doing so, we account for most of the influences of the lattice, so that the electrons and holes can be treated as "almost free" carriers in most computations. The calculation of effective mass must take into account the shape of the energy bands in three-dimensional k-space, taking appropriate averages over the various energy bands.

Example: Find the (E,k) relationship for a free electron and relate it to the electron mass.

Solution:

The electron momentum is $p = mv = \hbar k$. Then

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{p^2}{m} = \frac{\hbar^2}{2m}k^2$$



Thus the electron energy is parabolic with wave vector k.

The electron mass is inversely related to the curvature (second derivative) of the (E,k) relationship, since $\frac{d^2E}{dk^2} = \frac{\hbar^2}{m}$.

Although electrons in solids are not free, most energy bands are close to parabolic at their minima (for conduction bands) or maxima (for valence bands). We can also approximate effective mass near those band extrema from the curvature of the band.

The effective mass of an electron in a band with a given (E,k) relationship is given by

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

A particularly interesting feature is that the curvature d^2E/dk^2 is positive at the conduction band minima, and is negative at the valence band maxima. Thus, the electrons near the top of the valence band have ***negative effective mass***. Valence band electrons with negative charge and negative mass move in an electric field in the same direction as holes with positive charge and positive mass. We can fully account for charge transport in the valence band by considering hole motion.

In any calculation involving the mass of the charge carriers, we must use effective mass values for the particular material involved. Table given below lists the effective masses for Ge, Si, and $GaAs$ appropriate for one type of calculation. In this table and in all subsequent discussions, the electron effective mass is denoted by m_n^* and the hole effective mass by m_p^*. The n subscript indicates the electron as a negative charge carrier, and the p subscript indicates the hole as a positive charge carrier (The free electron rest mass is m_0).

	Ge	*Si*	*GaAs*
m_n^*	$0.55\,m_0$	$1.1\,m_0$	$0.067\,m_0$
m_p^*	$0.37\,m_0$	$0.56\,m_0$	$0.48\,m_0$

1.4 Intrinsic Material

A perfect semiconductor crystal with no impurities or lattice defects is called an intrinsic semiconductor. In such material there are no charge carriers at $0\,K$, since the valence band is filled with electrons and the conduction band is empty. At higher temperatures electron-hole pairs are generated as valence band electrons are excited thermally across the band gap to the conduction band. These EHPs are the only charge carriers in intrinsic material.

The generation of EHPs can be visualized in a qualitative way by considering the breaking of covalent bonds in the crystal lattice. If one of the *Si* valence electrons is broken away from its position in the bonding structure such that it becomes free to move about in the lattice, a conduction electron is created and a broken bond (hole) is left behind. The energy required to break the bond is the band gap energy E_g. This model helps in visualizing the physical mechanism of EHP creation, but the energy band mode is more productive for purposes of quantitative calculation. One Important difficulty in the "broken bond" model is that the free electron and the hole seem deceptively localized in the lattice. Actually, the positions of the free electron and the hole are spread out over several lattice spacing and should be considered quantum mechanically by probability distributions.

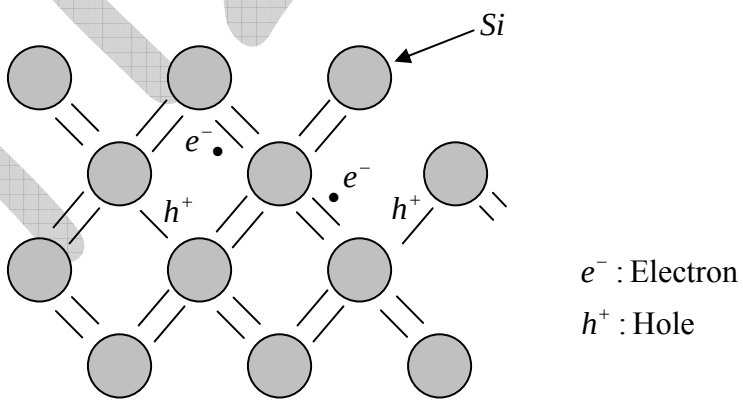


Figure 1.4: Electron-hole pairs in the covalent bonding model of the *Si* crystal.

Since the electrons and holes are created in pairs, the conduction band electron concentration n (electrons per cm^3) is equal to the concentration of holes in the valence band p (holes per cm^3). Each of these intrinsic carrier concentrations is commonly referred to as n_i. Thus for intrinsic material

$$n = p = n_i .$$

At a given temperature there is a certain concentration of electron-hole pairs n_i. Obviously, if a steady state carrier concentration is maintained, there must be *recombination* of EHPs at the same rate at which they are generated. Recombination occurs when an electron in the conduction band makes a transition (direct or indirect) to an empty state (hole) in the valence band, thus annihilating the pair. If we denote the generation rate of EHPs as g_i, (EHP/cm^3) and the recombination rate as r_i, equilibrium requires that: $r_i = g_i$

Each of these rates is temperature dependent. For example, $g_i(T)$ increases when the temperature is raised, and a new carrier concentration n_i, is established such that the higher recombination rate $r_i(T)$ just balances generation. At any temperature, we can predict that the rate of recombination of electrons and holes r_i is proportional to the equilibrium concentration of electrons n_0 and the concentration of holes p_0

$$r_i = \alpha_r n_0 p_0 = \alpha_r n_i^2 = g_i$$

The factor α_r is a constant of proportionality which depends on the particular mechanism by which recombination takes place.

1.5 Extrinsic Material

In addition to the intrinsic carriers generated thermally, it is possible to create carriers in semiconductor purposely by introducing impurities into the crystal. This process, called *doping* is the most common technique for varying the conductivity of semiconductors. By doping, a crystal can be altered so that it has a predominance of either electrons or holes. Thus there are two types of doped semiconductors, ***n*-type** (mostly electrons) and ***p*-type** (mostly holes).

When impurities or lattice defects are introduced into an otherwise perfect crystal, additional levels are created in the energy band structure usually within the band gap. For example, an impurity from column V of the periodic table (*P*, *As*, and *Sb*) introduces an energy level very near the conduction band in *Ge* or *Si*. This level is filled with electrons at $0\,K$, and very little thermal energy is required to excite these electrons to the conduction band. Thus at about $50\,K - 100\,K$ virtually all of the electrons in the impurity level are "donated" to the conduction band. Such an impurity level is called a **donor level** and the column *V* impurities in *Ge* or *Si* are called donor impurities. From figure 1.5, we note that the material doped with donor impurities can have a considerable concentration of electrons in the conduction band, even when the temperature is too low for the intrinsic EHP concentration to be appreciable. Thus semiconductors doped with a significant number of donor atoms will have $n_0 >> (n_i, p_0)$ at room temperature. This is ***n*-type** material.

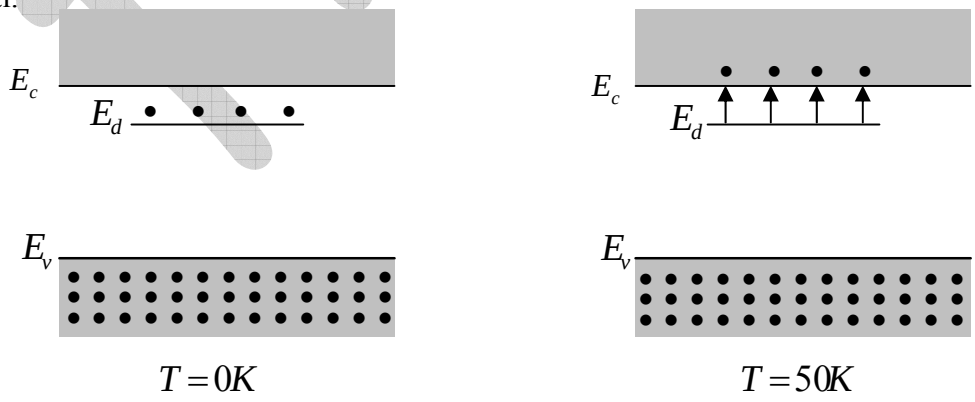


Figure 1.5: Donation of electrons from a donor level to the conduction band.

Atoms from column III (*B*, *Al*, *Ga*, and *In*) introduce impurity levels in *Ge* or *Si* near the valence band. These levels are empty of electrons at $0\,K$. At low temperatures, enough thermal energy is available to excite electrons from the valence band into the impurity level, leaving behind holes in the valence band, since this type of impurity level "accepts" electrons from the valence band, it is called an ***acceptor level***, and the column III impurities are acceptor impurities in *Ge* and *Si*. Figure 1.6 indicates, doping with acceptor impurities can create a semiconductor with a hole concentration p_0 much greater than the conduction band electron concentration n_0 (this is ***p*-type** material).

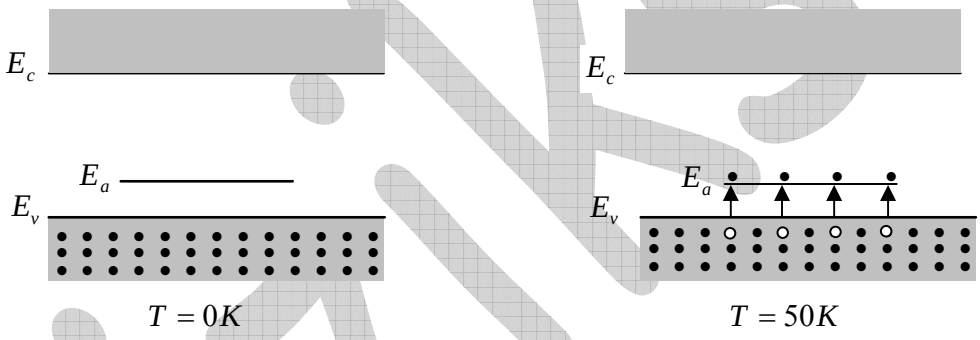


Figure 1.6: Acceptance of valence band electrons by an acceptor level, and the resulting creation of holes.

1.6 The Fermi Level

Electrons in solids obey *Fermi-Dirac* statistics. In the development of this type of statistics, one must consider the indistinguishability of the electrons, their wave nature, and the Pauli Exclusion Principle. The rather simple result of these statistical arguments is that the distribution of electrons over a range of allowed energy levels at thermal equilibrium is: $$f(E)=\frac{1}{1+e^{(E-E_F)/kT}}$$

where k is Boltzmann constant. The function $f(E)$, the *Fermi-Dirac distribution function*, gives the probability that an available energy state at E will be occupied by an electron at absolute temperature T. The quantity E_F is called the *Fermi Level*, and it represents an important quantity in the analysis of semiconductor behavior. We notice that, for an energy E equal to the Fermi level energy E_F , the occupation probability is

$$f(E_F)=\left[1+e^{(E_F-E_F)/kT}\right]^{-1}=\frac{1}{1+1}=\frac{1}{2}.$$

A closer examination of $f(E)$ indicates that at $0K$ the distribution takes the simple rectangular form shown in figure 1.7. With $T = 0$ in the denominator of the exponent, $f(E)$ is $1/(1 + 0) = 1$ when the exponent is negative ($E < E_F$), and is $1/ (1 + \infty) = 0$ when the exponent is positive ($E > E_F$). This rectangular distribution implies that at $0K$ every available energy state up to E_F is filled with electrons and all states above E_F are empty.

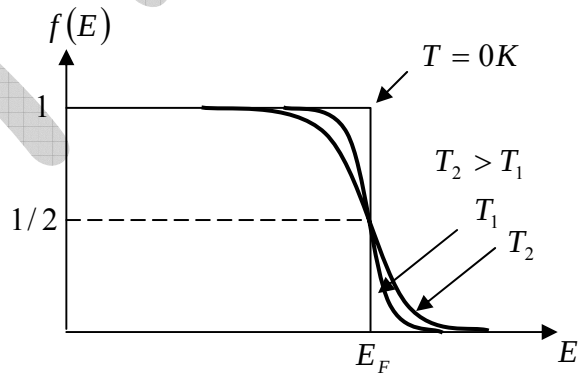


Figure 1.7: The Fermi Dirac distribution function.

At temperatures higher than $0\,K$, some probability exists for states above E_F to be filled. For example, at $T = T_1$ there is some probability $f(E)$ that states above E_F are filled, and there is a corresponding probability $[1 - f(E)]$ that states below E_F are empty.

The Fermi function is symmetrical about E_F for all temperatures; that is the probability $f(E_F + \Delta E)$ that a state ΔE above E_F is filled is the same as the probability $[1 - f(E_F - \Delta E)]$ that a state ΔE below E_F is empty. The symmetry of the distribution of empty and filled states about E_F makes the Fermi level a natural reference point in calculations of electron and hole concentrations in semiconductors.

For intrinsic material we know that the concentration of holes in the valence band is equal to the concentration of electrons in the conduction band. Therefore, the Fermi level E_F must lie at the middle of the band gap in intrinsic material [Figure 1.8a]. Since $f(E)$ is symmetrical about E_F, the electron probability "tail" of $f(E)$ extending into the conduction band is symmetrical with the hole probability tail $[1 - f(E)]$ in the valence band. The distribution function has values within the band gap between E_c and E_v but there are no energy states available, and no electron occupancy results from $f(E)$ in this range.

In *n*-type material there is a high concentration of electrons in the conduction band compared with the hole concentration in the valence band. Thus in *n*-type material the distribution function $f(E)$ must lie above its intrinsic position on the energy scale (figure 1.8 b). Since $f(E)$ retains its shape for a particular temperature, the larger concentration of electrons at E_c in *n*-type material implies a correspondingly smaller hole concentration at E_v. We notice that the value of $f(E)$ for each energy level in the conduction band (and therefore the total electron concentration n_0) increases as E_F moves closer to E_c. Thus the energy difference $(E_c - E_F)$ gives a measure of n.

For *p*-type material the Fermi level lies near the valence band (figure 1.8 c) such that the $\left[1-f\left(E\right)\right]$ tail below E_v is larger than the $f\left(E\right)$ tail above E_c. The value of $\left(E_F-E_v\right)$ indicates how strongly *p*-type the material is.

It is usually inconvenient to draw $f\left(E\right)$ *vs.* E on every energy band diagram to indicate the electron and hole distributions. Therefore, it is common practice merely to indicate the position of E_F in band diagrams.

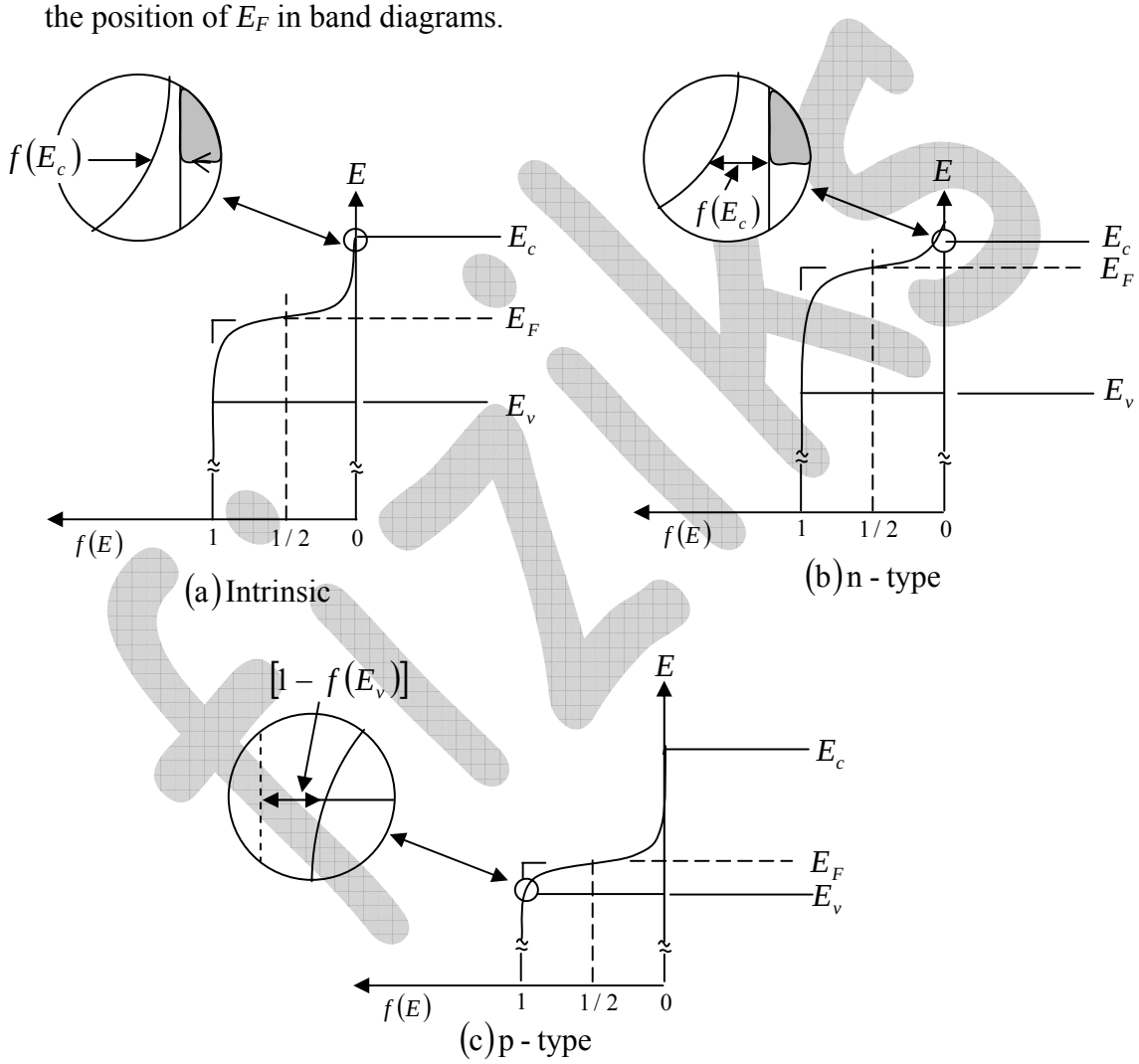


Figure 1.8: The Fermi distribution function applied to semiconductors:
(a) Intrinsic material; (b) *n*-type material; (c) *p*-type material.

1.6.1 Electron and Hole Concentrations at Equilibrium

The Fermi distribution function can be used to calculate the concentrations of electrons and holes in a semiconductor, if the densities of available states in the valence and conduction bands are known. For example, the concentration of electrons in the conduction band is

$$n_0 = \int_{E_c}^{\infty} f(E) N(E) dE$$

where $N(E)dE$ is the density of states (cm^{-3}) in the energy range dE. The subscript 0 used with the electron and hole concentration symbols (n_0, p_0) indicates equilibrium conditions. The number of electrons per unit volume in the energy range dE is the product of the density of states and the probability of occupancy $f(E)$. Thus the total electron concentration is the integral over the entire conduction band. The function $N(E)$ can be calculated by using quantum mechanics and the Pauli Exclusion Principle.

Since $N(E)$ is proportional to $E^{1/2}$, so the density of states in the conduction band increases with electron energy. On the other hand, the Fermi function becomes extremely small for large energies. The result is that the product $f(E)N(E)$ decreases rapidly above E_c and very few electrons occupy energy states far above the conduction band edge. Similarly, the probability of finding an empty state (hole) in the valence band $\left[1 - f(E)\right]$ decreases rapidly below E_v and most holes occupy states near the top of the valence band. This effect is demonstrated in figure 1.9, which shows the density of available states, the Fermi function, and the resulting number of electrons and holes occupying available energy states in the conduction and valence bands at thermal equilibrium (i.e., with no excitations except thermal energy). For holes, increasing energy points down, since the *E* scale refers to electron energy.

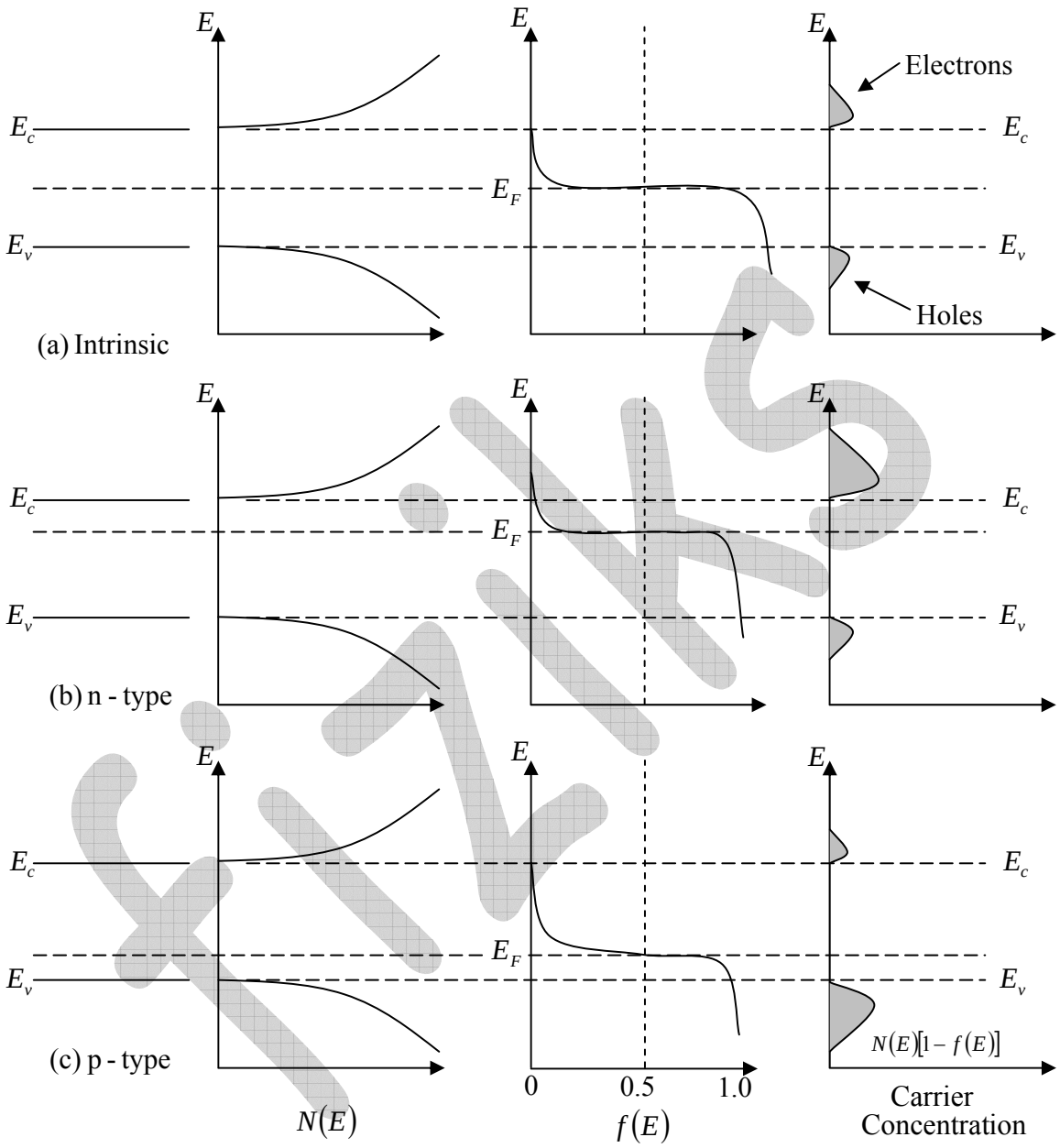


Figure 1.9: Schematic band diagram, density of states, Fermi-Dirac distribution, and the carrier concentrations for (a) intrinsic, (b) n-type, and (c) p-type semiconductors at thermal equilibrium.

The result of the integration of $n_0 = \int_{E_c}^{\infty} f(E) N(E) dE$ is the same as that obtained if we represent the entire distributed electron states in the conduction band by an *effective density of states* N_c located at the conduction band edge E_c. Therefore, the conduction band electron concentration is simply the effective density of states at E_c times the probability of occupancy at E_c

$$n_0 = f(E_c) N_c$$

In this expression we assume the Fermi level E_F lies at least several kT below the conduction band. Then the exponential term is large compared with unity and the Fermi function $f(E_c)$ can be simplified as

$$f(E_c) = \frac{1}{1 + e^{(E_c - E_F)/kT}} \approx e^{-(E_c - E_F)/kT}$$

Since kT at room temperature is only $0.026\ eV$, this is generally a good approximation. For this condition the concentration of electrons in the conduction band is

$$n_0 = N_c e^{-(E_c - E_F)/kT} \Rightarrow E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right)$$

The effective density of states $N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$

Thus electron concentration increases as E_F moves closer to the conduction band.

By similar arguments, the concentration of holes in the valence band is

$$p_0 = N_v \left[1 - f(E_v)\right]$$

where N_c is the effective density of states in the valence band.

The probability of finding an empty state at E_v is,

$$1 - f(E_v) = 1 - \frac{1}{1 + e^{-(E_v - E_F)/kT}} \approx e^{-(E_F - E_v)/kT}$$ for E_F larger than E_v by several kT.

From these equations, the concentration of holes in the valence band is

$$p_0 = N_v e^{-(E_F - E_v)/kT} \Rightarrow E_F - E_v = kT \ln\left(\frac{N_v}{p_0}\right)$$

The effective density of states in the valence band $N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$

Thus hole concentration increases as E_F moves closer to the valence band.

The electron and hole concentrations predicted by above equations are valid whether the material is intrinsic or doped, provided thermal equilibrium is maintained.

Thus for *intrinsic material*, E_F lies, at some intrinsic level E_i near the middle of the band gap, and the intrinsic electron and hole concentrations are

$$n_i = N_c e^{-(E_c - E_i)/kT}, \quad p_i = N_v e^{-(E_i - E_v)/kT}$$

$$n_i = p_i \Rightarrow E_i = \frac{E_c + E_v}{2} + \frac{kT}{2}\ln\left(\frac{N_v}{N_c}\right) \Rightarrow E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4}\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Note: The intrinsic level E_i is the middle of the band gap $\left(E_c - E_i = \frac{E_g}{2}\right)$, if the effective densities of states N_c and N_c are equal. There is usually some difference in effective mass for electrons and holes, however, and N_c and N_c are slightly different.

The product of n_0 and p_0 at equilibrium is a constant for a particular material and temperature, even if the doping is varied:

$$n_0 p_0 = \left(N_v e^{-(E_c - E_F)/kT}\right)\left(N_v e^{-(E_F - E_v)/kT}\right) = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

$$n_i p_i = \left(N_c e^{-(E_c - E_i)/kT}\right)\left(N_v e^{-(E_i - E_v)/kT}\right) = N_c N_v e^{-E_g/kT}$$

The intrinsic electron and hole concentrations are equal (since the carriers are created in pairs), $n_i = p_i$; thus the intrinsic concentration is

$$\boxed{n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}}$$

Law of Mass Action

The constant product of electron and hole concentrations can be written conveniently as

$$\boxed{n_0 p_0 = n_i^2}$$

For *n*-type material the minority concentration (holes)

$$p_n = \frac{n_i^2}{n_n} \approx \frac{n_i^2}{N_D}$$ where N_D is donor ion concentration.

For *p*-type material the minority concentration (electrons)

$$n_p = \frac{n_i^2}{p_p} \approx \frac{n_i^2}{N_A}$$ where N_A is acceptor ion concentration.

Another convenient way of writing electron and hole concentration is

$$\boxed{n_0 = n_i e^{(E_F - E_i)/kT} \quad \text{and} \quad p_0 = n_i e^{(E_i - E_F)/kT}}$$

This form of the equation indicates directly that the electron concentration is n_i when E_F is at the intrinsic level E_i and that n_0 increases exponentially as the Fermi level moves away from E_i toward the conduction band. Similarly, the hole concentration p_0 varies from n_i to larger values as E_F moves from E_i, toward the valence band. Since these equations reveal the qualitative features of carrier concentration so directly, they are particularly convenient to remember.

1.7 Temperature Dependence of Carrier Concentrations

The variation of carrier concentration with temperature is indicated by equations $n_0 = n_i e^{(E_F - E_i)/kT}$ and $p_0 = n_i e^{(E_i - E_F)/kT}$. Initially, the variation of n_0 and p_0 with T seems relatively straightforward in these relations. The problem is complicated, however, by the fact that n_i has strong temperature dependence $\left(n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}\right)$ and that E_F can also vary with temperature. Let us begin by examining the intrinsic carrier concentration. $n_i(T) = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} \left(m_n^* m_p^*\right)^{3/4} e^{-E_g/2kT}$

The exponential temperature dependence dominates $n_i(T)$ and a plot of $\ln(n_i)$ *vs* $1000/T$ appears almost linear (figure 1.10).

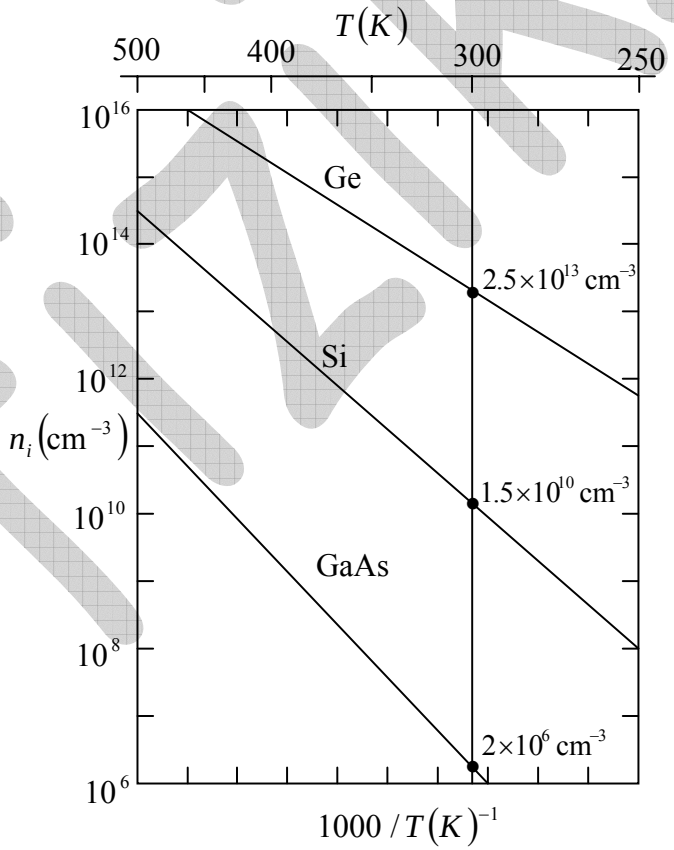


Figure 1.10: Intrinsic carrier concentration for *Ge*, *Si*, and *GaAs* as a function of inverse temperature. The room temperature values are marked for reference.

1.8 Compensation and Space Charge Neutrality

Figure 1.11 illustrates a semiconductor for which both donors and acceptors are present, but $N_D > N_A$. The predominance of donors makes the material *n*-type and the Fermi level is therefore in the upper part of the band gap. Since E_F is well above the acceptor level E_a, this level is essentially filled with electrons. However, with E_F above E_i we cannot expect a hole concentration in the valence band commensurate with the acceptor concentration. In fact, the filling of the E_a states occurs at the expense of the donated conduction band electrons.

The mechanism can be visualized as follows: Assume an acceptor state is filled with a valence band electron, with a hole resulting in the valence band. This hole is then filled by recombination with one of the conduction band electrons. Extending this logic to all the acceptor atoms, we expect the resultant concentration of electrons in the conduction band to be $N_D - N_A$ instead of the total N_D. This process is called *compensation*. By this process it is possible to begin with an *n*-type semiconductor and add acceptors until $N_A = N_D$ and no donated electrons remain in the conduction band. In such compensated material $n_0 = n_i = p_0$ and intrinsic conduction is obtained. With further acceptor doping the semiconductor becomes *p*-type with a hole concentration of essentially $N_A - N_D$.

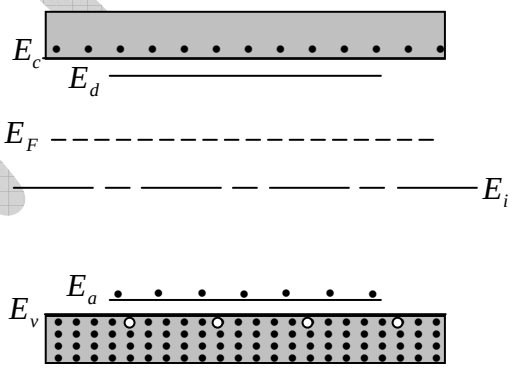


Figure 1.11: Compensation in an *n*-type semiconductor $(N_D > N_A)$.

The exact relationship among the electron, hole, donor, and acceptor concentrations can be obtained by considering the requirements for ***space charge neutrality***. If the material is to remain electrostatically neutral, the sum of the positive charges (holes and ionized donor atoms) must balance the sum of the negative charges (electrons and ionized acceptor atoms):

$$p_0 + N_D^+ = n_0 + N_A^-$$

Thus the net electron concentration in the conduction band is $n_0 = p_0 + (N_D^+ - N_A^-)$.

If the material is doped *n*-type $(n_0 \gg p_0)$ and all the impurities are ionized, we can approximate that $n_0 = N_D - N_A$.

Since the intrinsic semiconductor itself is electrostatically neutral and the doping atoms we add are also neutral, the requirement of equation $p_0 + N_D^+ = n_0 + N_A^-$ must be maintained at equilibrium.

Knowledge of carrier concentrations in a solid is necessary for calculating current flow in the presence of electric or magnetic fields. In addition to the values of *n* and *p,* we must be able to take into account the collisions of the charge carriers with the lattice and with the impurities. These processes will affect the ease with which electrons and holes can flow through the crystal, that is, their mobility within the solid. As should be expected, these collision and scattering processes depend on temperature, which affects the thermal motion of the lattice atoms and the velocity of the carriers.

Example: The donor concentration in a sample of n-type silicon is increased by a factor of 100. Find the shift in the position of the Fermi level at $300K$. $(k_BT = 25meV\ at\ 300K)$

Solution:

$$E_C - E_F = kT\ln\left(\frac{N_c}{N_d}\right) \text{ and } E_C - E_F' = kT\ln\left(\frac{N_c}{100N_d}\right) = kT\ln\left(\frac{N_c}{N_d}\right) - kT\ln(100)$$

Thus shift is $\Delta E = kT\ln(100) = 25\ln(100)meV = 115.15\,meV$

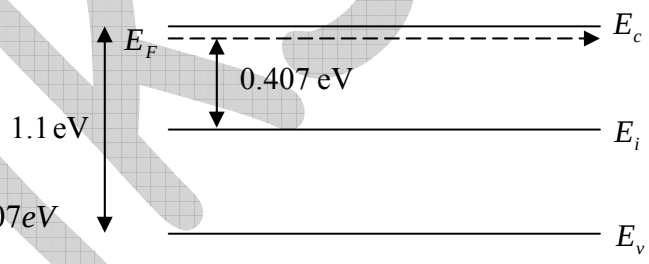
Example: A Si sample is doped with 10^{17} As atoms/cm^3. What is the equilibrium hole concentration p_0 at $300\ K$? Where is E_F relative to E_i? (where $n_i = 1.5\times10^{10}cm^{-3}$)

Solution:

Since $N_D \gg n_i$ we can approximate n_i and

$$p_0 = \frac{n_i^2}{n_0} = \frac{2.25\times10^{20}}{10^{17}} = 2.25\times10^3 cm^{-3}$$

$$E_F - E_i = kT\ln\frac{n_0}{n_i} = 0.0259\ln\frac{10^{17}}{1.5\times10^{10}} = 0.407eV$$



Example: A pure Si sample at 300K with intrinsic carrier concentration of $1.5\times10^{16}/m^3$ is doped with phosphorous. The equilibrium hole concentration and electron mobility is $2.25\times10^9/m^3$ and 1350 cm^2/Vs respectively. Find the position of Fermi-level relative to the intrinsic level at $300K$.

Solution:

Equilibrium electron concentration is

$$np = n_i^2 \quad (Law\ of\ mass) \Rightarrow n = \frac{n_i^2}{p} = \frac{\left(1.5\times10^{16}\right)^2}{2.25\times10^9} = 1.00\times10^{23}m^{-3}$$

$$E_F - E_i = k_BT\ln\left(\frac{n}{n_i}\right) = 8.67\times10^{-5}\times300\times\ln\left(\frac{10^{23}}{1.5\times10^{16}}\right) = 0.406\ eV$$

1.9 Current Components in Semiconductor

1.9.1 Drift Current (Conductivity and Mobility)

The charge carriers in a solid are in constant motion, even at thermal equilibrium. At room temperature, for example, the thermal motion of an individual electron may be visualized as random scattering from lattice atoms, impurities, other electrons, and defects (figure 1.12). Since the scattering is random, there is no net motion of the group of $n\ electrons/cm^3$ over any period of time. This is not true of an individual electron, of course. The probability of the electron in returning to its starting point after some time t is negligibly small. However, if a large number of electrons is considered (e.g. $10^{16}\ cm^{-3}$ in an n-type semiconductor), there will be no preferred direction of motion for the group of electrons and no net current flow.

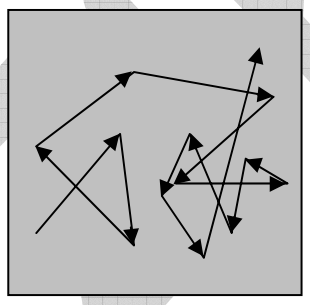


Figure 1.12: Thermal motion of an electron in a solid.

If an electric field E_x is applied in the x-direction, each electron experiences a net force $-qE_x$ from the field. This force may be insufficient to alter appreciably the random path of an individual electron; the effect when averaged over all the electrons, however, is a net motion of the group in the x-direction. If p_x is the x-component of the total momentum of the group, force of the field on the $n\ electrons/cm^3$ is

$$-nqE_x = \left.\frac{dp_x}{dt}\right|_{\text{field}} .$$

Initially, above equation seems to indicate a continuous acceleration of the electrons in the $-x$-direction. This is not the case, however, because the net acceleration is just balanced in steady state by the decelerations of the collision processes. Thus while the steady field E_x does produce a net momentum p_{-x}, the net rate of change of momentum when collisions are included must be zero in the case of steady state current flow.

To find the total rate of momentum change from collisions, we must investigate the collision probabilities more closely. If the collisions are truly random, there will be a constant probability of collision at any time for each electron. Let us consider a group of N_0 electrons at time $t=0$ and define $N(t)$ as the number of electrons that have not undergone a collision by time *t*. The rate of decrease in $N(t)$ at any time *t* is proportional to the number left unscattered at *t*,

$$-\frac{dN(t)}{dt}=\frac{1}{\bar{t}}N(t) \quad \text{where } \bar{t}^{-1} \text{ is a constant proportionality.}$$

The solution to above equation is an exponential function $N(t)=N_0e^{-t/\bar{t}}$ and $\bar{t}$ represents the mean time between scattering events, called the ***mean free time***.

The probability that any electron has a collision in the time interval dt is $\frac{dt}{\bar{t}}$.

Thus the differential change in *p*, due to collisions in time dt is $dp_x=-p_x\frac{dt}{\bar{t}}$.

The rate of change of p_x, due to the decelerating effect of collisions is $\left.\frac{dp_x}{dt}\right|_{\text{collision}}=-\frac{p_x}{\bar{t}}$

The sum of acceleration and deceleration effects must be zero for steady state. Thus

$$-\frac{p_x}{\bar{t}}-nqE_x=0\,.$$

The average momentum per electron is $<p_x>=\frac{p_x}{n}=-q\bar{t}\,E_x$ where the angular brackets

indicate an average over the entire group of electrons. As expected for steady state, the above equation indicates that the electrons have *on the average* a constant net velocity in the negative *x*-direction: $\langle v_x \rangle = \frac{\langle p_x \rangle}{m_n^*} = -\frac{q\bar{t}}{m_n^*} E_x$

Actually, the individual electrons move in many directions by thermal motion during a given time period, but $\langle v_x \rangle$ tells us the *net* ***drift*** of an average electron in response to the electric field. The drift speed $\langle v_x \rangle$ is usually much smaller than the random speed due to thermal motion v_{th}.

The **current density** resulting from this net drift is just the number of electrons crossing a unit area per unit time $\left(n\langle v_x \rangle\right)$ multiplied by the charge on the electron $(-q)$:

$$J_x = -qn\langle v_x \rangle = \frac{nq^2\bar{t}}{m_n^*} E_x \quad ampere/cm^2 .$$

Thus the current density is proportional to the electric field, as we expect from Ohm's law: $J_x = \sigma E_x$ where $\sigma \equiv \frac{nq^2\bar{t}}{m_n^*}$.

The **conductivity** $\sigma(\Omega - cm)^{-1}$ can be written $\boxed{\sigma = qn\mu_n}$ where $\mu_n \equiv \frac{q\bar{t}}{m_n^*}$.

The quantity μ_n, called the ***electron mobility,*** describes the ease with which electrons drift in the material. Mobility is a very important quantity in characterizing semiconductor materials and in device development.

The mobility can be expressed as the average particle drift velocity per unit electric field.

Thus $\mu_n = -\frac{\langle v_x \rangle}{E_x}$, and units of mobility are $(cm/s)/(V/cm) = cm^2/V\text{-}s$. The minus sign in the definition results in a positive value of mobility, since electrons drift opposite to the field.

The current density can be written in terms of mobility as $J_x = qn\mu_n E_x$.

This derivation has been based on the assumption that the current is carried primarily by

electrons. For hole conduction we change n to p, $-q$ to $+q$, μ_n to μ_p

where $\mu_p = +\dfrac{\langle v_x \rangle}{E_x}$ is the mobility for holes.

If both electrons and holes participate, then

where $\boxed{\sigma = q\left(n\mu_n + p\mu_p\right)}$.

For *N*-type semiconductor

$\sigma_n = e\left(n_n\mu_n + p_n\mu_p\right) \approx n_n e\mu_n$ since $n_n >> p_n$ where n_n *and* p_n are electron and hole concentration in *N*-type.

For *P*-type semiconductor

$\sigma_p = e\left(n_p\mu_n + p_p\mu_p\right) \approx p_p e\mu_p$ since $p_p >> n_p$ where n_p *and* p_p are electron and hole concentration in *P*-type.

Example: The following data are given for intrinsic Germanium at $300\,K$. $n_i = 2.4\times10^{19}/m^3$, $\mu_e = 0.39 m^2V^{-1}s^{-1}$, $\mu_p = 0.19 m^2V^{-1}s^{-1}$. Find the conductivity of the Germanium.

Solution:

$\sigma = en_i\left(\mu_n + \mu_p\right) = 1.6\times10^{-19}\times2.4\times10^{19}\left(0.39+.19\right) = 2.227\left(\Omega m\right)^{-1}$.

Example: A sample of *Si* has electron and hole mobilities of 0.13 and 0.05 $m^2V^{-1}s^{-1}$ respectively at 300K. It is doped with *P* and *Al* with doping densities of $1.5\times10^{21}/m^3$ and $2.5\times10^{21}/m^3$ respectively. The resistivity of doped *Si* sample at 300K is

(a) $0.125\,\Omega m$ (b) $8.0\,\Omega m$ (c) $2.125\,\Omega m$ (d) $0.225\,\Omega m$

Solution:

Resulting doped crystal is *p*-type and $p_p = \left(2.5-1.5\right)\times10^{21}/m^3 = 1\times10^{21}/m^3$

$\sigma = e\left(n_p\mu_n + p_p\mu_p\right) \approx ep_p\mu_p = 1.6\times10^{-19}\times1\times10^{21}\times0.05 = 8\,\Omega^{-1}m^{-1}$

$\rho = \dfrac{1}{\sigma} = \dfrac{1}{8} = 0.125\,\Omega m$

1.9.2 Diffusion Current

In addition to a conduction current, the transport of charges in a semiconductor may be accounted for a mechanism called diffusion. It is possible to have non-uniform concentration of particles in a semiconductor. As indicated in the figure 1.13, the concentration p of holes varies with distance x in the semiconductor, and there exist a concentration gradient, $\frac{dp}{dx}$ in the density of the carriers.

The existence of a gradient implies that if an imaginary surface is drawn in the semiconductor, the density of the holes immediately on one side of the surface is larger than the density on the other side. The holes are in random motion as a result of their thermal energy. Accordingly, holes will continue to move back and forth across this surface. We may then expect that, in a given time interval, more holes will cross the surface

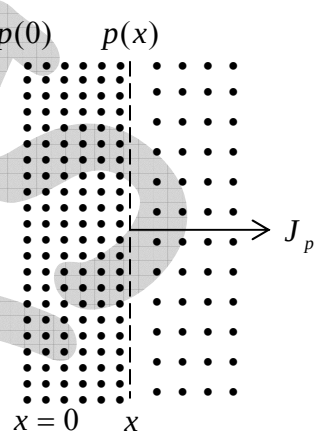


Figure 1.13

Note: It should be noted that this net transport of charge is not the result of mutual repulsion among charges of like sign, but is simply the result of a statistical phenomenon. This diffusion is exactly analogous to that which occurs in a neutral gas if concentration gradient exists in the gaseous container.

The diffusion hole-current density J_p (ampere per square meter) is proportional to the concentration gradient, and is given by: $J_p = -qD_p \frac{dp}{dx}$

where D_p (Square meters/second) is called diffusion constant. Since p decreases with increasing x, then $\frac{dp}{dx}$ is negative and the minus sign needed, so that J_p is positive in the positive x-direction.

Similarly, $J_n = qD_n \frac{dn}{dx}$

1.9.3 Einstein Relationship

Since both diffusion and mobility are statistical thermodynamic phenomena, D and μ are not independent. The relationship between them is given by

$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = V_T$ where V_T is the 'Volt-equivalent of temperature'.

$$V_T = \frac{kT}{q} = \frac{T}{11,600} V$$

$k \rightarrow$ Boltzmann constant in electron volts per degree Kelvin

At room temperature $T = 300^0\ K$, $V_T = 0.026\ V \Rightarrow \mu = 39D$

1.9.4 Total Current in a Semiconductor

It is possible for both a potential gradient and a concentration gradient to exist simultaneously within a semiconductor. In such a situation, the total hole current is the sum of the drift current and the diffusion current, $J_p = q\mu_p pE - qD_p \frac{dp}{dx}$

Similarly the net electron current is: $J_n = q\mu_n nE + qD_n \frac{dn}{dx}$

1.10 Effects of Temperature and Doping on Mobility

The two basic types of scattering mechanisms that influence electron and hole mobility are ***lattice scattering*** and ***impurity scattering***. In lattice scattering a carrier moving through the crystal is scattered by a vibration of the lattice, resulting from the temperature (Collective vibrations of atoms in the crystal are called *phonons*. Thus lattice scattering is also known as *phonon scattering*). The frequency of such scattering events increases as the temperature increases, since the thermal agitation of the lattice becomes greater. Therefore, we should expect the mobility to decrease as the sample is heated. On the other hand, scattering from crystal defects such as ionized impurities becomes the dominant mechanism at low temperatures. Since the atoms of the cooler lattice are less agitated, lattice scattering is less important; however, the thermal motion of the carriers is also slower.

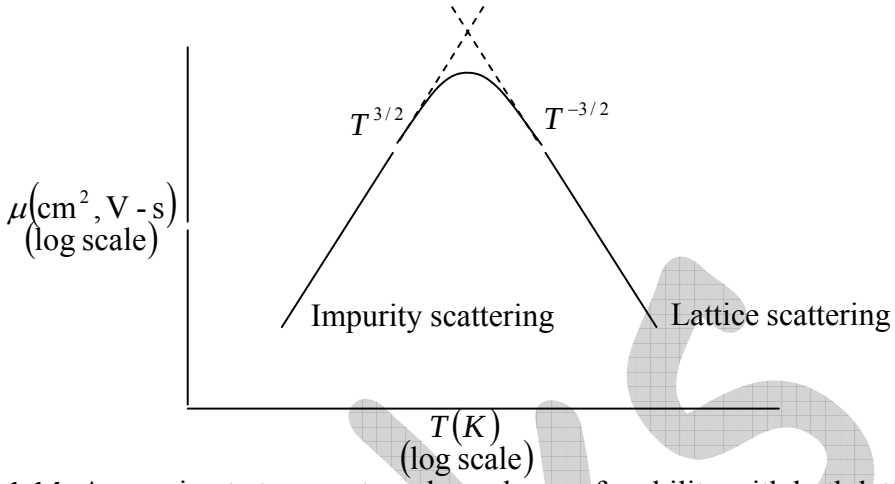


Figure 1.14: Approximate temperature dependence of mobility with both lattice and impurity scattering.

Since a slowly moving carrier is likely to be scattered more strongly by an interaction with a charged ion than is a carrier with greater momentum, impurity scattering events cause a decrease in mobility with decreasing temperature. The approximate temperature dependencies are $T^{-3/2}$ for lattice scattering and $T^{3/2}$ for impurity scattering.

1. 11 The Potential Variation within a Graded Semiconductor

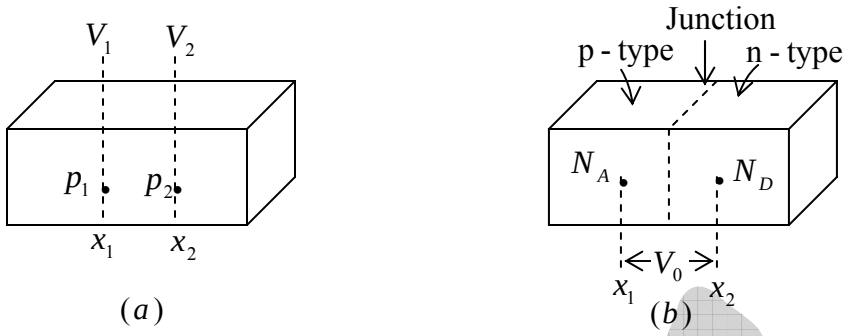


Figure 1.15 (a): A graded semiconductor: *p(x)* is not constant

(b): One portion is doped with (uniformly) acceptor ions and the other section is doped uniformly with donor ions so that a metallurgical junction is formed.

Consider a semiconductor where the hole concentration p is a function of x; that is, the doping is non-uniform or graded. Assume a steady-state situation and zero excitation; that is, no carriers are injected into the specimen from any external source. With no excitation there can be no steady movement of charge in the bar, although the carriers possess random motion due to thermal agitation. Hence the total hole current must be zero (also, the total electron current must be zero). Since p is not constant, we expect a non-zero hole diffusion current. In order for the total hole current to vanish there must exist a hole drift current which is equal and opposite to the diffusion current. However, conduction current requires an electric field and hence we conclude that, as a result of the non-uniform doping, an electric field is generated within the semiconductor. We shall now find this field and the corresponding potential variation throughout the bar.

Since $J_p = q\mu_p pE - qD_p \frac{dp}{dx} \Rightarrow E = \frac{V_T}{p}\frac{dp}{dx}$ $\quad \because J_p = 0$ and then use $D_p = \mu_p V_T$

If the doping concentration $p(x)$ is known, this equation allows the built in field $E(x)$ to be calculated.

$\because E = -\frac{dV}{dx} \Rightarrow dV = -V_T \frac{dp}{p}$.

If this equation is integrated between x_1, where the concentration is p_1 and the potential is V_1 and x_2 where $p = p_2$ and $V = V_2$, the result is: $V_{21} \equiv V_2 - V_1 = V_T \ln \frac{p_1}{p_2}$

Note: The potential difference between two points depends only upon the concentration at these points and is independent of their separation $(x_2 - x_1)$.

Above equation can be put in the form $p_1 = p_2 e^{V_{21}/V_T}$

This is the Boltzmann relationship of kinetic gas theory.

Starting with $J_n = 0$ and proceeding as above, the Boltzmann equation for electrons is obtained as $n_1 = n_2 e^{-V_{21}/V_T}$. Now $n_1 p_1 = n_2 p_2$.

This equation states that the product np is a constant independent of x, and hence the amount of doping, under thermal equilibrium.

For an intrinsic semiconductor $n = p = n_i$ and hence $np = n_i^2$.

1.11.1 An Open-Circuited Step-graded Junction

Consider the special case indicated in figure 1.15 (b). The left half of the bar is *p*-type with a constant concentration N_A, whereas the right-half is *n*-type with a uniform density N_D. The dashed plane is a metallurgical $(p-n)$ junction separating the two sections with different concentrations. This type of doping where the density changes abruptly from *p* to *n* type is called step-grading. The step graded junction is located at the plane where the concentration is zero. The above theory indicates that there is built-in potential between these two sections (called the contact difference of potential V_o.)

Thus $V_o = V_{21} = V_T \ln \frac{p_{p_o}}{p_{n_o}}$

Because $p_1 = p_{p_o}$ = thermal-equilibrium hole concentration in *p*-side

$p_2 = p_{n_o}$ = thermal equilibrium hole concentration in *n*-side

since $p_{p_o} = N_A$ and $p_{n_o} = \frac{n_i^2}{N_D} \Rightarrow$ $\boxed{V_o = V_T \ln \frac{N_A N_D}{n_i^2}}$

Summary

1. In a semiconductor two types of mobile charge carriers are available. The bipolar nature of a semiconductor is to be contrasted with the unipolar property of a metal, which possesses only free electrons.

2. A semiconductor may be fabricated with donor (acceptor) impurities. So it contains mobile charges which are primarily electrons (holes).

3. The intrinsic carrier concentration is a function of temperature. At room temperature, essentially all donors or acceptors are ionized.

4. Current is due to two distinct phenomenons:

(a) Carriers drift in an electric field (this conduction current is also available in metals).

(b) Carriers diffuse if a concentration gradient exists (a phenomenon, which does not take place in metals).

5. Carriers are continuously being generated (due to thermal creation of hole-electron pairs) and are simultaneously disappearing (due to recombination).

6. The fundamental law governing the flow of charges is called the *continuity equation*. It is formulated by considering that charges can neither be created nor destroyed if generation, recombination, drift and diffusion are all taken into account.

7. If the minority carriers are injected into a region containing majority carriers, then usually the injected minority concentration is very small compared with the density of the majority carries. For this low-level injection condition the *minority current is predominantly due to diffusion*; in other words, the minority drift current may be neglected.

8. The total majority-carrier flow is the sum of a drift and diffusion current. The majority conduction current results from a small electric field internally created within the semiconductor because of the injected carriers.

9. The minority-carrier concentration injected into one end of a semiconductor bar decreases exponentially with distance into the specimen (as a result of diffusion and recombination).

10. Across an open-circuited p-n junction there exists a contact difference of potential.

Multiple Choice Questions (MCQ)

Q1. Consider the following statements: Electrical conductivity of a metal has negative temperature coefficient since

1. Electron concentration increases with temperature
2. Electron mobility decreases with temperature
3. Electron lattice scattering increases with temperature.

Which of the following statements given above correct?

(a) 1, 2, 3 (b) only 1 and 2 (c) only 2 and 3 (d) only 1 and 3

Q2. A piece of copper and a piece of germanium are cooled from room temperature to $100\ K$. Then which one of the following is correct?

(a) Resistance of each will increase

(b) Resistance of each will decrease

(c) Resistance of copper will increase while that of germanium will decrease.

(d) Resistance of copper will decrease while that of germanium will increase.

Q3. The probability of electrons to be found in the conduction band of an intrinsic semiconductor at a finite temperature

(a) Increases exponentially with increasing band gap.

(b) Decreases exponentially with increasing band gap.

(c) Decreases with increasing temperature.

(d) is independent of the temperature and the band gap.

Q4. Pure silicon at $300\ K$ has equal electron and hole concentration of $2\times10^{16}m^{-3}$. It is doped by by indium to the extent one part in 10^7 silicon atom. If the density of silicon is $4\times10^{29}m^{-3}$, then the electron concentration in the doped silicon is

(a) $10^5 m^{-3}$ (b) $10^7 m^{-3}$ (c) $10^9 m^{-3}$ (d) $10^{10} m^{-3}$

Q5. Two pure specimen of a semiconductor material are taken. One is doped with $10^{18} cm^{-3}$ numbers of donors and the other is doped with $10^{16} cm^{-3}$ numbers of acceptors. The minority carrier density in the first specimen is $10^{7} cm^{-3}$. What is the minority carrier density in the other specimen?

(a) $10^{16} cm^{-3}$ (b) $10^{27} cm^{-3}$ (c) $10^{18} cm^{-3}$ (d) $10^{9} cm^{-3}$

Q6. The donor concentration in a sample of n-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at $300\,K$, assuming the sample to non degenerate is $\left(k_B T = 25\ meV \text{ at } 300K\right)$

(a) 105 meV (b) 110 meV (c) 115 meV (d) 120 meV

Q7. A sample of Si has electron and hole mobility's of 0.13 and 0.05 $m^2 V^{-1} s^{-1}$ respectively at $300\,K$. It is doped with P and Al with doping densities of $1.5\times10^{21}/m^3$ and $2.5\times10^{21}/m^3$ respectively. The conductivity of doped Si sample at $300\,K$ is

(a) $8\ \Omega^{-1}m^{-1}$ (b) $32\ \Omega^{-1}m^{-1}$ (c) $20.8\ \Omega^{-1}m^{-1}$ (d) $83.2\ \Omega^{-1}m^{-1}$

Q8. A sample of Si has electron and hole mobility's of 0.13 and 0.05 $m^2 V^{-1} s^{-1}$ respectively at $300\,K$. It is doped with P and Al with doping densities of $2.5\times10^{21}/m^3$ and $1.5\times10^{21}/m^3$ respectively. The conductivity of doped Si sample at $300\,K$ is

(a) $8\ \Omega^{-1}m^{-1}$ (b) $32\ \Omega^{-1}m^{-1}$ (c) $20.8\ \Omega^{-1}m^{-1}$ (d) $83.2\ \Omega^{-1}m^{-1}$

Q9. Mobility of electrons as well as holes for intrinsic germanium is given by $3900\ cm^2/V-\text{sec}$ and $1900\ cm^2/V-\text{sec}$ with intrinsic concentration $2.5\times10^{13} cm^{-3}$. Then the intrinsic resistivity of the material is

(a) 43 Ωcm (b) 64 Ωcm (c) 86 Ωcm (d) 131 Ωcm

Q10. Consider an extrinsic semiconductor with intrinsic concentration of n_i. If μ_p and μ_n are mobility of holes and electron then the electron concentration at which semiconductor have minimum conductivity and $\sigma_{\min}$ are

(a) $n_i\sqrt{\mu_p\mu_n}$, $n_i e\sqrt{\mu_p\mu_n}$ (b) $n_i\sqrt{\mu_p/\mu_n}$, $2n_i e\sqrt{\mu_p\mu_n}$

(c) $n_i\sqrt{\mu_n/\mu_p}$, $n_i e\sqrt{\mu_n/\mu_p}$ (d) $n_i/\sqrt{\mu_n\mu_p}$, $2n_i e/\sqrt{\mu_n\mu_p}$

Q11. The Fermi-level in an *n*-type and *p*-type semiconductor material is expressed as (where N_D, N_A are donor and acceptor ion concentration and N_C, N_V are effective density of states in conduction and valance band)

(a) $E_C + kT\ln\left(\frac{N_D}{N_C}\right), E_V + kT\ln\left(\frac{N_A}{N_V}\right)$ (b) $E_C - kT\ln\left(\frac{N_D}{N_C}\right), E_V - kT\ln\left(\frac{N_A}{N_V}\right)$

(c) $E_C + kT\ln\left(\frac{N_D}{N_C}\right), E_V - kT\ln\left(\frac{N_A}{N_V}\right)$ (d) $E_C - kT\ln\left(\frac{N_D}{N_C}\right), E_V + kT\ln\left(\frac{N_A}{N_V}\right)$

Q12. In an *n*-type semiconductor the minority hole concentration is p_n and intrinsic carrier concentration is n_i. If the effective density of state in conduction band is n_c at temperature T^0K. Then relative position of the Fermi-level with respect to level (E_c) is

(a) $KT\ln\frac{n_c}{n_i^2 p_n}$ (b) $KT\ln\frac{n_i^2\cdot n_c}{p_n}$

(c) $KT\ln\frac{p_n\cdot n_c}{n_i^2}$ (d) $KT\ln\frac{n_i^2}{p_n\cdot n_c}$

Q13. A *Si* sample $\left(n_i = 1.5\times10^{10}cm^{-3}\right)$ is doped with 10^{17} *As atoms*$/cm^3$. Then relative position of Fermi-level (E_F) with respect to intrinsic level (E_i) is

(a) 0.12 *eV* (b) 0.14 *eV* (c) 0.16 *eV* (d) 0.41 *eV*

Q14. In an *n*-type semiconductor, the Fermi level is $0.24\,eV$ below the conduction band at room temperature of $300\,K$. If the temperature is increased to $350\,K$, then the new position of the Fermi-level is (Assume effective density of states to be independent of temperature):

(a) $0.28\,eV$ (b) $0.38\,eV$ (c) $0.48\,eV$ (d) $0.58\,eV$

Q15. A p-type semiconductor (acceptor ion concentration is N_A) is doped with donor ion (concentration is N_D) and $N_D > N_A$. If the intrinsic concentration is n_i, then the concentration of minority carrier in the doped specimen will be:

(a) $\frac{n_i^2}{N_D}$ (b) $\frac{n_i^2}{N_A}$ (c) $\frac{n_i^2}{(N_D - N_A)}$ (d) $\frac{n_i^2}{(N_A - N_D)}$

Numerical Answer Type Questions (NAT)

Q16. Pure silicon at $300\,K$ has equal electron and hole concentration of $1.5\times10^{16}\,m^{-3}$. Doping by indium increases hole concentration to $4.5\times10^{22}\,m^{-3}$. Then the electron concentration in the doped silicon is……..$\times10^{9}\,m^{-3}$

(a) $9\times10^{5}\,m^{-3}$ (b) $5\times10^{9}\,m^{-3}$ (c) $9\times10^{-5}\,m^{-3}$ (d) $5\times10^{-9}\,m^{-3}$

Q17. A pure Si sample at $300\,K$ with intrinsic carrier concentration of $1.5\times10^{16}/m^3$ is doped with phosphorous. The equilibrium hole concentration and electron mobility is $2.25\times10^{9}/m^3$ and 1350 cm^2/Vs respectively. Then the Position of Fermi-level relative to the intrinsic level at $300\,K$ is.....eV

Q18. A sample of Si has electron and hole mobility's of 0.13 and 0.05 $m^2V^{-1}s^{-1}$ respectively at $300\,K$. It is doped with P and Al with doping densities of $2.5\times10^{21}/m^3$ and $1.5\times10^{21}/m^3$ respectively. The resistivity of doped Si sample at $300\,K$ is.....Ωm

Q19. The following data are given for intrinsic Germanium at $300\,K$. $n_i = 2.4\times10^{19}/m^3$, $\mu_e = 0.39 m^2V^{-1}s^{-1}$, $\mu_p = 0.19 m^2V^{-1}s^{-1}$. The resistivity of the Germanium will turn out to be .....Ωm

Q20. A semiconductor has following parameters $\mu_n = 7500\ cm^2/Vs$, $\mu_p = 300\ cm^2/Vs$ and $n_i = 3.6\times10^{12}cm^{-3}$. When the conductivity is minimum, the hole concentration is .....$\times10^{13}cm^{-3}$

Multiple Select Type Questions (MSQ)

Q21. Which of the following are true regarding Fermi-Dirac distribution function

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

(a) An energy state at the Fermi level has a probability of $1/2$ of being occupied by an electron.

(b) At $0\,K$, every available energy state up to E_F is filled with electrons, and all states above E_F are empty.

(c) At temperatures higher than $0\,K$, there is some probability $f(E)$ that states above E_F are filled and there is a corresponding probability $[1-f(E)]$ that states below E_F are empty.

(d) The Fermi function is unsymmetrical about E_F for all temperatures.

Q22. Which of the following statement are true regarding semiconductors?

(a) An n-type semiconductor behaves as an intrinsic semiconductor at very high temperature.

(b) The breaking of the covalent bonds becomes a significant phenomenon at high temperatures.

(c) The carriers mobility increases with increase of temperature.

(d) The carriers mobility decreases with increase of temperature.

Q23. Which one of the following are true?

(a) Metals have positive temperature coefficient of resistance.

(b) Semiconductors have negative temperature coefficient of resistance.

(c) Conductivity of metals decreases with increase in temperature.

(d) Conductivity of semiconductor decreases with increase in temperature.

Q24. Which one of the following are not true?

(a) The diffusion constants D_n and D_p for electrons and holes respectively are related to their mobility by Einstein equation $\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{kT}{e}$

(b) The expression $n.p = n_i^2$ is valid for semiconductors at all temperature.

(c) $n_i(T) = AT^{3/2}\exp\left(-\frac{E_g}{2kT}\right)$, correctly describe the temperature (T) variation of the intrinsic carrier density of a semiconductor

(d) Gallium Arsenide $(GaAs)$ is an indirect band gap semiconductor with $E_g = 1.43eV$ at room temperature

Q25. Which of the following are true?

(a) Si and Ge are indirect band gap semiconductor.

(b) At 300^0K the band gap energies of Si and Ge are $1.1eV$ and $0.72eV$.

(b) At 0^0K the band gap energies of Si and Ge are $1.1eV$ and $0.72eV$.

(d) At 300^0K mobility (μ) and diffusion constant (D) is related by $\mu = 39D$.

Solution

MCQ

Ans.1: (c)

Ans.2: (d)

Ans.3: (b)

Ans.4: (d)

Acceptor ion concentration $N_A \approx p = \dfrac{4\times10^{29}\,m^{-3}}{10^7} = 4\times10^{22}\,m$

According to Law of Mass Action, $n.p = n_i^2$

$$n.p = n'.p' \Rightarrow n' = \frac{n.p}{p'} = \frac{\left(2\times10^{16}\right)\times\left(2\times10^{16}\right)}{\left(4\times10^{22}\right)} = 10^{10}\ m^{-3}$$

Ans.5: (d)

According to law of mass action,

$$n_1.p_1 = n_2.p_2 \Rightarrow n_2 = \frac{n_1.p_1}{p_2} = \frac{10^{18}\times10^7}{10^{16}} = 10^9\,cm^{-3}$$

Ans.6: (c)

$$E_C - E_F = kT\ln\left(\frac{N_c}{N_d}\right) \text{ and } E_C - E_F' = kT\ln\left(\frac{N_c}{100N_d}\right) = kT\ln\left(\frac{N_c}{N_d}\right) - kT\ln(100)$$

Thus shift is $\Delta E = kT\ln(100) = 25\ln(100)\,meV = 115.15\,meV$

Ans.7: (a)

Resulting doped crystal is *p*-type and $p_p = (2.5-1.5)\times10^{21}/m^3 = 1\times10^{21}/m^3$

$$\sigma = e\left(n_p\mu_n + p_p\mu_p\right) \approx ep_p\mu_p = 1.6\times10^{-19}\times1\times10^{21}\times0.05 = 8\ \Omega^{-1}m^{-1}$$

Ans.8: (c)

Resulting doped crystal is *n*-type and $n_n = (2.5-1.5)\times10^{21}/m^3 = 1\times10^{21}/m^3$

$$\sigma = e\left(n_n\mu_n + p_n\mu_p\right) \approx en_n\mu_n = 1.6\times10^{-19}\times1\times10^{21}\times0.13 = 20.8\ \Omega^{-1}m^{-1}$$

Ans.9: (a)

$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{n_i e\left(\mu_e + \mu_h\right)} = \frac{1}{2.5 \text{ x } 10^{13} \times 1.6 \text{ x } 10^{-19} \times \left(5800\right)} = 43\ \Omega cm$$

Ans.10: (b)

$$\text{Conductivity } \sigma = e\left(n\mu_n + p\mu_p\right) = e\left(n\mu_n + \frac{n_i^2}{n}\mu_p\right)$$

$$\text{For minimum conductivity, } \frac{d\sigma}{dn} = 0 \Rightarrow n = n_i\sqrt{\mu_p / \mu_n}$$

$$\text{Thus} \quad \sigma_{\min} = 2n_i e\sqrt{\mu_p \mu_n}$$

Ans.11: (c)

Ans.12: (c)

$$n_n = N_c e^{-\left(E_c - E_F\right)/kT} \Rightarrow E_c - E_F = kT \ln\left(\frac{N_c}{n_n}\right) \text{ and } n_n . p_n = n_i^2$$

Ans.13: (d)

$$p_n = \frac{n_i^2}{n_n} = \frac{2.25 \times 10^{20}}{10^{17}} = 2.25 \times 10^3\ cm^{-3} \Rightarrow E_F - E_i = kT \ln\frac{n_n}{n_i} = 0.407\, eV$$

Ans.14: (a)

$$E_C - E_F = kT \ln\frac{N_C}{N_D} \Rightarrow 0.24 = 300k \ln\frac{N_C}{N_D} \text{ and } E_C - E_F^{'} = 350k \ln\frac{N_C}{N_D}$$

$$\Rightarrow E_C - E_F^{'} = 0.28\, eV$$

Ans.15: (c)

NAT

Ans.16: 5

According to Law of Mass Action, $n.p = n_i^2$

$$n.p = n'.p' \Rightarrow n' = \frac{n.p}{p'} = \frac{\left(1.5\times10^{16}\right)\times\left(1.5\times10^{16}\right)}{\left(4.5\times10^{22}\right)} = 5\times10^{9}\ m^{-3}$$

Ans.17: 0.41

Equilibrium electron concentration is

$$np = n_i^2 \quad (Law\ of\ \ mass) \Rightarrow n = \frac{n_i^2}{p} = \frac{\left(1.5\times10^{16}\right)^2}{2.25\times10^{9}} = 1.00\times10^{23}\,m^{-3}$$

$$E_F - E_i = k_B T \ln\left(\frac{n}{n_i}\right) = 8.67\times10^{-5}\times300\times\ln\left(\frac{10^{23}}{1.5\times10^{16}}\right) = 0.406\ eV$$

Ans.18: 0.05

Resulting doped crystal is *n*-type and $n_n = (2.5-1.5)\times10^{21}/m^3 = 1\times10^{21}/m^3$

$$\sigma = e\left(n_n\mu_n + p_n\mu_p\right) \approx en_n\mu_n = 1.6\times10^{-19}\times1\times10^{21}\times0.13 = 20.8\ \Omega^{-1}m^{-1}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{20.8} = 0.048\,\Omega m$$

Ans.19: 0.45

$$\sigma = en_i\left(\mu_n + \mu_p\right) = 1.6\times10^{-19}\times2.4\times10^{19}\left(0.39+.19\right) = 2.227\left(\Omega m\right)^{-1}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{2.227} = 0.449\,\Omega m$$

Ans.20: 2

$$p = n_i\sqrt{\mu_n/\mu_p} = 3.6\times10^{12}\sqrt{7500/300} = 18\times10^{12} = 2\times10^{13}\,cm^{-3}$$

MSQ

Ans.21: (a), (b) and (c)

Ans.22: (a), (b) and (d)

Ans.23: (a), (b) and (c)

Ans.24: (a), (b) and (c)

Ans.25: (a), (c) and (d)

2. P-N Junction Diode

2.1 Semiconductor Diode

In an *n*-type material the electron is called the majority carrier and the hole the minority carrier. In an *p*-type material the holes are the majority carrier and the electrons are minority carrier.

If the two materials are "joined" the electrons and holes in the region of the junction will combine resulting in a lack of carriers in the region near the junction. This region of uncovered positive and negative ions is called the depletion region due to the depletion of mobile carriers in this region.

Since the diode is two terminal devices, the application of a voltage across its terminals leaves three possibilities: **no bias** $(V_D = 0V)$, **forward bias** $(V_D > 0V)$ and **reverse bias** $(V_D < 0V)$. Each is a condition that will result in a response that one must clearly understand if the device is to be applied effectively.

2.1.1 No Applied Bias $(V_D = 0V)$

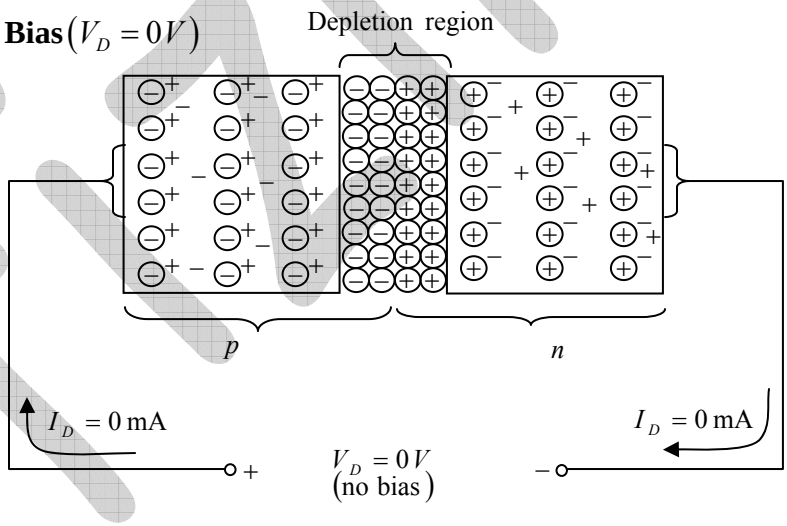


Figure 2.1: *p-n* junction with no applied bias.

Under no bias condition, any minority carriers (holes) in the *n*-type material that find themselves within the depletion region will pass directly into the *p*-type material. The closer the minority carrier is to the junction, the greater the attraction for the layer of negative ions and the less the opposition of the positive ions in the depletion region of the *n*-type material. For further discussions we shall assume that all the minority carriers in

the *n*-type material that find themselves in the depletion region due to their random motion will pass directly into the *p*-type material. Similar discussion can be applied to the minority carriers (electrons) of the *p*-type material.

The majority carriers (electrons) of the *n*-type material must overcome the attractive forces of the layer of positive ions in the *n*-type material and the shield of negative ions in the *p*-type material in order to migrate into the area beyond the depletion region of the *p*-type material. Again the same type of discussion can be applied to the majority carriers (holes) of the *p*-type material.

In the absence of an applied bias voltage, the net flow of charge in any one direction for a semiconductor diode is zero.

2.1.2 Reverse Bias Condition $(V_D < 0V)$

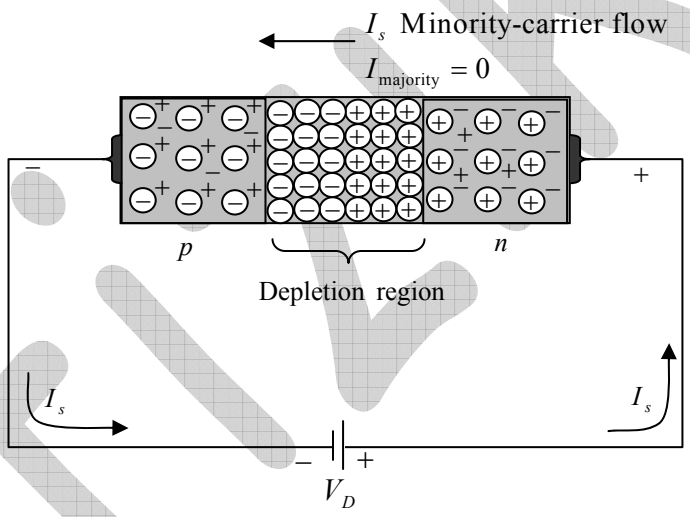


Figure 2.2: Reversed biased *p*-*n* junction.

If an external potential of *V* volts is applied across the *p*-*n* junction such that the positive terminal is connected to the *n*-type material and the negative terminal is connected to the *p*-type material, the number of uncovered positive ions in the depletion region of the *n*-type material will increase due to the large number of "free" electrons drawn to the positive potential of the applied voltage.

For similar reasons, the number of uncovered negative ions will increase in the *p*-type material. The net effect, therefore, is a widening of the depletion region. This widening of the depletion region will establish too great a barrier for the majority carriers to overcome, effectively reducing the majority carrier flow to zero.

The number of minority carriers, however, that find themselves entering the depletion region will not change, resulting in minority-carrier flow. *The current that exists under reverse bias conditions is called the reverse saturation current and is represented by* I_s or I_0.

2.1.3 Forward Bias Condition $(V_D > 0V)$

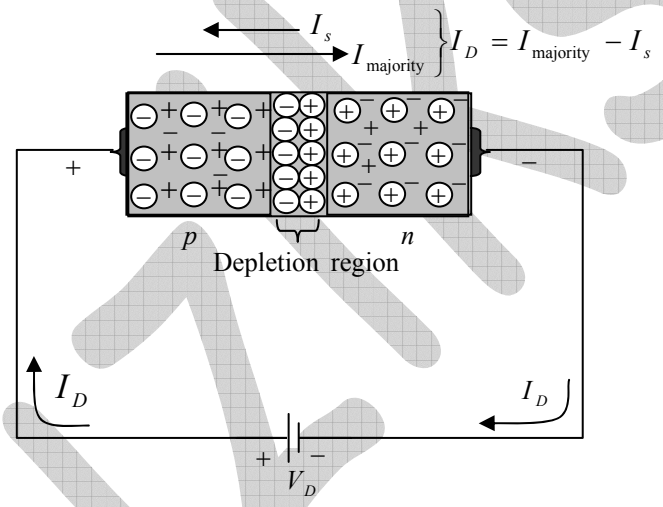


Figure 2.3: Forward biased *p-n* junction.

A forward-bias or "on" condition is established by applying the positive potential to the *p*-type material and the negative potential to the *n*-type material as shown in figure 2.3. Thus *a semiconductor diode is forward-biased when the association p-type positive and n-type negative has been established.*

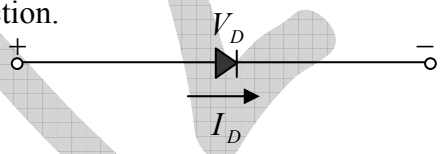
The application of a forward-bias potential V_D will "pressure" electrons in the *n*-type material and holes in the *p*-type material to recombine with the ions near the boundary and reduce the width of the depletion region. The resulting minority-carrier flow of electrons from the *p*-type material to the *n*-type material (and of holes from the *n*-type material to the *p*-type material) has not changed in magnitude (since the conduction level

is controlled primarily by the limited number of impurities in the material), but the reduction in the width of the depletion region has resulted in a heavy majority flow across the junction. An electron in the *n*-type material now "sees" a reduced barrier at the junction due to the reduced depletion region and a strong attraction for the positive potential applied to the *p*-type material. As the applied bias increases in magnitude the depletion region will continue to decrease in width until a flood of electrons can pass through the junction, resulting in an exponential rise in current as shown in the forward-bias region of the characteristics.

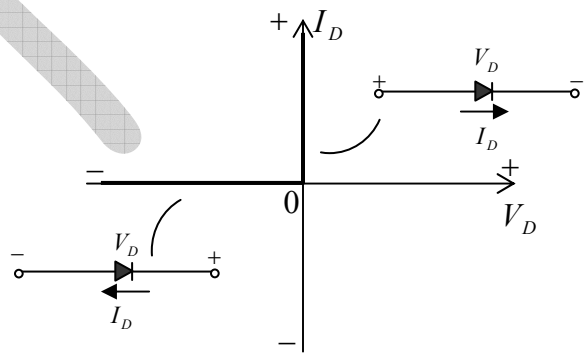
2.1.4 Ideal Diode

Before examining the construction and characteristics of an actual device, we first consider the ideal device, to provide a basis for comparison. The ideal diode is a two-terminal device having the symbol and characteristics shown in figure 2.4(a) and 2.4(b), respectively.

Ideally, a diode will conduct current in the direction defined by the arrow in the symbol and act like an open circuit to any attempt to establish current in the opposite direction. In essence: The characteristics of an ideal diode are those of a switch that can conduct current in only one direction.



(a)



(b)

Figure 2.4: Ideal Diode: (a) Symbol; (b) Characteristics.

The ideal diode, therefore, is a short circuit in the region of conduction and is an open circuit in the region of non conduction.

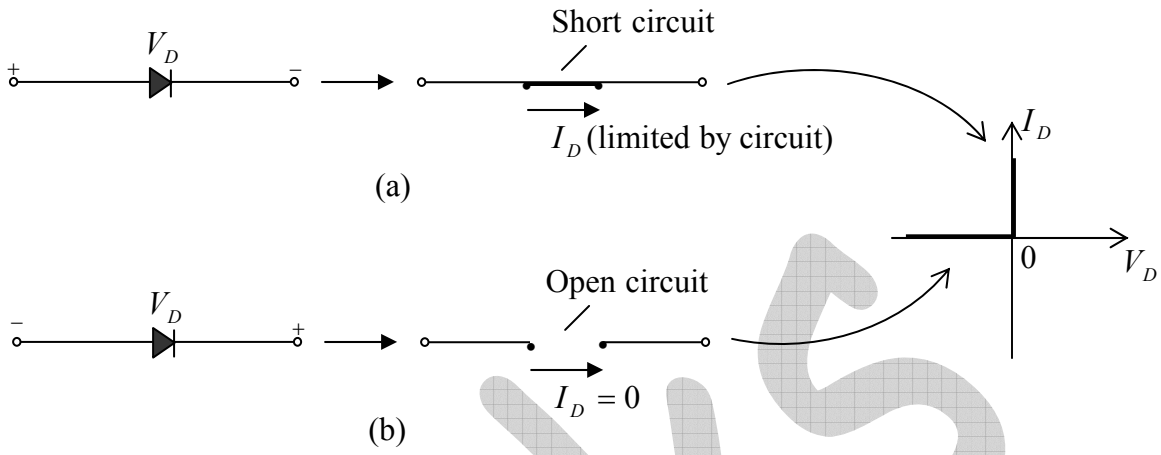


Figure 2.5: (a) Conduction and (b) non-conduction states of the ideal diode as determined by the applied bias.

2.1.5 Diode Characteristics ($V_\gamma = 0.7V$ for *Si* and $V_\gamma = 0.3V$ for *Ge*)

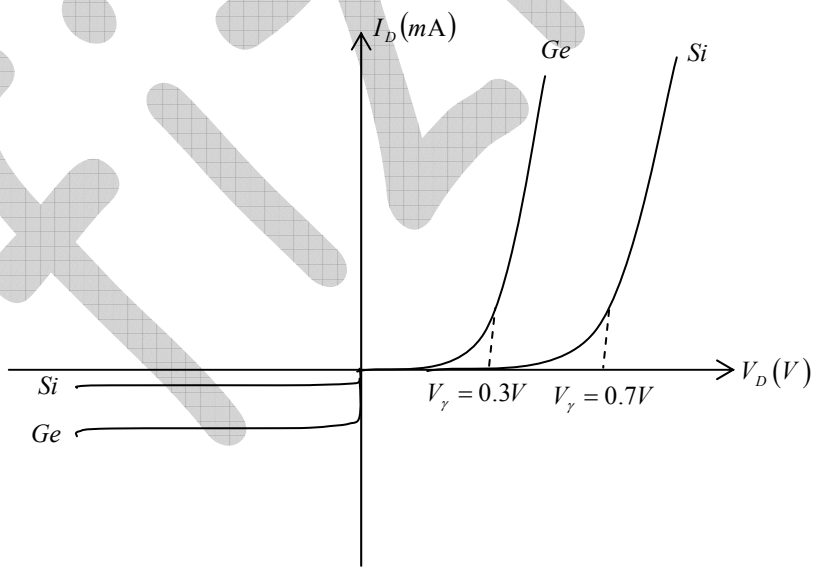


Figure 2.6: Comparison of *Si* and *Ge* semiconductor diodes.

It is clear from the characteristics of figure 2.6 that to forward bias the diode minimum voltage of V_γ is required. This voltage V_γ is called cut-in voltage of the diode. The closer the upward swing is to the vertical axis, the more "ideal" the device. However, the other characteristics of silicon as compared to germanium still make it the choice in the majority of commercially available units.

2.1.6 Diode Equation

$I_D = I_s\left(e^{V_D/\eta V_T} - 1\right)$ where I_s = reverse saturation current,

$V_T = \dfrac{kT}{e}$ is volt equivalent of temperature and $\eta = 1$ for *Ge* and $\eta = 2$ for *Si* devices.

Note: The reverse saturation current in a germanium diode is normally larger by a factor of about 1000 than the reverse saturation current in a silicon diode of comparable ratings. I_s is in the range of μA for a *Ge* diode and *nA* for a silicon diode at room temperature.

2.1.7 Breakdown Diodes

Diodes which are designed with adequate power-dissipation capabilities to operate in the breakdown region may be employed as voltage-reference or constant-voltage devices. Such diodes are known as avalanche, breakdown or Zener diodes. They are used characteristically in the manner indicated in the figure 2.7.

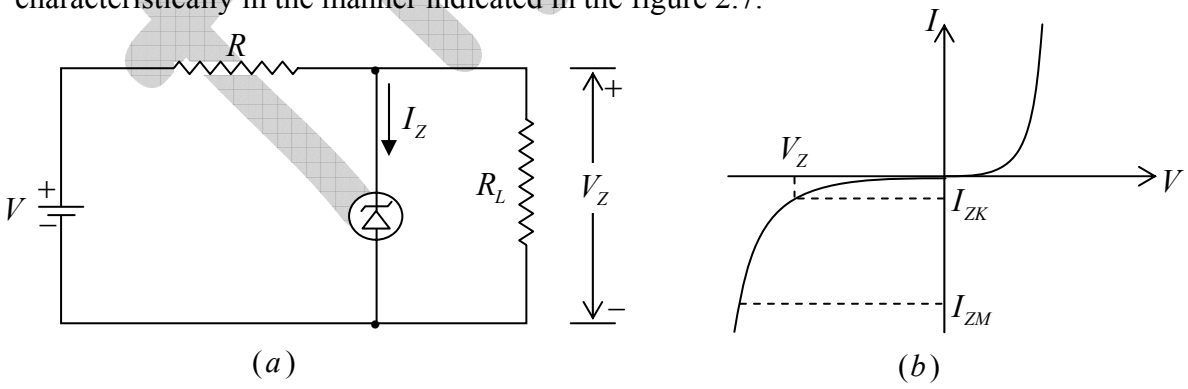


Figure 2.7: (a) Zener diode as voltage regulator (b) Zener charachteristics.

The source V and resistor R are selected so that initially, the diode is operating in the breakdown region. Here the diode voltage, which is also the voltage across the load R_L, is V_Z, and the diode current is I_Z. The diode will now regulate the load voltage against variations in load current and against variations in supply V, because in the break-down region only small changes in diode voltage produce large changes in diode current. Moreover, as load current or supply voltage changes, the diode current will accommodate itself to these changes to maintain a nearly constant load voltage. The diode will continue to regulate until the circuit operation requires the diode current to fall to I_{ZK}, in the neighborhood of the knee of the diode volt-ampere curve. The upper limit on diode current is determined by the power dissipation rating of the diode.

2.1.8 The Temperature Dependence of the *V-I* Characteristics

The volt-ampere relationship contains the temperature implicitly in the two symbols V_T and I_s. If the temperature is increased at a fixed voltage, the current increases. However if we now reduce V, then I may be brought back to its previous value. It is found that for either silicon or germanium (at room temperature) $\frac{dV}{dT} \approx -2.5 \, mV/^0C$ in order to maintain a constant value of I. It should be noted that $\left|\frac{dV}{dT}\right|$ decreases with increasing T.

2.2 Diode Resistances

2.2.1 DC or Static Resistance

The application of a dc voltage to a circuit containing a semiconductor diode will result in an operating point on the characteristic curve that will not change with time. The resistance of the diode at the operating point can be found simply by finding the corresponding levels of V_D and I_D as shown in figure 2.8 and applying the following equation:

$$R_D = \frac{V_D}{I_D}$$

The dc resistance levels at the knee and below will be greater than the resistance levels obtained for the vertical rise section of the characteristics. The resistance levels in the reverse-bias region will naturally be quite high. Since ohmmeters typically employ a relatively constant current source, the resistance determined will be at a preset current level (typically, a few milliamperes).

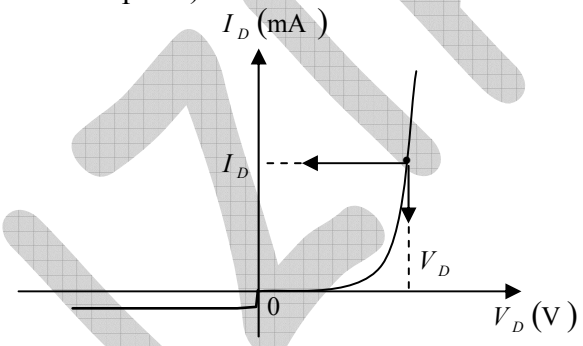


Figure 2.8: Determining the *dc* resistance of a diode at a particular operating point.

2.2.2 AC or Dynamic Resistance

The dc resistance of a diode is independent of the shape of the characteristic in the region surrounding the point of interest. If a sinusoidal rather than dc input is applied, the situation will change completely. The varying input will move the instantaneous operating point up and down a region of the characteristics and thus defines a specific change in current and voltage as shown in figure 2.9(a). With no applied varying signal, the point of operation would be the *Q*-point appearing on figure determined by the applied dc levels. The designation *Q*-point is derived from the word quiescent which means "still or unvarying."

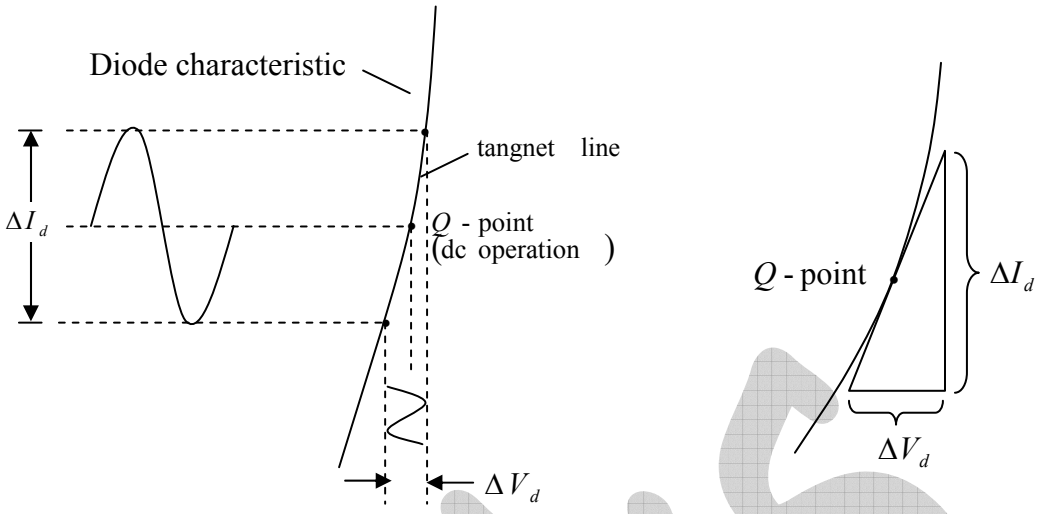


Figure 2.9: (a) Defining ac Resistance. (b) Determining the ac Resistance at a *Q*-point.

A straight line drawn tangent to the curve through the *Q*-point as shown in figure 2.9(b) will define a particular change in voltage and current that can be used to determine the *ac* or *dynamic* resistance for this region of the diode characteristics. An effort should be made to keep the change in voltage and current as small as possible and equidistant to either side of the *Q*-point. In equation form $r_d = \frac{\Delta V_d}{\Delta I_d}$ where Δ signifies a finite change in the quantity.

For small-signal operation the dynamic resistance *r* is an important parameter and is defined as the reciprocal of the slope of the volt ampere characteristic $r \equiv \frac{dV}{dI}$.

The dynamic resistance is not a constant, but depends upon the operating voltage.

For a semiconductor diode, the dynamic conductance $g \equiv \frac{1}{r}$

$$g \equiv \frac{dI}{dV} = \frac{I_s e^{V/\eta V_T}}{\eta V_T} = \frac{I + I_s}{\eta V_T} \Rightarrow \boxed{r \approx \frac{\eta V_T}{I}} \quad \because I = I_s\left(e^{V/\eta V_T} - 1\right) \text{ and } \left|\frac{V}{\eta V_T}\right| >> 1,\ I >> I_S .$$

At room temperature, for η = 1, $r = \frac{26}{I}$, where *I* is in *mA* and *r* is in Ω. For a forward current of 26 *mA* the dynamic resistance is 1Ω.

2.3 Diode Capacitances

2.3.1 Space-Charge or Transition Capacitance

A reverse bias causes majority carriers to move away from the junction, thereby uncovering more immobile charges. Hence the thickness of the space charge layer at the junction increases with reverse voltage. The increased in uncovered charge with applied voltage may be considered a capacitive effect. We may define an incremental capacitance C_T by: $C_T = \left|\frac{dQ}{dV}\right|$ where dQ is the increase in charge caused by a change dV in voltage.

It follows from this definition that a change in voltage dV in a time dt will result in a current $i = \frac{dQ}{dt}$ given by $i = C_T \frac{dV}{dt}$ where C_T is not a constant, but depends upon the magnitude of the reverse voltage.

2.3.2 Diffusion Capacitance

For a forward bias a capacitance which is much larger than the transition capacitance lies in the injected charge stored near the junction outside the transition region. It is convenient to introduce an incremental capacitance, defined as the rate of change of injected charge with voltage, called the diffusion or storage, capacitance C_D.

Charge control description of a diode

If the bias is in the forward direction, the potential barrier at the junction is lowered and holes from the *p*-side enter the *n*- side. Similarly electrons from the *n*-side move into the *p*-side. This process of minority-carrier injection has been discussed earlier. The excess hole density falls off exponentially with distance.

Assume that one side of the diode, say the *p* material, is so heavily doped in comparison with the *n* side that the current I is carried across the junction entirely by holes moving from the *p* to the *n* side or $I = I_{pn}(0)$. The excess minority charge Q will then exist only on the *n* side.

Now $I = \frac{Q}{\tau}$, where $\tau = \tau_p =$ mean life for holes.

The above equation states that the diode current (which consists of holes crossing the junction from the *p* to the *n* side) is proportional to the stored charge *Q* of excess minority carriers. The factor of proportionality is the reciprocal of the decay time constant (the mean lifetimeτ) of the minority carriers. Thus in the steady state, the current *I* supplies minority carriers at the rate at which these carriers are disappearing because of the process of recombination.

Static Derivation of C_D

Since, $I = \frac{Q}{\tau}$ and $C_D = \frac{dQ}{dV} = \tau \frac{dI}{dV} = \tau g = \frac{\tau}{r}$

where $g = \frac{dI}{dV}$ is the diode incremental conductance and $r = \frac{1}{g}$ is the diode incremental resistance.

Thus $\boxed{C_D = \frac{\tau I}{\eta V_T}}$.

We see that the diffusion capacitance is proportional to the current *I*. In the above derivation we have assumed that the diode current *I* is due to holes only. If this assumption is not satisfied then above equation gives the diffusion capacitance C_{D_P} due holes only and a similar expression can be obtained for the diffusion capacitance C_{D_n} due to electrons. The total diffusion capacitance can then be obtained as the sum of C_{D_P} and C_{D_n}.

Note: For a reverse bias, *g* is very small and C_D may be neglected compared with C_T.

For a forward current, on the other hand, C_D is usually much larger than C_T.

Despite the large value of C_D, the time constant rC_D may not be excessive because the dynamic forward resistance $r = \frac{1}{g}$ is small. Thus $rC_D = \tau$. Hence, the diode time constant equals the mean lifetime of minority carriers, which lies in the range of nanoseconds (*ns*) to hundreds of microseconds(μs).

2.4 Load Line Analysis

The applied load will normally have an important impact on the point or region of operation of a device. If the analysis is performed in a graphical manner, a line can be drawn on the characteristics of the device that represents the applied load. The intersection of the load line with the characteristics will determine the point of operation of the system. Such an analysis is, for obvious reasons, called *load-line analysis*.

Consider the network of figure 2.10(a) employing a diode having the characteristics of figure 2.10(b). Note in figure 2.10(a) that the "pressure" established by the battery is to establish a current through the series circuit in the clockwise direction. The fact that this current and the defined direction of conduction of the diode are a "match" reveals that the diode is in the "on" state and conduction has been established. The resulting polarity across the diode will be as shown in figure 2.10(a) and the first quadrant (V_D and I_D positive) of figure 2.10 (b) will be the region of interest – the forward-bias region.

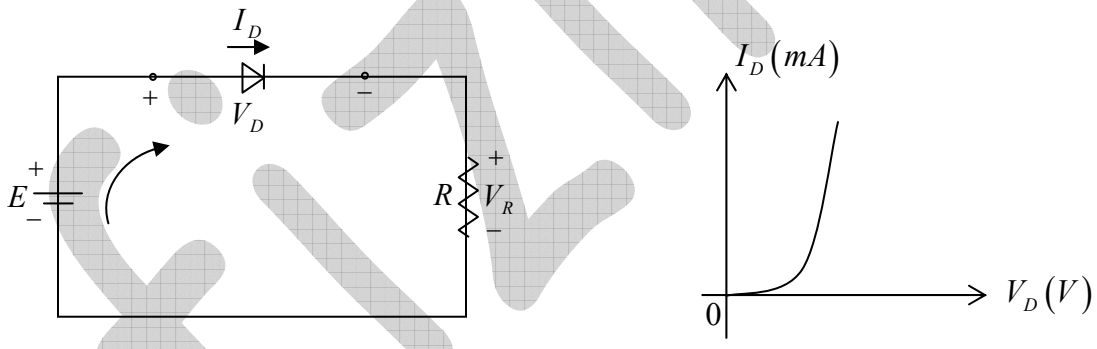


Figure 2.10: (a) Series diode configuration circuit; (b) Characteristics.

Applying Kirchhoff's voltage law to the series circuit of figure 2.10 (a) will result in

$$-E + V_D + V_R = 0 \Rightarrow E = V_D + V_R$$

$$\Rightarrow E = V_D + I_D R$$

The two variables of above equation (V_D and I_D) are the same as the diode axis variables of figure 2.10(a). This similarity permits a plotting of the equation on the same characteristics of figure 2.10 (b).

The intersections of the load line on the characteristics can easily be determined if one simply employs the fact that anywhere on the horizontal axis $I_D = 0A$ and anywhere on the vertical axis $V_D = 0\ V$.

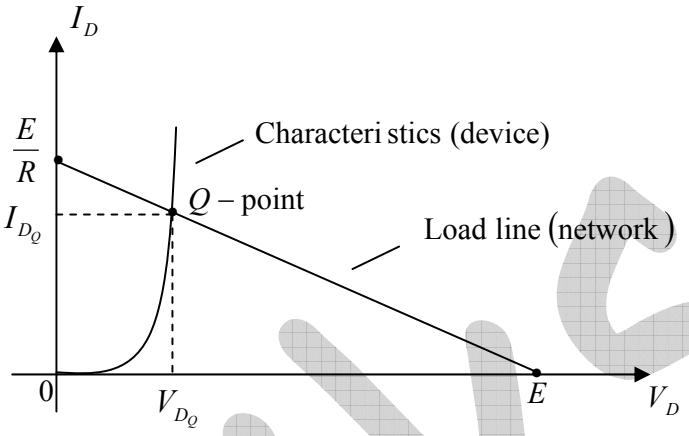


Figure 2.11: Drawing the load line and finding the point of operation.

If we set $V_D = 0\ V$ in equation $E = V_D + I_D R$ and solve for I_D, we have the magnitude of I_D on the vertical axis. Thus $E = V_D + I_D R = 0\ V + I_D R$ and $I_D = \left.\frac{E}{R}\right|_{V_D = 0V}$.

If we set $I_D = 0\,A$ in equation $E = V_D + I_D R$ and solve for V_D, we have the magnitude of V_D on the horizontal axis. Thus $E = V_D + (0A)R$ and $V_D = E\big|_{I_D = 0\,A}$.

A straight line drawn between the two points will define the load line as depicted in figure 2.11. Change the level of R (the load) and the intersection on the vertical axis will change. The result will be a change in the slope of the load line and a different point of intersection between the load line and the device characteristics.

We now have a load line defined by the network and a characteristics curve defined by the device. The point of intersection between the two is the point of operation for this circuit. By simply drawing a line down to the horizontal axis the diode voltage V_{D_Q} can be determined, whereas a horizontal line from the point of intersection to the vertical axis will provide the level of I_{D_Q}. The current I_D is actually the current through the entire

series configuration of figure 2.10(a). The point of operation is usually called the *quiescent point* (abbreviated "*Q*-point") to reflect its "still, unmoving" qualities as defined by a dc network.

The solution obtained at the intersection of the two curves is the same that would be obtained by a simultaneous mathematical solution of equations $E = V_D + I_D R$ and $I_D = I_s\left(e^{V_D/\eta V_T} - 1\right)$.

2.5 Series Diode Configurations with DC Inputs

In this section the approximate model is utilized to investigate a number of series diode configurations with dc inputs. The procedure described can, in fact, be applied to networks with any number of diodes in a variety of configurations.

For each configuration the state of each diode must first be determined. Which diodes are "on" and which are "off"? Once determined, the appropriate equivalent can be substituted and the remaining parameters of the network determined.

In general, a diode is in the "on" state if the current established by the applied sources is such that its direction matches that of the arrow in the diode symbol, and $V_D \geq 0.7$ *V for silicon and* $V_D \geq 0.3$ *V for germanium.*

For each configuration, mentally replace the diodes with resistive elements and note the resulting direction as established by the applied voltages ("pressure"). If the resulting direction is a "match" with the arrow in the diode symbol, conduction above is, of course, contingent on the supply having a voltage greater than the "turn-on" voltage (V_γ) of each diode.

If a diode is in the "on" state, one can either place a 0.7 *V* drop across the element, or the network can be redrawn with the V_γ equivalent circuit. In time the preference will probably simply be to include the 0.7 *V* drop across each "on" diode and draw a line through each diode in the "off" or open state. Initially, however, the substitution method will be utilized to ensure that the proper voltage and current levels are determined.

The series circuit of figure 2.12(a) will be used to demonstrate the approach described in the paragraphs above. The state of the diode is first determined by mentally replacing the diode with a resistive element as shown in figure 2.12(b). The resulting direction of I is a match with the arrow in the diode symbol and since $E > V_\gamma$ the diode is in the "on" state. The network is then redrawn as shown in figure 2.12(c) with the appropriate equivalent model for the forward-biased silicon diode. Note for future reference that the polarity of V_D is the same as would result if in fact the diode were a resistive element.

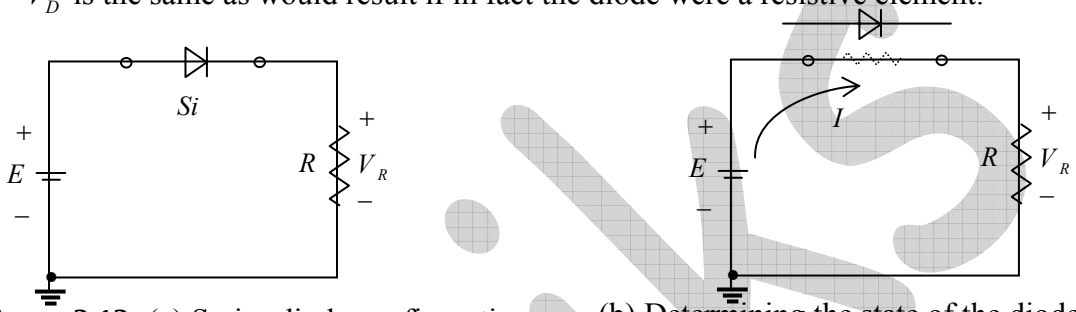


Figure 2.12: (a) Series diode configuration. (b) Determining the state of the diode of figure (a).

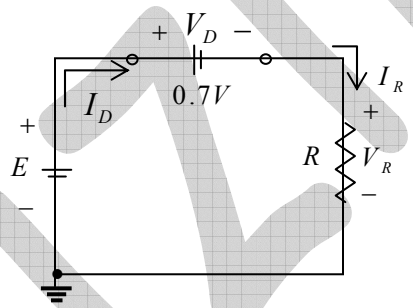


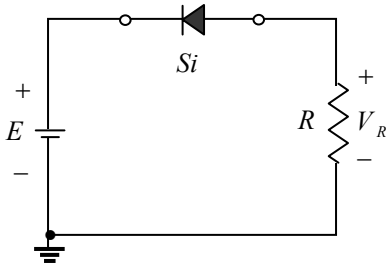
Figure 2.12: (c) Substituting the equivalent model for the "on" diode of figure (a).

The resulting voltage and current levels are the following.

$$V_D = V_\gamma,\ V_R = E - V_\gamma \ \text{ and } I_D = I_R = \frac{V_R}{R}.$$

In figure 2.13(a) the diode of figure 2.12(a) has been reversed. Mentally replacing the diode with a resistive element as shown in figure 2.13(b) will reveal that the resulting current direction does not match the arrow in the diode symbol. The diode is in the "off" state, resulting in the equivalent circuit of figure 2.13(c). Due to the open circuit, the diode current is 0 A and the voltage across the resistor R is the following:

$$V_R = I_R R = I_D R = 0V$$



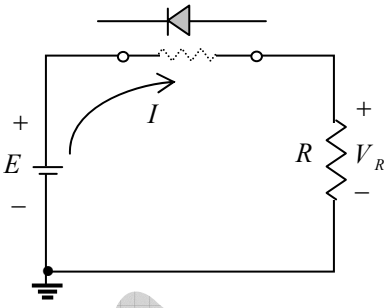


Figure 2.13: (a) Reversing the diode of figure 2.12(a). (b) Determining the state of diode of figure (a).

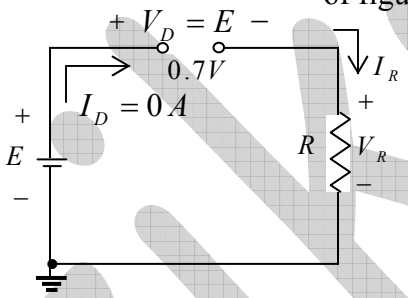
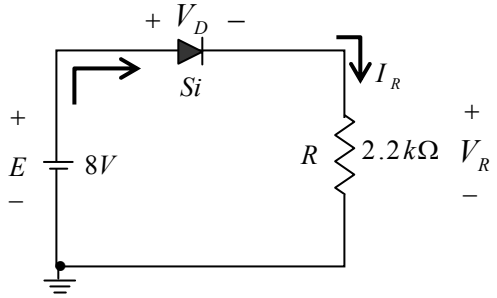


Figure 2.13(c): Substituting the equivalent model for the "off" diode of figure (a).

The fact that $V_R = 0V$ will establish E volts across the open circuit as defined by Kirchhoff's voltage law. *Always keep in mind that under any circumstances–dc, ac instantaneous values, pulses, and so on – Kirchhoff's voltage law must be satisfied!*

Example: For the series diode configuration of figure shown below, determine V_D, V_R, and I_D.



Solution: Since the applied voltage establishes a current in the clockwise direction to match the arrow of the symbol and the diode is in the "on" state.

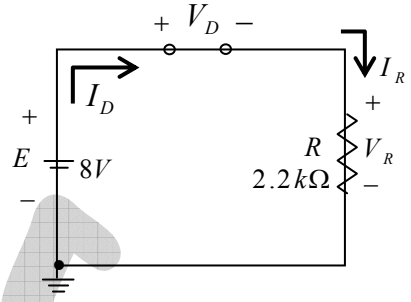
$V_D = 0.7\ V, \quad V_R = E - V_D = 8\ V - 0.7\ V = 7.3\ V$

$$I_D = I_R = \frac{V_R}{R} = \frac{7.3V}{2.2k\Omega} \cong 3.32\ mA$$

Example: Repeat above example with the diode reversed.

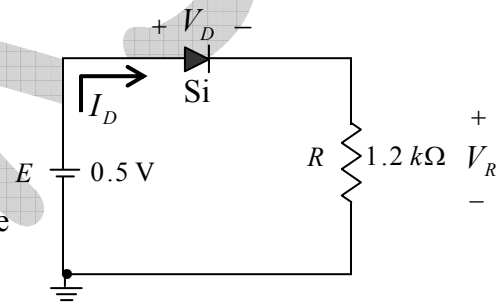
Solution: Removing the diode, we find that the direction of I is opposite to the arrow in the diode symbol and the diode equivalent is the open circuit no matter which model is employed. The result is the network of figure shown below, where $I_D = 0\ A$ due to the open circuit. Since $V_R = I_R R$, $V_R = (0)\ R = 0\ V$. Applying Kirchhoff's voltage law around the closed loop yields $-E + V_D + V_R = 0$

and $V_D = E - V_R = E - 0 = E = 8\ V$



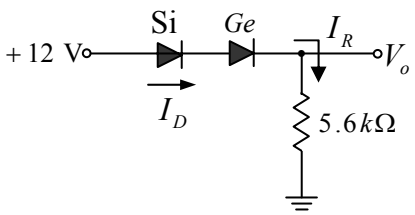
Example: For the series diode configuration of figure shown below, determine V_D, V_R, and I_D.

Solution: Although the "pressure" establishes a current with the same direction as the arrow symbol the level of applied voltage is insufficient to turn the silicon diode "on". The resulting voltage and current levels are therefore the following:



$I_D = 0\ A$, $V_R = I_R R = I_D R = (0\ A)\ 1.2\ k\Omega = 0\ V$ and $V_D = E = 0.5\ V$

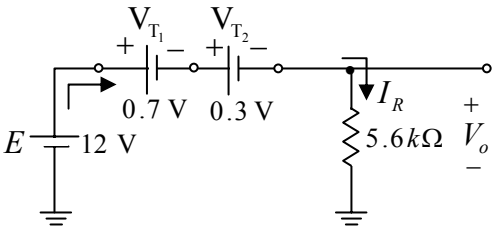
Example: Determine V_0 and I_D for the series circuit of figure shown below.



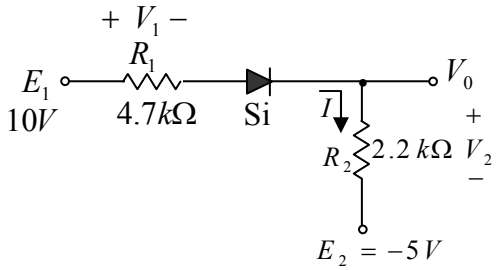
Solution: The resulting current has the same direction as the arrowheads of the symbols of both diodes, and the network of figure shown below results because $E = 12V > (0.7\ V + 0.3\ V) = 1\ V$ .Note the redrawn supply of 12 V and the polarity of V_0 across the 5.6 $k\Omega$ resistor. The resulting voltage

$V_0 = E - V_{\gamma_1} - V_{\gamma_2} = 12V - 0.7V - 0.3V = 11V$

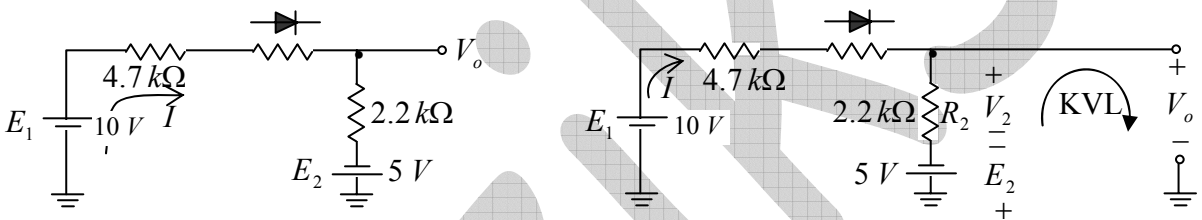
and $I_D = I_R = \dfrac{V_R}{R} = \dfrac{V_0}{R} = \dfrac{11V}{5.6k\Omega} \cong 1.96mA$



Example: Determine I, V_1, V_2 and V_0 for the series *dc* configuration of figure shown below.



Solution: The sources are drawn and the current direction indicated as shown in figure below. Note that the "on" state is noted simply by the additional V_D= 0.7 V on the figure. This eliminates the need to redraw the network and avoids any confusion that may result from the appearance of another source.



The resulting current through the circuit is,

$$I = \frac{E_1 + E_2 - V_D}{R_1 + R_2} = \frac{10V + 5V - 0.7V}{4.74k\Omega + 2.2k\Omega} = \frac{14.3V}{6.9k\Omega} \cong 2.07mA$$

and the voltages are $V_1 = IR_1 = (2.07\ mA)\ (4.7\ k\Omega) = 9.73\ V$

$V_2 = IR_2 = (2.07\ mA)\ (2.2\ k\Omega) = 4.55\ V$

Applying Kirchhoff's voltage law to the output section in the clockwise direction will result in

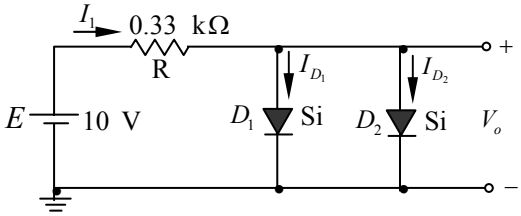
$+E_2 - V_2 + V_0 = 0$ and $V_0 = V_2 - E_2 = 4.55\ V - 5\ V = -0.45\ V$.

The minus sign indicates that V_0 has a polarity opposite to that appearing in figure.

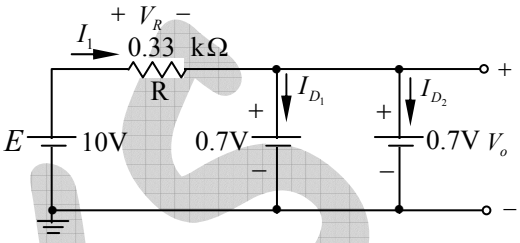
2.6 Parallel and Series–Parallel Configurations

The methods applied in series configurations can be extended to the analysis of parallel and series-parallel configurations. For each area of application, simply match the sequential series of steps applied to series diode configurations.

Example: Determine V_0, I_1, I_{D_1} and I_{D_2} for the parallel diode configuration as shown in figure.



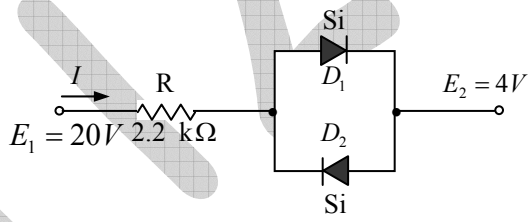
Solution: For the applied voltage the "pressure" of the source is to establish a current through each diode in the same direction as shown in figure below. Since the resulting current direction matches that of the arrow in each diode symbol and the applied voltage is greater than 0.7 V, both diodes are in the "on" state. The voltage across parallel elements is always the same and $V_0 = 0.7\ V$.



The current $I_1 = \dfrac{V_R}{R} = \dfrac{E - V_D}{R} = \dfrac{10V - 0.7V}{0.33k\Omega} = 28.18mA$

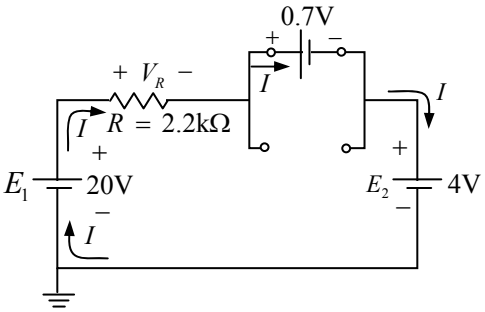
Assuming diodes of similar characteristics, $I_{D_1} = I_{D_2} = \dfrac{I_1}{2} = \dfrac{28.18mA}{2} = 14.09mA$

Example: Determine the current I for the network shown in figure below.



Solution:

Redrawing the network as shown in figure reveals that the resulting current direction is such as to turn on diode D_1 and turn off diode D_2. The resulting current I is then



$$I = \frac{E_1 - E_2 - V_D}{R} = \frac{20V - 4V - 0.7V}{2.2k\Omega} \cong 6.95mA$$

2.7 Rectifiers

2.7.1 Half-Wave Rectification

This circuit will generate a waveform v_o that will have an average value of particular use in the ac-to-dc conversion process.

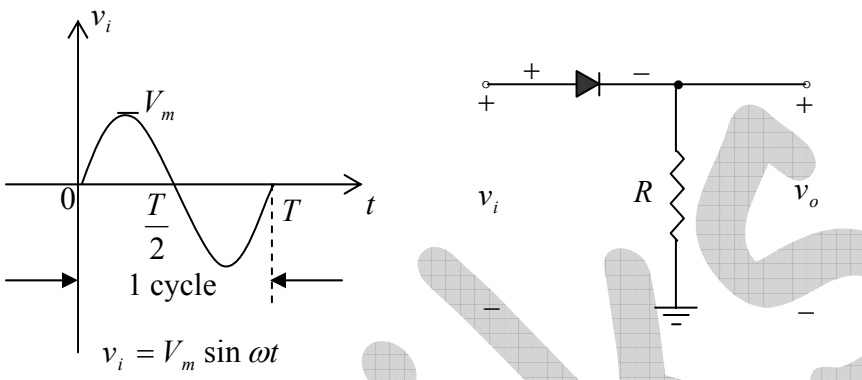


Figure 2.14: Half-Wave Rectification.

During the interval $t = 0 \rightarrow T/2$ the polarity of the applied voltage is such as to establish "pressure" in the direction indicated and turn on the diode with the polarity appearing above the diode. Substituting the short-circuit equivalence for the ideal diode will result in the equivalent circuit shown in figure 2.15, where it is fairly obvious that the output signal is an exact replica of the applied signal. The two terminals defining the output voltage are connected directly to the applied signal via the short-circuit equivalence of the diode.

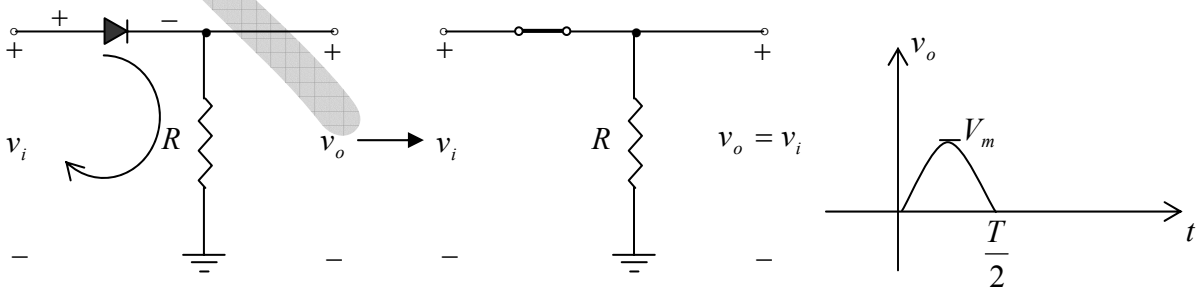


Figure 2.15: Conduction region ($0 \rightarrow T/2$).

For the period $t = T/2 \rightarrow T$, the polarity of input v_i is shown in figure 2.16 and the resulting polarity across the ideal diode produces an "off" state with an open circuit equivalent. The result is the absence of a path for charge to flow and $v_0 = 0V$ for the period $T/2 \rightarrow T$.

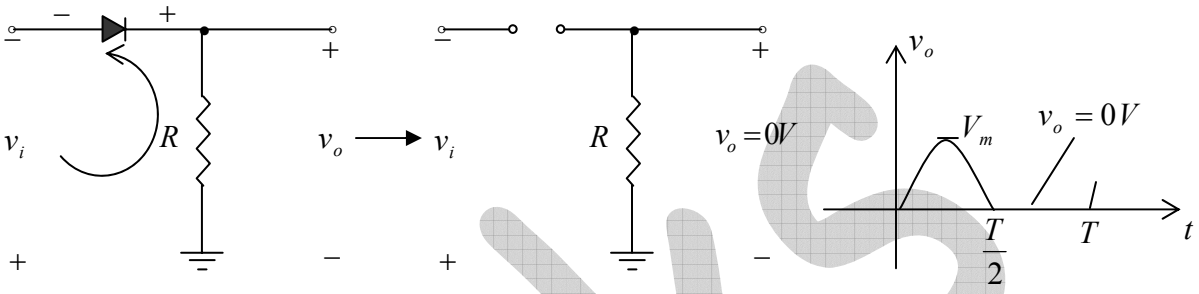


Figure 2.16: Nonconduction region ($T/2 \rightarrow T$).

The process of removing one-half the input signal to establish a dc level is aptly called half-wave rectification.

The input v_i and the output v_0 were sketched together in figure 2.17 for comparison purposes. The output signal v_0 now has a net positive area above the axis over a full period and an average value determined by $V_{dc} = \frac{V_m}{\pi} = 0.318 V_m$

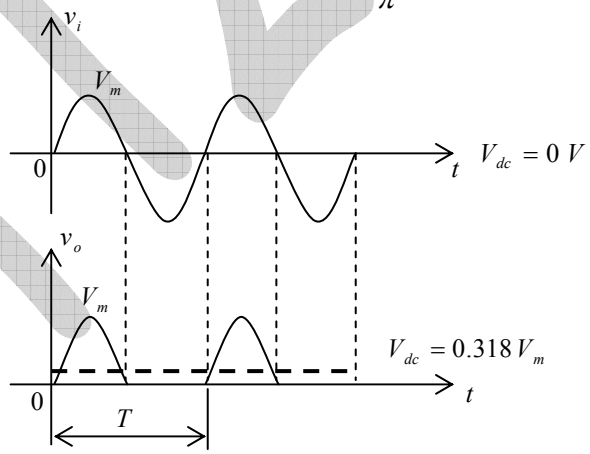


Figure 2.17: Half-wave rectified signal.

The process of removing one-half the input signal to establish a dc level is aptly called *half-wave rectification.*

NOTE: The effect of using a silicon diode with $V_\gamma = 0.7V$ is demonstrated in figure 2.18 for the forward-bias region. The applied signal must now be at least $0.7V$ before the diode can turn "on." For levels of v_i less than $0.7V$ the diode is still in an open-circuit state and $v_o = 0V$. When conducting, the difference between v_o and v_i is a fixed level of $V_\gamma = 0.7V$ and $v_0 = v_i - V_\gamma$. The net effect is a reduction in area above the axis, which naturally reduces the resulting dc voltage level. For situations where $V_m >> V_\gamma$ equation given below can be applied to determine the average value with a relatively high level of accuracy. $V_{dc} \cong 0.318(V_m - V_\gamma)$

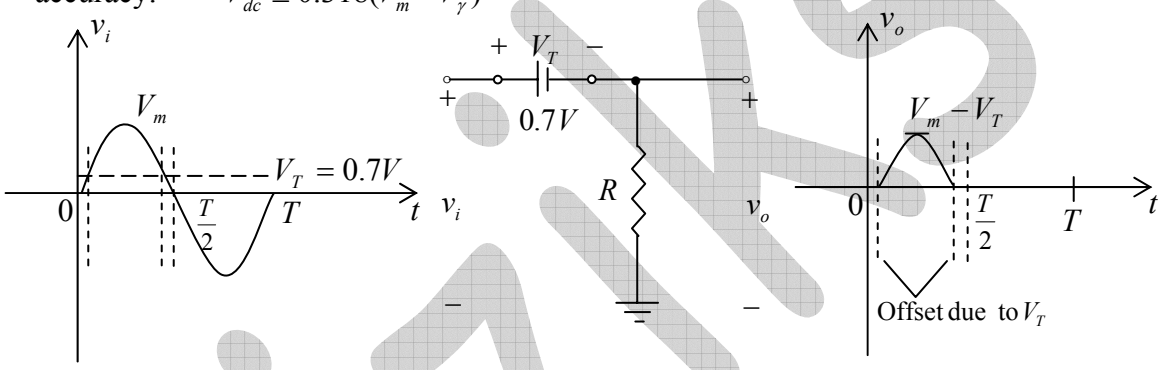


Figure 2.18: Effect of V_γ on half-wave rectified signal.

Peak Inverse Voltage (PIV) or Peak Reverse Voltage (PRV)

It is the voltage rating that must not be exceeded in the reverse-bias region. The required PIV rating for the half-wave rectifier can be determined from figure 2.19, which displays the reverse-biased diode with maximum applied voltage. Applying Kirchhoff's voltage law, it is fairly obvious that the PIV rating of the diode must equal or exceed the peak value of the applied voltage. Therefore, PIV rating $\geq V_m$

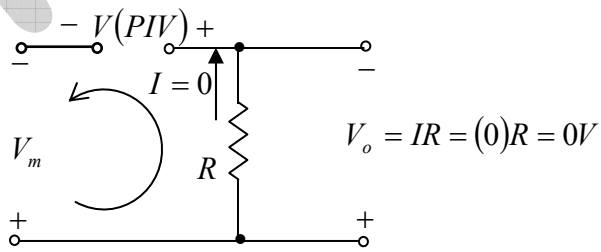


Figure 2.19: Determining the required *PIV* rating for the half-wave rectifier.

2.7.2 Full Wave Rectification

(i) Bridge Network

The dc level obtained from a sinusoidal input can be improved 100% using a process called *full-wave rectification.* The most familiar network for performing such function appears in figure 2.20 with its four diodes in a bridge configuration.

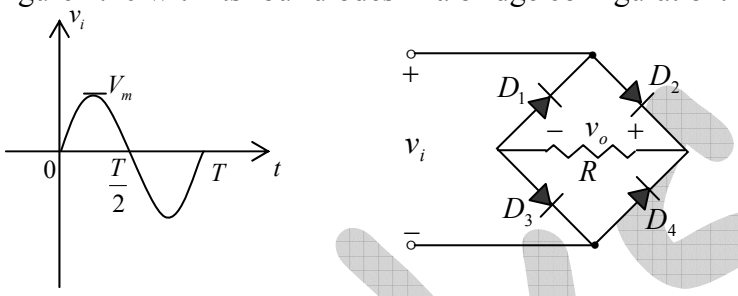


Figure 2.20: Full-wave bridge rectifier.

During period $t = 0 \rightarrow T/2$ the polarity of the input is shown in figure 2.21. The resulting polarities across the ideal diodes are shown to reveal that D_2 and D_3 are conducting while D_1 and D_4 are in the "off" state.

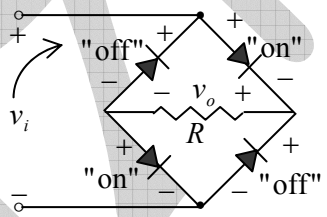


Figure 2.21: FWR for the period $0 \rightarrow T/2$ of the input voltage v_i.

The net result is the configuration of figure 2.22, with its indicated current and polarity across R. Since the diodes are ideal the load voltage $v_0 = v_i$.

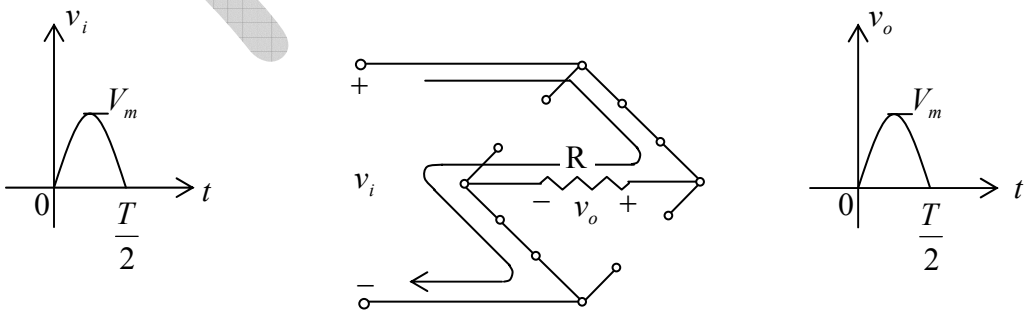


Figure 2.22: Conduction path for the positive region of v_i.

For the negative region of the input the conducting diodes are D_1 and D_4, resulting in the configuration of figure 2.23. The important result is that the polarity across the load resistor R is the same as during positive half cycle.

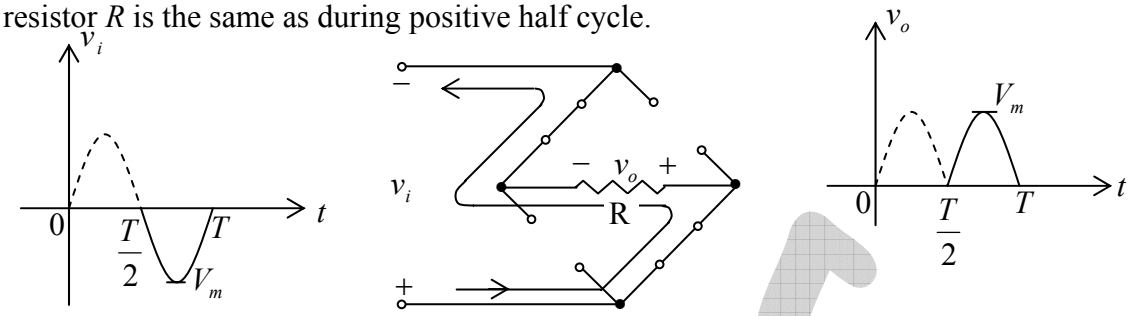


Figure 2.23: Conduction path for the negative region of v_i.

Over one full cycle the input and output voltages will appear as shown in figure 2.24.

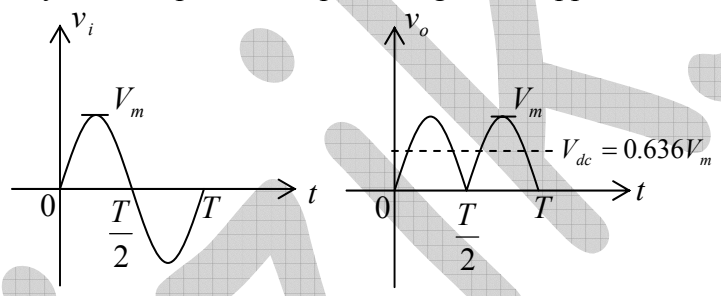


Figure 2.24: Input and output waveforms for a full-wave rectifier.

Since the area above the axis for one full cycle is now twice that obtained for half-wave system, the dc level has also been doubled and $V_{dc} = \frac{2V_m}{\pi} = 0.636V_m$.

NOTE: If silicon rather than ideal diodes are employed as shown in figure 2.25, an application of Kirchhoff's voltage law around the conduction path would result in

$$-v_i + V_\gamma + v_o + V_\gamma = 0 \text{ And } v_o = v_i - 2V_\gamma$$

The peak value of the output voltage v_0 is therefore $V_{0_{max}} = V_m - 2V_\gamma$

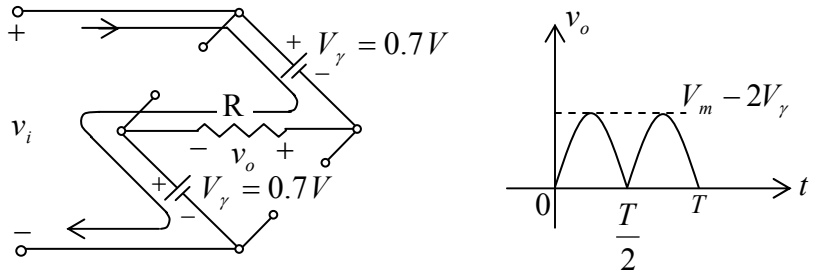


Figure 2.25: Determining $V_{o_{max}}$ for silicon diodes in the bridge configuration.

For situations where $V_m >> 2V_\gamma$ equation given below can be applied to determine the average value with a relatively high level of accuracy.

$$V_{dc} \cong 0.636(V_m - 2V_\gamma)$$

PIV

The required PIV of each diode (ideal) can be determined from figure 2.26 obtained at the peak of the positive region of the input signal. For the indicated loop the maximum voltage across R is V_m and the PIV rating is defined by $\text{PIV} \geq V_m$

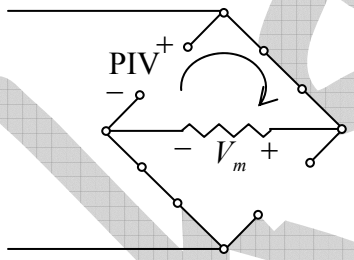


Figure 2.26: Determining the required PIV rating for the bridge configurations.

(ii) Center-Tapped transformer

A second popular full-wave rectifier appears in figure 2.27 with only two diodes but requiring a center-tapped (CT) transformer to establish the input signals across each section of the secondary of the transformer.

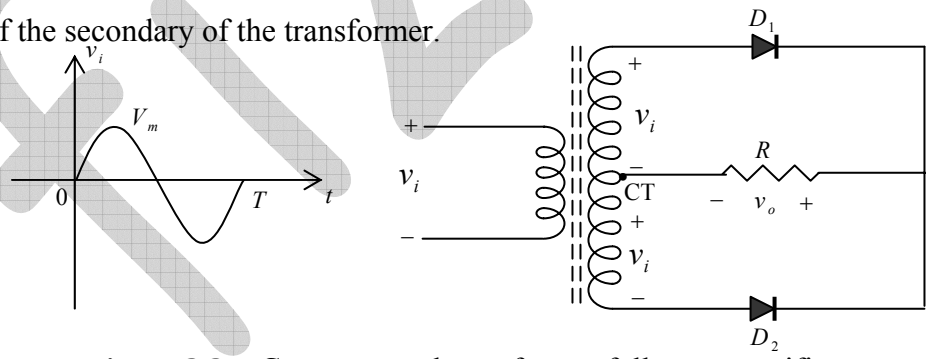


Figure 2.27: Centre-tapped transformer full-wave rectifier.

During the positive portion of v_i applied to the primary of the transformer, the network will appear as shown in figure 2.28. D_1 assumes the short-circuit equivalent and D_2 the open circuit equivalent as detrmined by the secondary voltages and the resulting current directions.The output voltage v_0 appears as shown in figure 2.28.

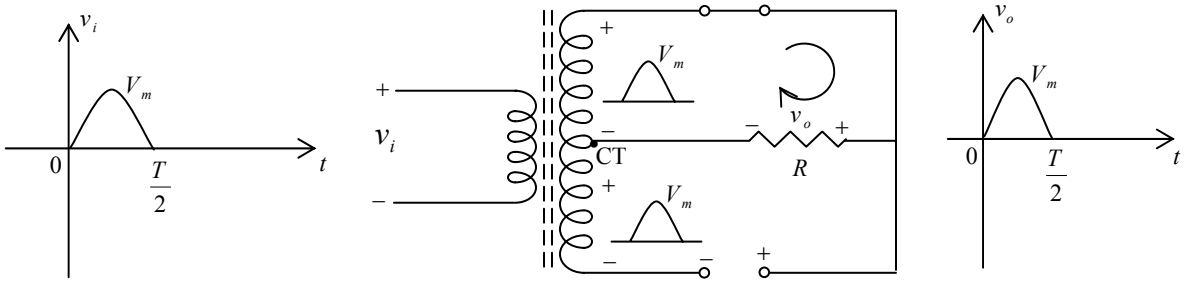


Figure 2.28: Network conditions for the positive region of v_i.

During the negative portion of the input the network appears as shown in figure 2.29, reversing the roles of the diodes but maintaining the same polarity for the voltage across the load resistor R. The net effect is the same output as that appearing in positive half cycle with the same dc levels.

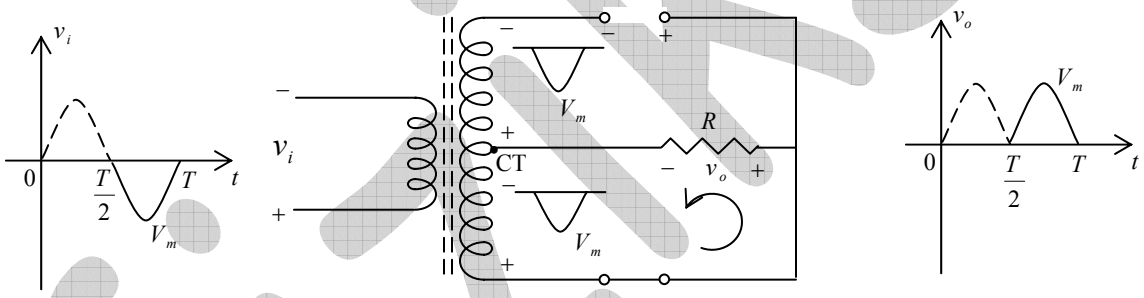


Figure 2.29: Network conditions for the negative region of v_i.

PIV: The network of figure 2.30 will help us determine the net PIV for each diode for this full-wave rectifier. Inserting the maximum voltage for the secondary voltage and V_m as established by the adjoining loop will result in

$$\text{PIV} = V_{\text{secondary}} + V_R = V_m + V_m \quad \text{and} \quad \text{PIV} \geq 2V_m .$$

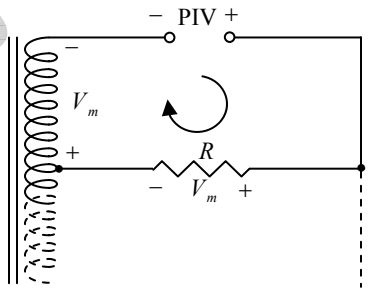


Figure 2.30: Determining the PIV level for the diodes.

2.8 Clippers

There are a variety of diode networks called clippers that have the ability to "clip" off a portion of the input signal without distorting the remaining part of the alternating waveform. The half-wave rectifier is an example of the simplest form of diode clipper– one resistor and diode. Depending on the orientation of the diode, the positive or negative region of the input signal is "clipped" off.

2.8.1 Series Clippers (Positive and Negative)

The response of the series configuration of figure 2.31(a) to a variety of alternating waveforms is provided in figure 2.31(b). Although first introduced as a half-wave rectifier (for sinusoidal waveforms), there are no boundaries on the type of signals that can be applied to a clipper.

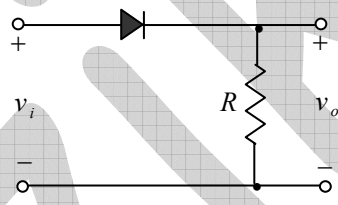


Figure 2.31(a): Series clipper.

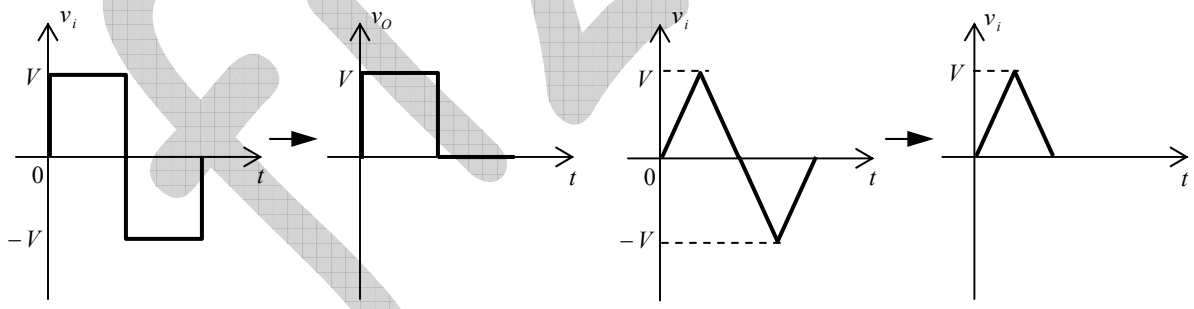


Figure 2.31(b): Input and output waveforms.

The addition of dc supply such as shown in figure 2.32 can have a pronounced effect on the output of a clipper. Our initial discussion will be limited to ideal diodes and the effect of V_γ will be discussed later.

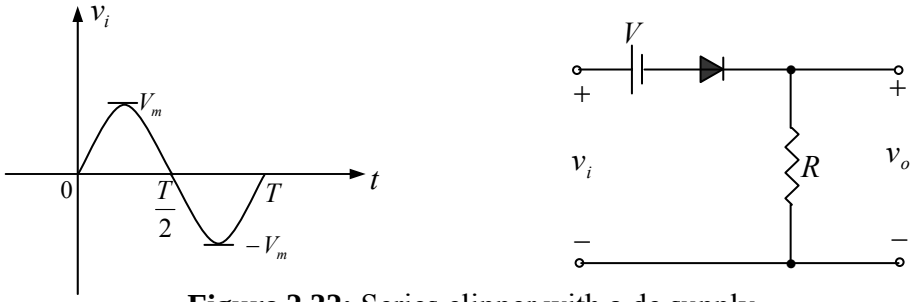


Figure 2.32: Series clipper with a dc supply.

There is no general procedure for analyzing above networks, but there are a few thoughts to keep in mind before analyzing these circuits.

1. *Make a mental sketch of the response of the network based on the direction of the diode and the applied voltage levels.*

The direction of the diode suggests that the signal v_i must be positive to turn it on. The dc supply further requires that the voltage v_i be greater than V volts to turn the diode on. The negative region of the input signal is "pressuring" the diode into the "off" state, supported further by the dc supply. In general, therefore, we can be quite sure that the diode is an open circuit ("off" state) for the negative region of the input signal.

2. *Determine the applied voltage (transition voltage) that will cause change in state for the diode.*

For the ideal diode the transition between states will occur at the point on the characteristic where $v_d = 0V$ and $i_d = 0A$. Applying the condition $i_d = 0A$ at $v_d = 0$ will result in the configuration of figure 2.33, where it is recognized that the level of v_i that will cause a transition in state is: $v_i = V$.

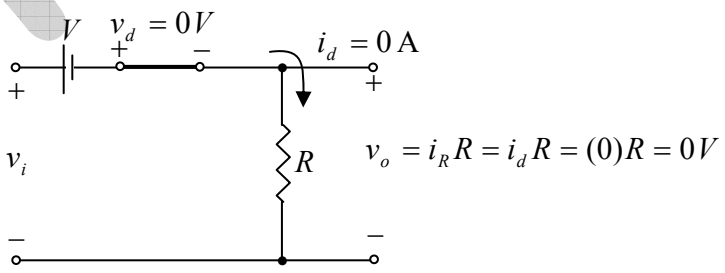


Figure 2.33: Determining the transition level.

For an input voltage greater than V volts the diode is in the short-circuit state, while for input voltage less than V volts it is in the open-circuit or "off" state.

3. *Be continually aware of the defined terminals and polarity of v_o.*

When the diode is in the short-circuit state such as shown in figure 2.34, the output voltage v_o can be determined by applying Kirchhoff's voltage law in the clockwise direction. $-v_i + V + v_o = 0$ and $v_o = v_i - V$.

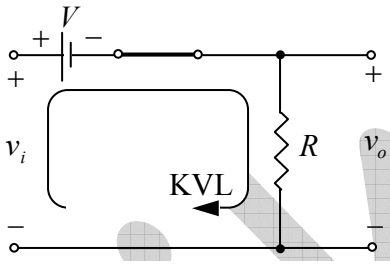


Figure 2.34: Determining v_o.

4. *It can be helpful to sketch the input signal above the output and determine the output at instantaneous values of the input.*

It is then possible that the output voltage can be sketched from the resulting data points of v_o as demonstrated in figure 2.35.

Keep in mind that at an instantaneous value of v_i the input can be treated as a dc supply of that value and the corresponding dc value (the instantaneous value) of the output determined. For instance, at $v_i = V_m$ the network to be analyzed appears in figure 2.36.

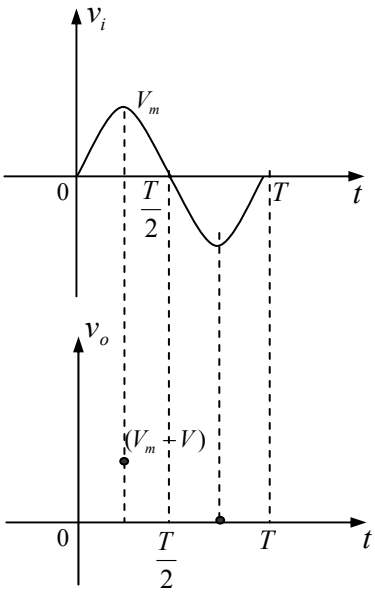


Figure 2.35: Determining levels of v_o.

For $V_m > V$ the diode is in the short-circuit state and $v_o = V_m - V$. At $v_i = V$ the diodes change state and at $v_i = -V_m$, $v_o = 0V$ and the complete curve for v_o can be sketched as shown in figure 2.37.

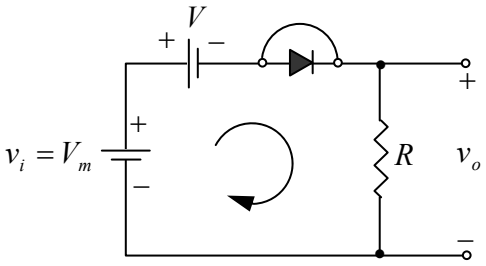


Figure 2.36: Determining v_o when $v_i = V_m$.

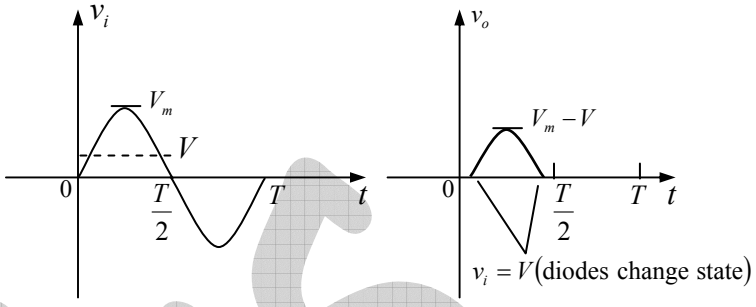
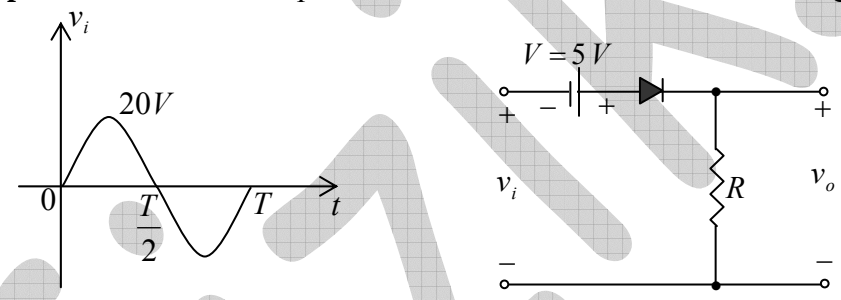


Figure 2.37: Sketching v_o.

Example: Determine the output waveform for the network shown in figure below.



Solution: The diode will be in "on" state for the positive region of v_i, especially when we note the aiding effect of V= 5V. The network will then appear as shown in figure (a) and $v_0 = v_i + 5V$. Substituting $i_d = 0$ at $v_d = 0$ for the transition levels, we obtain the network of figure (b) and $v_i = -5V$.

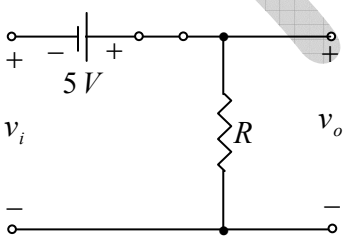


Figure (a): v_0 with diode in the "on" state.

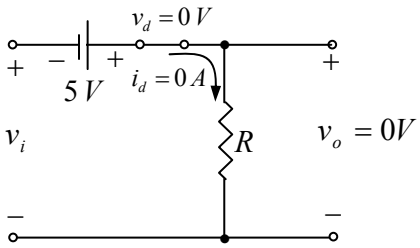


Figure (b): Determining the transition level.

For v_i more negative than $-5V$ the diode will enter its open-circuit state, while for voltages more positive than $-5V$ the diode is in the short-circuit state. The input and output voltage appear in figure shown below.

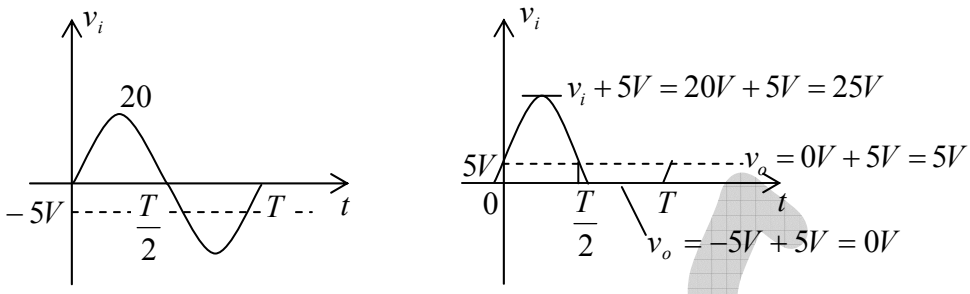
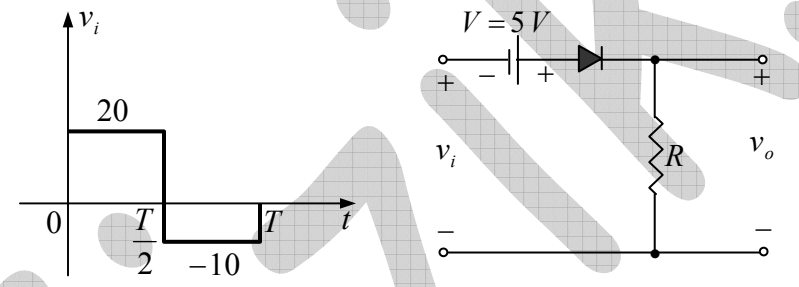


Figure: Sketching v_0.

Example: Repeat above example for the square-wave input shown in figure below.



Solution: For $v_i = 20V\ (0 \to T/2)$ the network of figure (a) will result. The diode is in the short-circuit state and $v_o = 20V + 5V = 25V$. For $v_i = -10V$ the network of figure (b) will result, placing the diode in the "off" state and $v_o = i_R R = 0 \times R = 0V$. The resulting output voltage appears in figure (c).

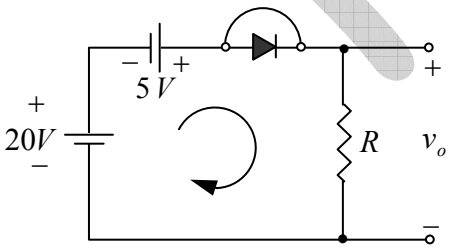


Figure (a): v_o at $v_i = 20V$

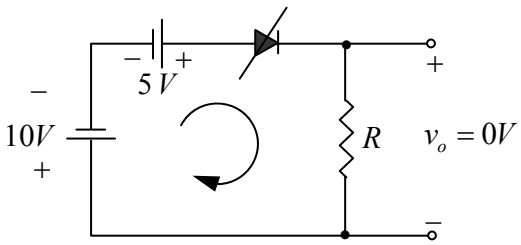
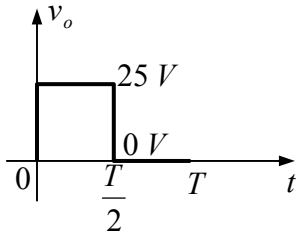


Figure (b): v_o at $v_i = -10V$



Figure(c): sketching v_o

2.8.2 Parallel Clippers (Positive and Negative)

The network of figure 2.38 is the simplest of parallel diode configurations with the output for the same inputs. The analysis of parallel configurations is very similar to that applied to series configurations, as demonstrated in the next example.

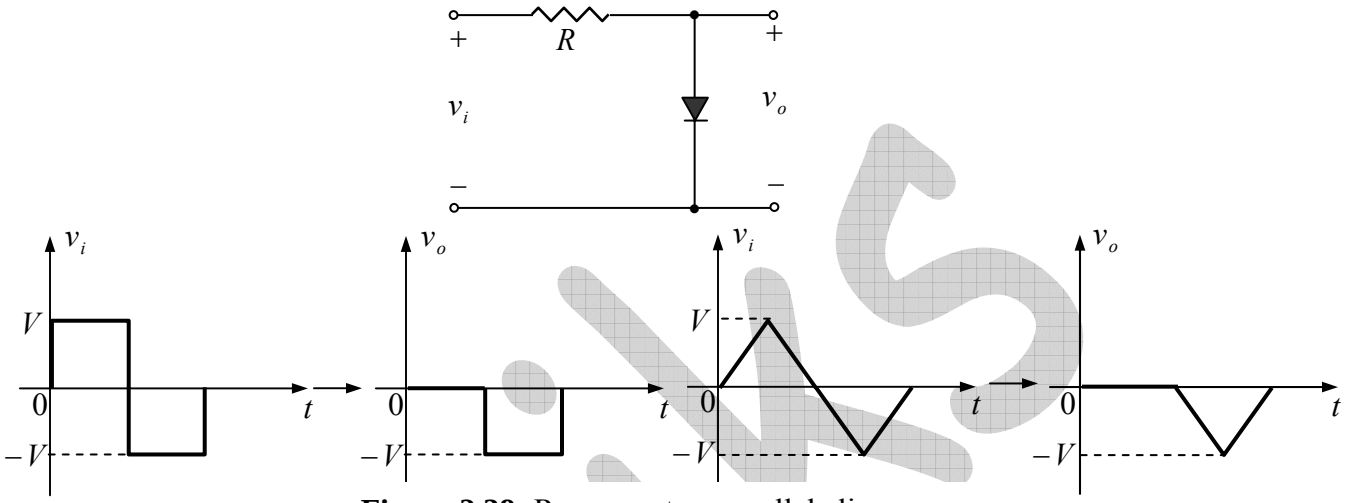
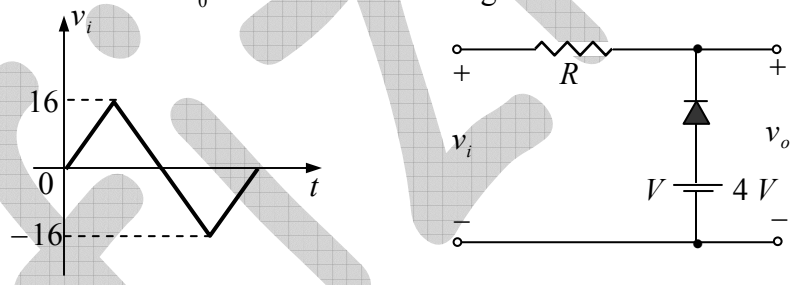


Figure 2.38: Response to a parallel clipper.

Example: Determine v_0 for the network of figure shown below.



Solution: The polarity of the dc supply and the direction of the diode strongly suggest that the diode will be in the "on" state for the negative region of the input signal. For this region the network will appear as shown in figure below where the defined terminals for v_0 require that $v_0 = V = 4V$.

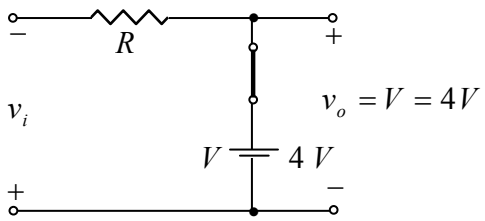


Figure: v_0 for the negative region of v_i.

The transition state can be determined from figure shown below, where the condition $i_d = 0A$ at $v_d = 0V$ has been imposed. The result is v_i (transition) $= V = 4\ V$.

Since the dc supply is obviously "pressuring" the diode to stay in the short-circuit state, the input voltage must be greater than 4 V for the diode to be in the "off" state. Any input voltage less than $4V$ will result in a short-circuited diode.

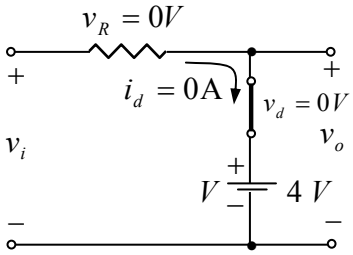


Figure: Determining the transition level.

For the open-circuit state the network will appear as shown in figure below, where $v_0 = v_i$. Completing the sketch of v_0 results in the waveform of figure shown below.

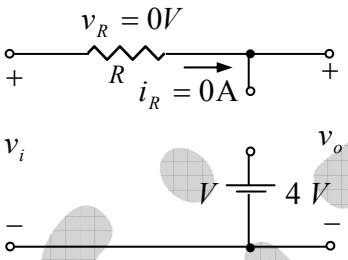


Figure: Determining v_0 for the open state of the diode

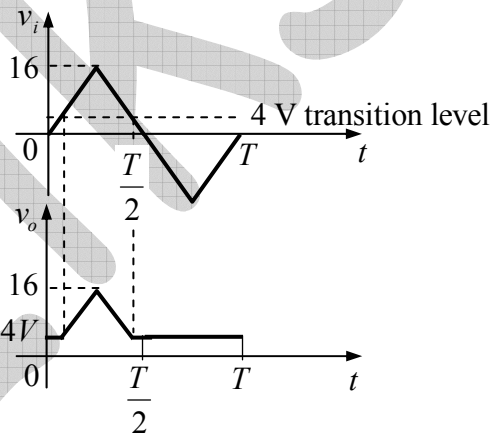


Figure: Sketching v_0.

To examine the effect of V_γ on the output voltage, the next example will specify a silicon diode rather than an ideal diode equivalent.

Example: Repeat above example using a silicon diode with $V_\gamma = 0.7\ V$.

Solution: The transition voltage can first be determined by applying the condition $i_d = 0A$ at $v_d = 0.7V$ and obtaining the network of figure shown below. Applying Kirchhoff's voltage law around the output loop in the clockwise direction, we find that

$$-v_i - V_\gamma + V = 0$$

and $v_i = V - V_\gamma = 4 - 0.7 = 3.3V$

$$v_R = i_d R = 0 \times R = 0V$$

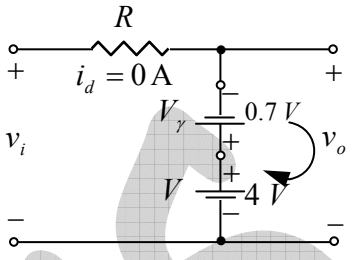


Figure: Determining the transition level.

For input voltages greater than $3.3V$, the diode will be an open circuit and $v_0 = v_i$. For input voltages of less than 3.3 V, the diode will be in the "on" state and the network of figure shown below results, where $v_0 = 3.3V$.

Note that the only effect of V_γ was to drop the transition level to 3.3 V from 4 V.

There is no question that including the effects of V_γ was to drop the transition level to 3.3 V from $4V$.

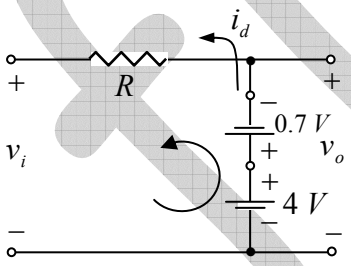


Figure: Determining v_o for the diode of figure 2.83 in the "on" state.

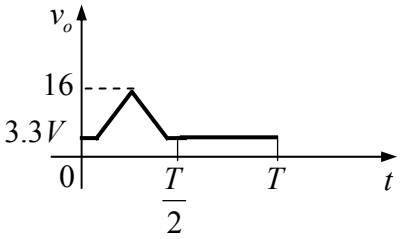


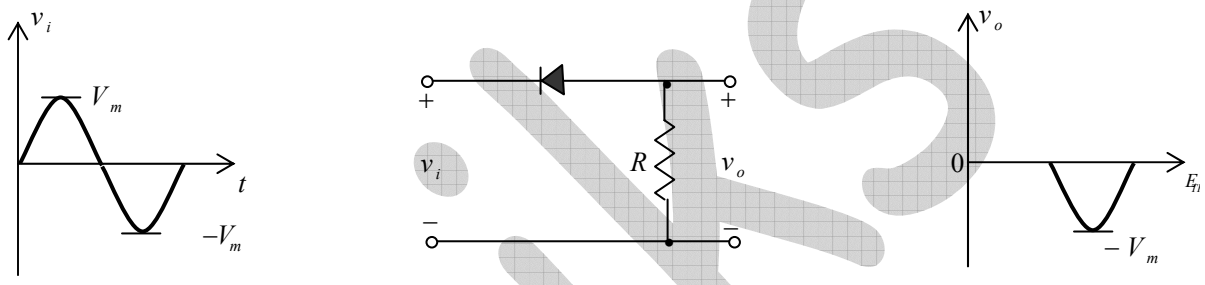
Figure: Sketching v_0.

Summary Clippers Networks

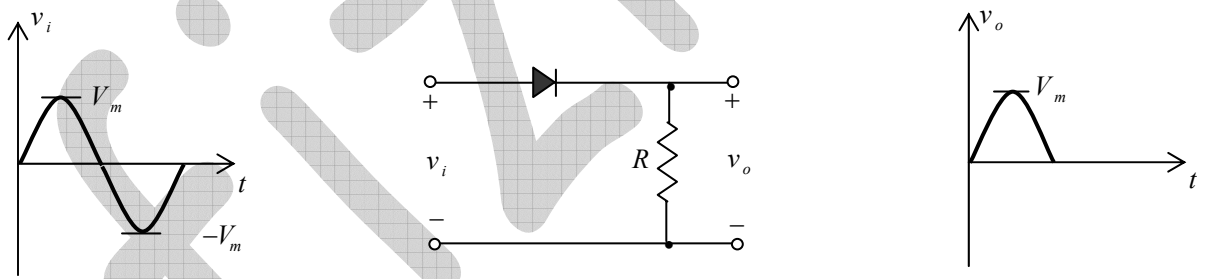
A variety of series and parallel clippers with the resulting output for the sinusoidal input are provided in figures shown below. In particular, note the response of the last configuration, with its ability to clip off a positive and a negative section as determined by the magnitude of the dc supplies.

Simple Series Clippers (Ideal Diodes)

Positive:

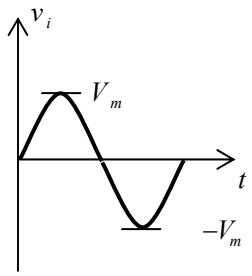


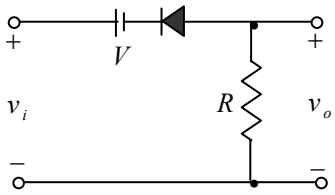
Negative:

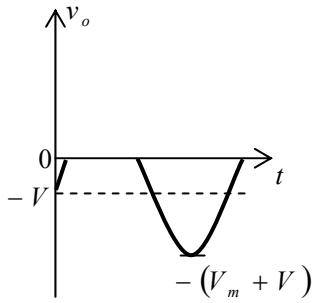


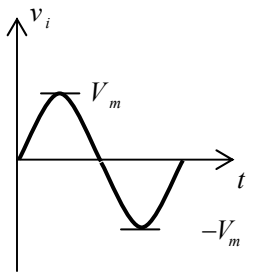
Biased Series Clippers (Ideal Diodes)

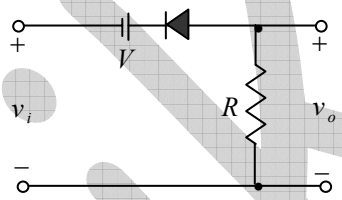
Positive:

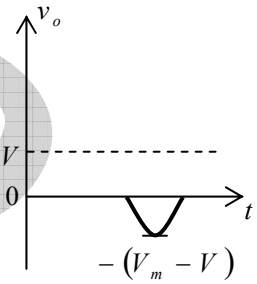




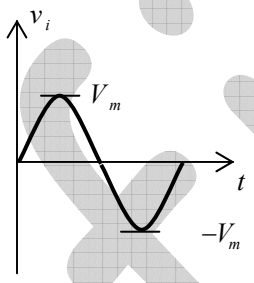


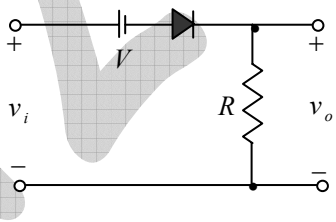


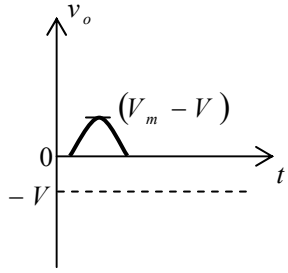


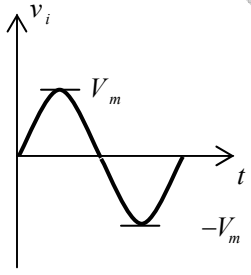


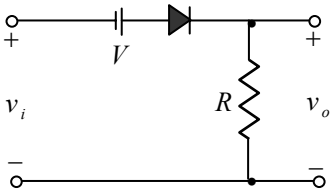
Negative

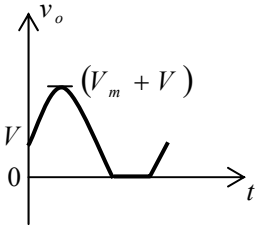






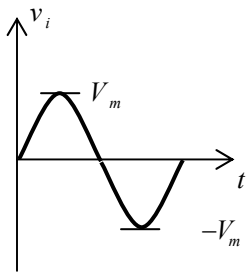


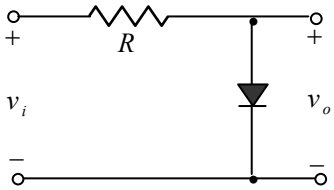


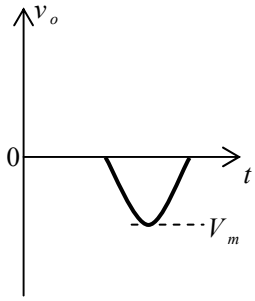


Simple Parallel Clippers (Ideal Diodes)

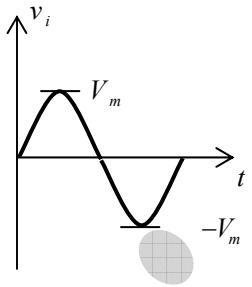
Positive:

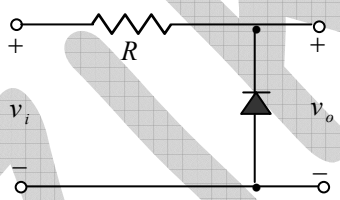


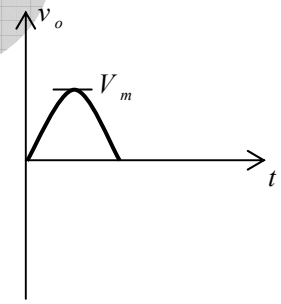




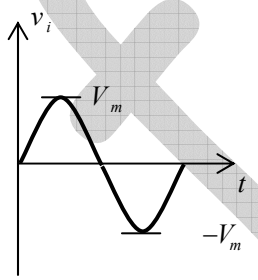
Negative:

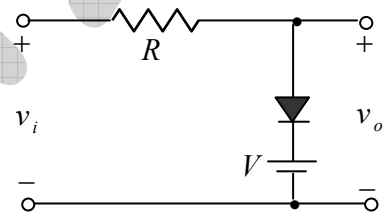


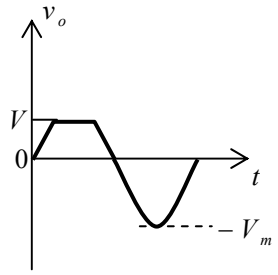


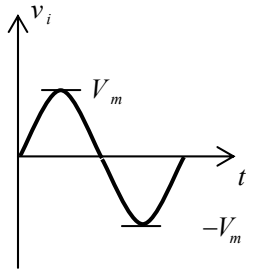


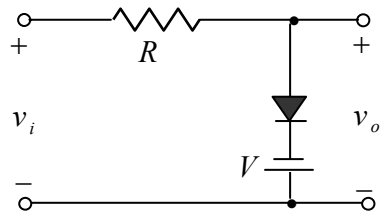
Biased Parallel Clippers (Ideal Diodes)

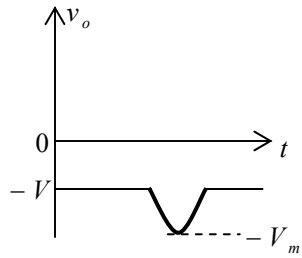




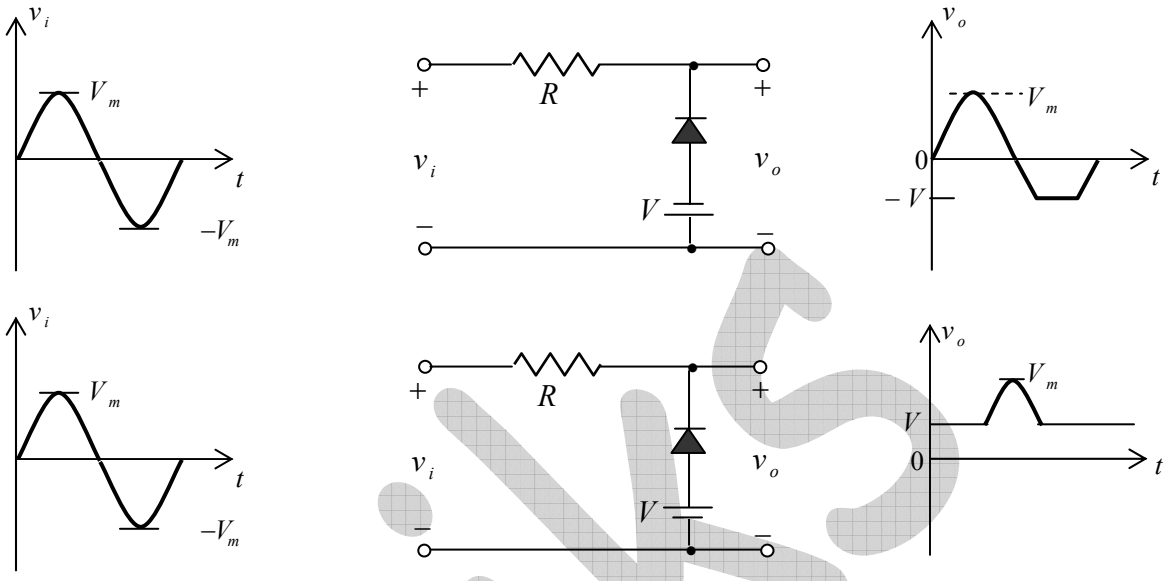




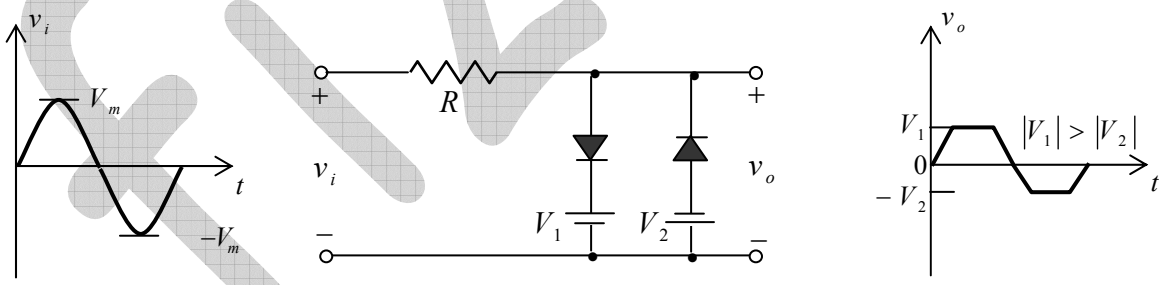




Negative:



Miscellaneous



2.9 Clampers

The *clamping* network is one that will "clamp" a signal to a different dc level. The network must have a capacitor, a diode, and a resistive element, but it can also employ an independent dc supply to introduce an additional shift. The magnitude of R and C must be chosen such that the ***time constant τ = RC is large enough*** to ensure that the voltage across the capacitor does not discharge significantly during the interval the diode is non-conducting. Throughout the analysis we will assume that for all practical purposes the capacitor will fully charge or discharge in five time constants.

The network of figure 2.39 will clamp the input signal to the zero level (for ideal diodes).

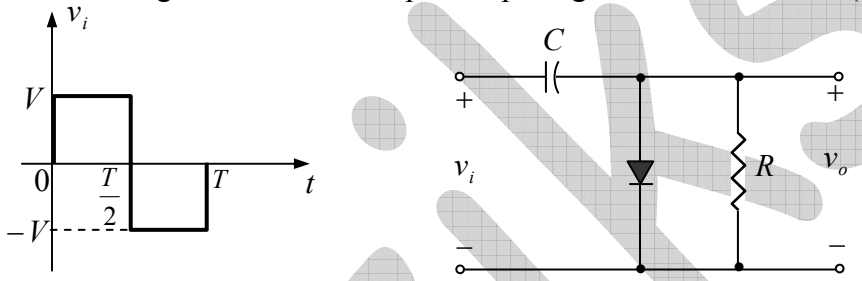


Figure 2.39: Clamper.

During the interval $0 \rightarrow T/2$ the network will appear as shown in figure 2.40, with the diode in the "on" state effectively "shorting out" the effect of resistor R. The resulting RC time constant is so small (R is determined by the inherent resistance of the network) that the capacitor will charge to V volts very quickly.

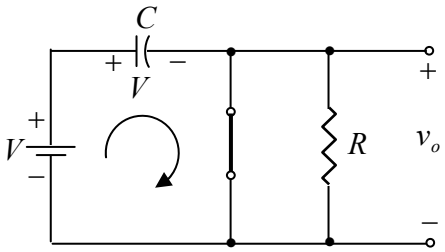


Figure 2.40: Diode "on" and the capacitor charging to V volts.

During this interval the output voltage is directly across the short circuit and $v_o = 0$ V.

When the input switches to $-V$ state, the network will appear as shown in figure 2.41, with the open-circuit equivalent for the diode determined by the applied signal and stored voltage across the capacitor-both "pressuring" current through the diode from cathode to anode.

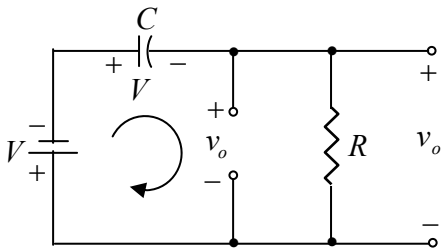


Figure 2.41: Determining v_o with the diode "off".

Now that R is back in the network the time constant determined by the RC product is sufficiently large to establish a discharge period 5τ much greater than the period $T/2 \rightarrow T$ and it can be assumed on an approximate basis that the capacitor holds onto all its charge and therefore voltage (since $V = Q/C$) during this period.

Applying Kirchhoff's voltage law around the input loop will result in

$$+V + V + v_o = 0 \text{ and } \quad v_o = -2V.$$

The negative sign resulting from the fact that the polarity of $2V$ is opposite to the polarity defined for v_o. The resulting output waveform appears in figure 2.42 with the input signal. The output signal is clamped to 0 V for the interval 0 to $T/2$ but maintains the same total swing ($2V$) as the input.

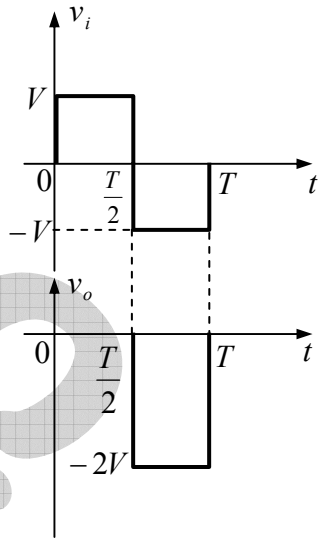


Figure 2.42: Sketching v_o.

For a clamping network:

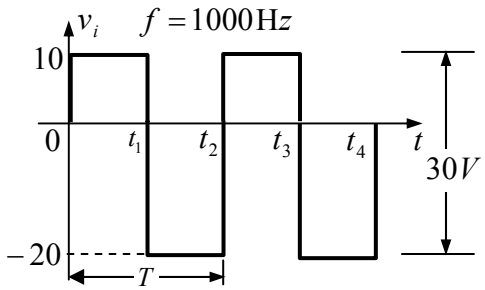
The total swing of the output is equal to the total swing of the input signal.

This fact is an excellent checking tool for the result obtained.

In general, the following steps may be helpful when analyzing clamping networks:

1. Start the analysis of clamping networks by considering that part of the input signal that will forward bias the diode.
2. During the period that the diode is in the "on" state, assume that the capacitor will charge up instantaneously to a voltage level determined by the network.
3. Assume that during the period when the diode is in the "off" state the capacitor will hold on to its established voltage level.
4. Throughout the analysis maintain a continual awareness of the location and reference polarity for v_o to ensure that the proper levels for v_o are obtained.
5. Keep in mind the general rule that the total swing of the total output must match the swing of the input signal.

Example: Determine v_o for the network of figure (a) for the input indicated.



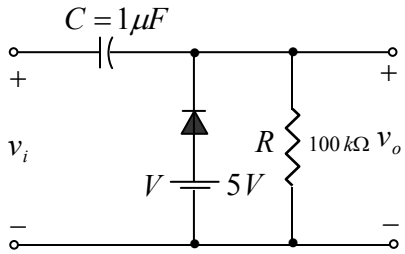


Figure (a): Applied signal and network.

Solution: Note that the frequency is 1000 *Hz*, resulting in a period of 1 *ms* and an interval of 0.5 *ms* between levels. The analysis will begin with the period $t_1 \rightarrow t_2$ of the input signal since the diode is in it short-circuit state as recommended by comment 1.

For this interval the network will appear as shown in figure (b). The output is across R, but it is also directly across the $5V$ battery if you follow the direct connection between the defined terminals for v_o and the battery terminals. The result is $v_o = 5V$ for this interval.

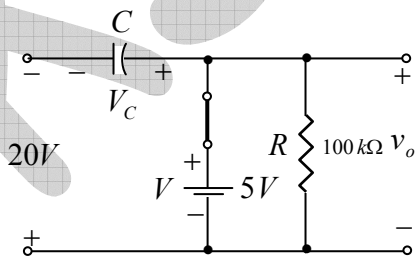


Figure (b): Determining v_o and V_C with the diode in the "on" state.

Applying Kirchhoff's voltage law around the input loop will result in

$$+20V - V_C + 5V = 0 \text{ and } V_C = 25\ V.$$

The capacitor will therefore, charge up to $25V$, as stated in comment 2. In this case the resistor R is not shorted out by the diode but a Thevenin's equivalent circuit of that portion of the network which includes the battery and the resistor will result in $R_{Th} = 0\,\Omega$ with $E_{Th} = \text{V} = 5\ V$.

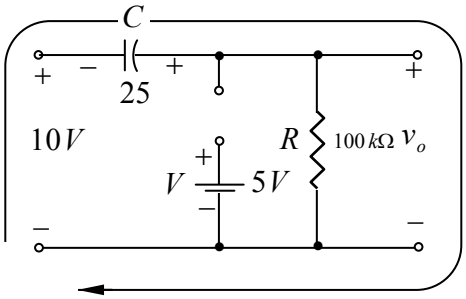


Figure (c): Determining v_o with the diode in the "off" state.

For the period $t_2 \rightarrow t_3$ the network will appear as shown in figure (c). The open-circuit equivalent for the diode will remove the $5V$ battery from having any effect on v_o, and applying Kirchhoff's voltage law around the outside loop of the network will result in

$$-10\ V - 25\ V - v_o = 0 \quad \text{and} \quad v_o = 35\ V.$$

The time constant of the discharging network of figure (c) is determined by the product RC and has the magnitude: $\tau = RC = (100\ k\Omega)\ (0.1\ \mu F) = 0.01\ \text{s} = 10\ ms$

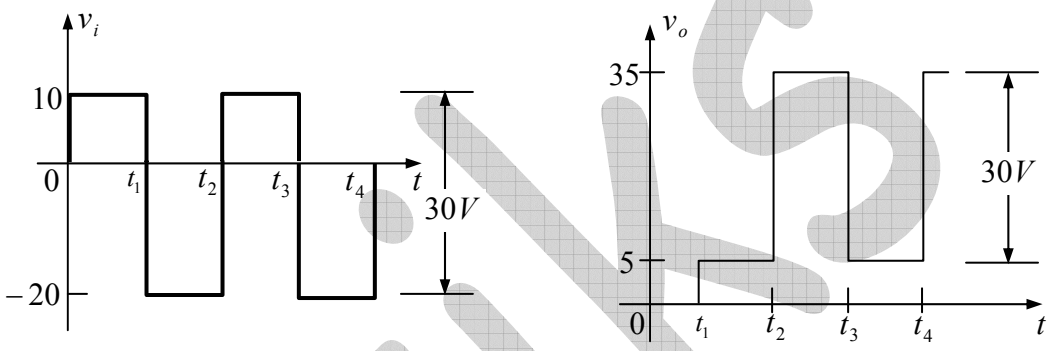


Figure (d): v_i and v_o for the clamper.

The total discharge time is therefore $5\tau = 5\ (10\ \text{ms}) = 50\ \text{ms}$.

Since the interval $t_2 \rightarrow t_3$ will only last for 0.5 ms, it certainly a good approximation that the capacitor will hold its voltage during the discharge period between pulses of the input signals. The resulting output appears in figure (d) with the input signal. Note that the output swing of 30 V matches the input swing as noted in step 5.

Example: Repeat above example using a silicon diode with $V_\gamma = 0.7$ V.

Solution: For the short-circuit state the network now takes form of figure (a) and v_o can be determined by Kirchhoff's voltage law in the output section.

$$-5V+0.7\ V+ v_o = 0$$

and $$v_o = 5V - 0.7\ V = 4.3\ V$$

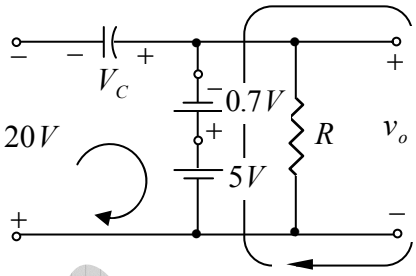


Figure (a): Determining v_o and V_C with the diode in the "on" state

For the input section Kirchhoff's voltage law will result in

$$+20V-V_C-0.7V+5V = 0$$

and $V_C = 25\ V - 0.7\ V = 24.3\ V$.

For the period $t_2 \rightarrow t_3$ the network will now appear as in figure (b) with the only change being the voltage across the capacitor. Applying Kirchhoff's voltage law yields

$$-10V-24.3\ V+ v_o = 0 \text{ and } v_o = 34.3\ V.$$

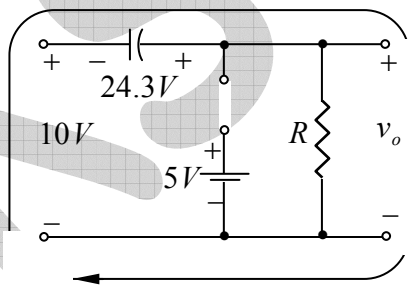


Figure (b): Determining v_o with the diode in the open state.

The resulting output appears in figure (c) verifying the statements that the input and output swings are the same.

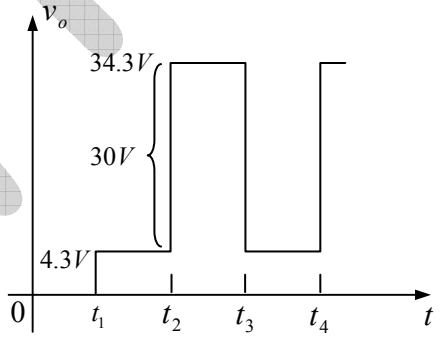
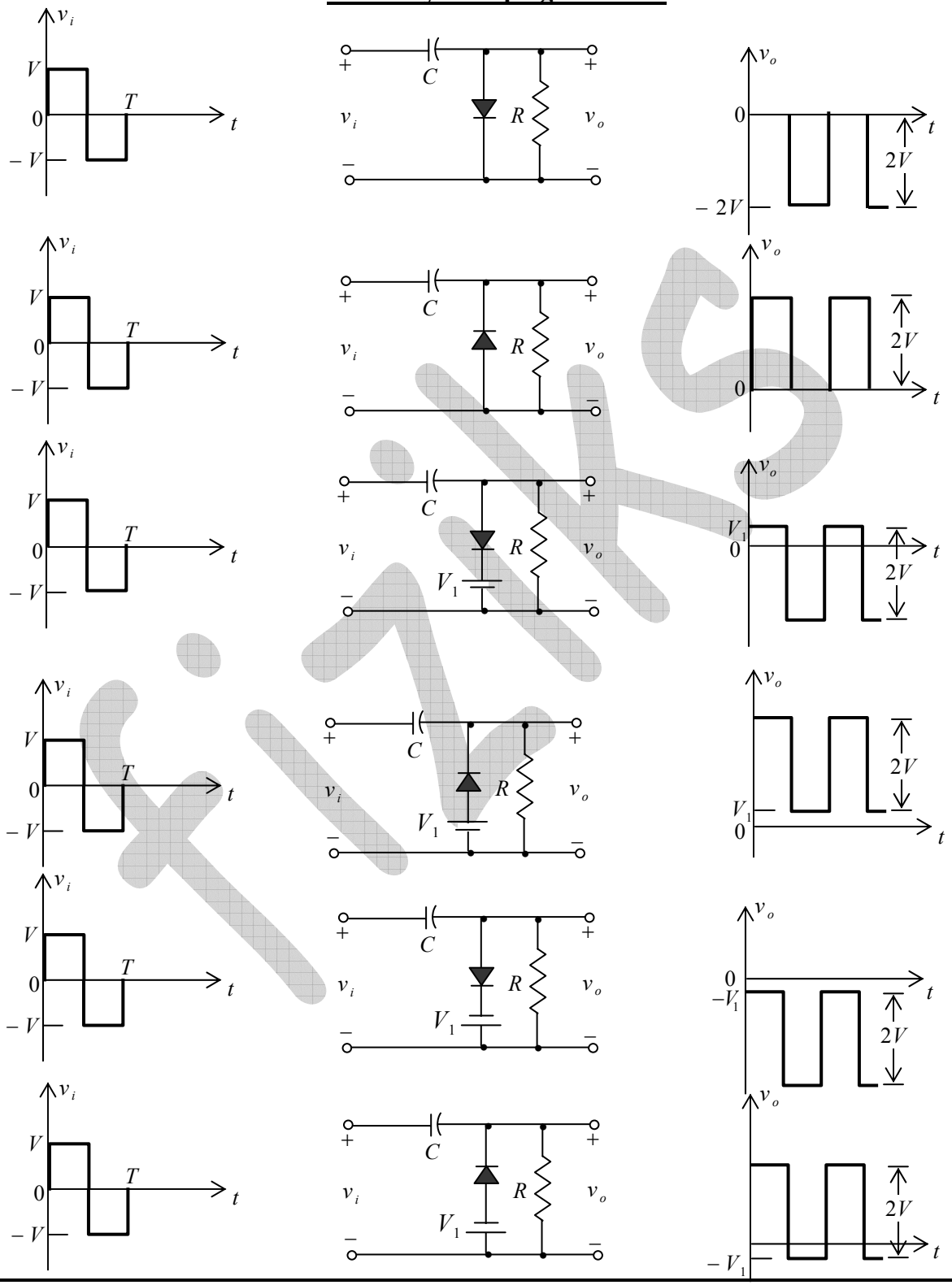


Figure (c): Sketching v_o.

Summary Clamping Networks



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2.10 Zener Diode

The analysis of networks employing Zener diodes is quite similar to that applied to the analysis of semiconductor diodes. First the state of the diode must be determined followed by a substitution of the appropriate model and a determination of the other unknown quantities of the network. The Zener model to be employed for the "on" state and for the "off" state as defined by a voltage less than V_Z but greater than $0V$ with the polarity indicated as shown in figure 2.43.

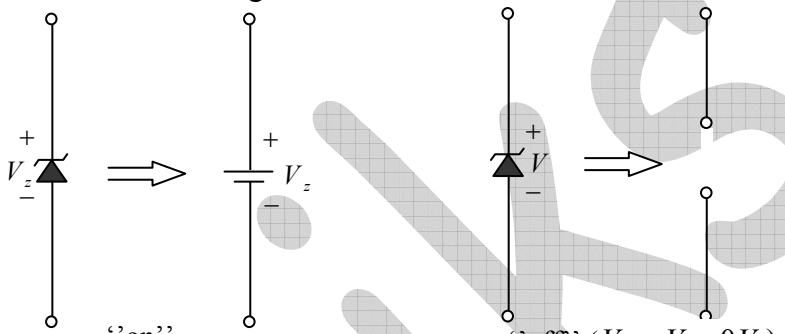


Figure 2.43: Zener diode equivalents for the (a) "on" and (b) "off" states.

2.10 .1 Case-I (V_i and R_L fixed)

The simplest of Zener diode networks appears as shown in figure 2.44.

The applied dc voltage is fixed, as is the load resistor. The analysis can fundamentally be broken down into two steps.

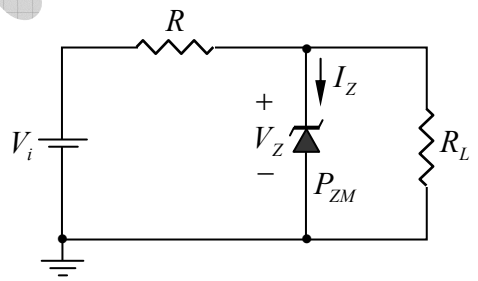


Figure 2.44: Basic Zener regulator.

1. *Determine the state of the Zener diode by removing it from the network and calculating the voltage across the resulting open circuit.*

Open circuit voltage $V = V_L = \dfrac{R_L V_i}{R + R_L}$.

If $V \geq V_Z$, the Zener is "on" and if $V < V_Z$, the Zener is "off."

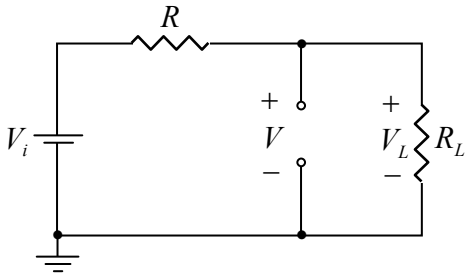


Figure 2.45: Determining the state of the Zener diode.

2. *Substitute the appropriate equivalent circuit and solve for the desired unknowns.*

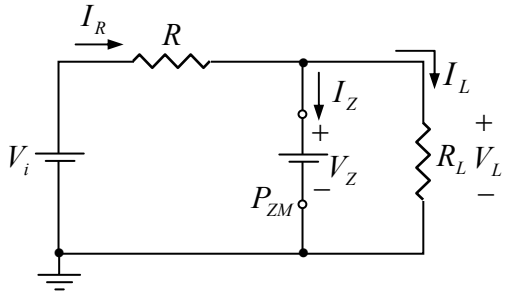


Figure 2.46: Substituting the Zener equivalent for the "on" situation.

When the Zener is "on" then $V_L = V_Z$ and $I_R = I_Z + I_L \Rightarrow I_Z = I_R - I_L$

where $I_L = \dfrac{V_L}{R_L}$ and $I_R = \dfrac{V_R}{R} = \dfrac{V_i - V_L}{R}$.

Power dissipated by the Zener diode $P_Z = V_Z I_Z \le P_{ZM}$.

Note: If the Zener diode is in the "on" state, the voltage across the diode is not V volts. When the system is turned on the Zener diode will turn "on" as soon as the voltage across the Zener diode is V_Z volts . It will then "lock in" at this level and never reach the higher level of V volts.

Zener diodes are most frequently used in *regulator* network or a *reference* voltage. A simple regulator designed to maintain a fixed voltage across the load R_L . For values of applied voltage greater than required to turn the Zener diode "on," the voltage across the load will be maintained at V_Z volts . If the Zener diode is employed as a reference voltage, it will provide a level for comparison against other voltages.

Example: (a) For the Zener diode network of figure (a), determine V_L, V_R, I_Z and P_Z.

(b) Repeat part (a) with $R_L = 3k\Omega$.

Solution: (a) Following the suggested procedure the network is redrawn as shown in figure (b).

Open circuit voltage

$$V = \frac{R_L V_i}{R + R_L} = \frac{1.2k\Omega(16\ V)}{1\ k\Omega + 1.2\ k\Omega} = 8.73\ V$$

Since $V = 8.73\ V$ is less than $V_Z = 10\ V$ the diode is in the "off" state. Substituting the open-circuit equivalent we will find that

$V_L = V = 8.73\ V$

$V_R = V_i - V_L = 16\ V - 8.73\ V = 7.27\ V$

$I_Z = 0\ A$ and $P_Z = V_Z I_Z = 0\ W$.

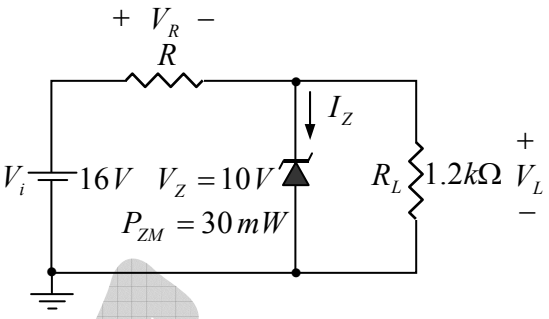


Figure (a)

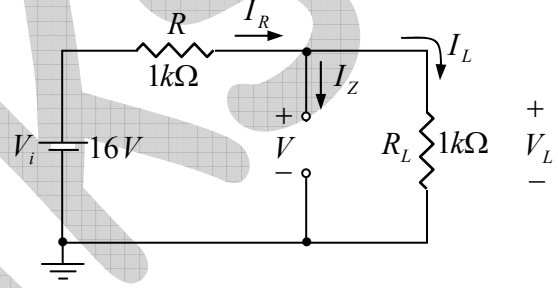


Figure (b): Determining V for the regulator of figure (a).

(b) Open circuit voltage

$$V = \frac{R_L V_i}{R + R_L} = \frac{3\ k\Omega(16\ V)}{1\ k\Omega + 3\ k\Omega} = 12\ V$$

Since $V = 12\ V$ is greater than $V_Z = 10\ V$, the diode is in the "on" state and the network of figure (c) will result.

$V_L = V_Z = 10\ V$ and $V_R = V_i - V_L = 6\ V$

with $I_L = \frac{V_L}{R_L} = \frac{10V}{3k\Omega} = 3.33\ mA$

and $I_R = \frac{V_R}{R} = \frac{6V}{1k\Omega} = 6\,mA$.

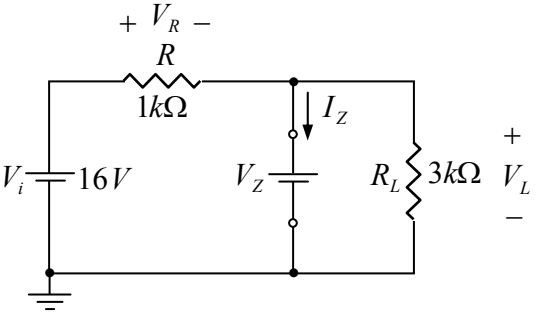


Figure (c): Network of figure (a) in the "on" state.

$I_Z = I_R - I_L = 6\ mA - 3.33\ mA = 2.67\ mA$. The power dissipated, $P_Z = V_Z I_Z = 26.7\ mW$ which is less than the specified $P_{ZM} = 30\ mW$.

2.10.2 Case-II (fixed V_i and variable R_L)

Due to the offset voltage V_Z, there is a specific range of resistor values (and therefore load current) which will ensure that the Zener is in the "on" state. Too small a load resistance R_L will result in a voltage V_L across the load resistor less than V_Z and the Zener device will be in the "off" state.

To determine the minimum load resistance that will turn the Zener diode on, simply calculate the value of R_L that will result in a load voltage $V_L = V_Z$.

That is, $V_L = V_Z = \dfrac{R_L V_i}{R_L + R}$. Solving for R_L we have $R_{L_{\min}} = \dfrac{RV_Z}{V_i - V_Z}$.

Any load resistance value greater than the R_L obtained from above equation will ensure that the Zener diode is in the "on" sate and the diode can be replaced by its V_Z source equivalent.

This condition establishes the minimum R_L, but in turn specifies the maximum I_L as

$$I_{L_{\max}} = \frac{V_L}{R_L} = \frac{V_Z}{R_{L_{\min}}}.$$

Once the diode is in the "on" state, the voltage across R remains fixed at

$$V_R = V_i - V_Z$$

and I_R remains fixed at $I_R = \dfrac{V_R}{R}$

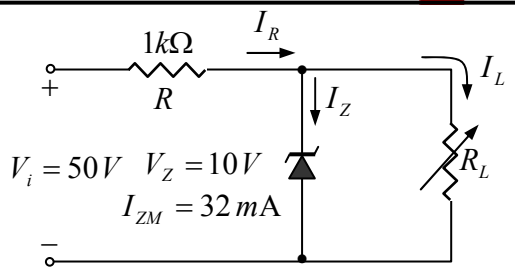
The Zener current $I_Z = I_R - I_L$ resulting in a minimum I_Z when I_L is a maximum and a maximum I_Z when I_L is a minimum value since I_R is constant.

Since I_Z is limited to I_{ZM} as provided on the data sheet, it does affect the range of R_L and therefore I_L. Substituting I_{ZM} for I_Z establishes the minimum I_L as

$$I_{L_{\min}} = I_R - I_{ZM}$$

and the maximum load resistance as $R_{L_{\max}} = \dfrac{V_Z}{I_{L_{\min}}}$.

Example: (a) For the network of figure shown determines the range of R_L and I_L that will result in V_{R_L} being maintained at 10 V.



(b) Determine the maximum voltage rating of the diode.

Solution:

(a) To determine the value of R_L that will turn the Zener diode on, apply equation

$$R_{L_{\min}} = \frac{RV_Z}{V_i - V_Z} = \frac{(1\,k\Omega)(10V)}{50V - 10V} = \frac{10\,k\Omega}{40} = 250\,\Omega$$

The voltage across the resistor R is then determined by equation

$$V_R = V_i - V_Z = 50\ V - 10\ V = 40\ V \quad \text{and} \quad I_R = \frac{V_R}{R} = \frac{40\ \ V}{1\ \ k\Omega} = 40\ mA.$$

The minimum level of I_L is then determined by equation

$$I_{L_{\min}} = I_R - I_{ZM} = 40\ m\text{A} - 32\ m\text{A} = 8\ mA$$

with $$R_{L_{\max}} = \frac{V_Z}{I_{L_{\min}}} = \frac{10V}{8\,mA} = 1.25\,k\Omega.$$

(b) $P_{\max} = V_Z I_{ZM} = (10\ V)(32\ mA) = 320\ mW$.

2.10.3 Case-III (fixed R_L and variable V_i)

For fixed values of R_L, the voltage V_i must be sufficiently large to turn the Zener diode on. The minimum turn-on voltage $V_i = V_{i_{\min}}$ is determined by

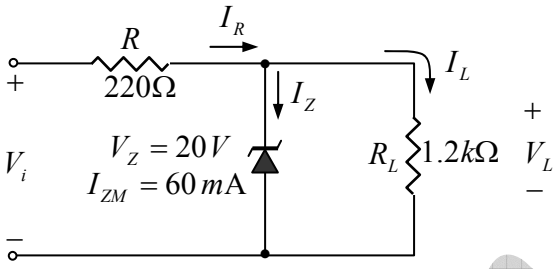
$$V_L = V_Z = \frac{R_L V_i}{R_L + R} \text{ and } V_{i_{\min}} = \frac{(R_L + R)V_Z}{R_L}.$$

The maximum value of V_i is limited by the maximum Zener current I_{ZM}.

Since $I_{ZM} = I_R - I_L$, $\quad I_{R_{\max}} = I_{ZM} + I_L$

Since I_L is fixed at V_Z / R_L and I_{ZM} is the maximum value of I_Z, the maximum V_i is defined by $\quad V_{i_{\max}} = V_{R_{\max}} + V_Z = I_{R_{\max}} R + V_Z$.

Example: Determine the range of values of V_i that will maintain the Zener diode in the "on" state.



Solution:

$$V_{i_{\min}} = \frac{(R_L + R)V_Z}{R_L} = \frac{(1200\,\Omega + 220\Omega + 20V)}{1200\,\Omega} = 23.67V$$

$$I_L = \frac{V_L}{R_L} = \frac{V_Z}{R_L} = \frac{20\ V}{1.2\ k\Omega} = 16.67\ mA$$

$$I_{R_{\max}} = I_{ZM} + I_L = 60\ \ mA + 16.67\ \ mA = 76.67\ mA$$

$$V_{i_{\max}} = I_{R_{\max}} R + V_Z = (76.67\ mA)\,(0.22\ k\Omega) + 20\ V = 16.87\ V + 20\ V = 36.87\ V$$

2.10.4 Zener as a Reference Levels

Two or more reference levels can be established by placing Zener diodes in series as shown in figure 2.47. As long as V_i is greater than the sum of V_{Z_1} and V_{Z_2}, both diodes will be in the "on" state and the three reference voltages will be available.

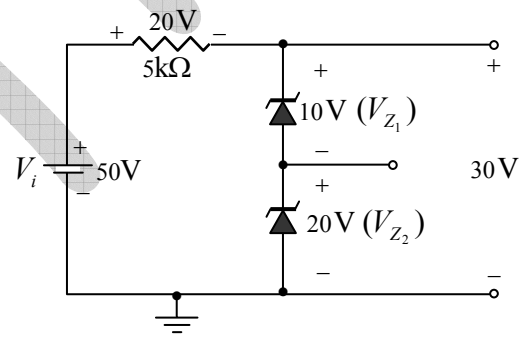


Figure 2.47: Establishing three reference voltage levels.

Two back-to-back Zener can also be used as an ac regulator as shown in figure 2.48. For the sinusoidal signal v_i the circuit will appear as shown in figure 2.49 at the instant $v_i = 10\ V$.

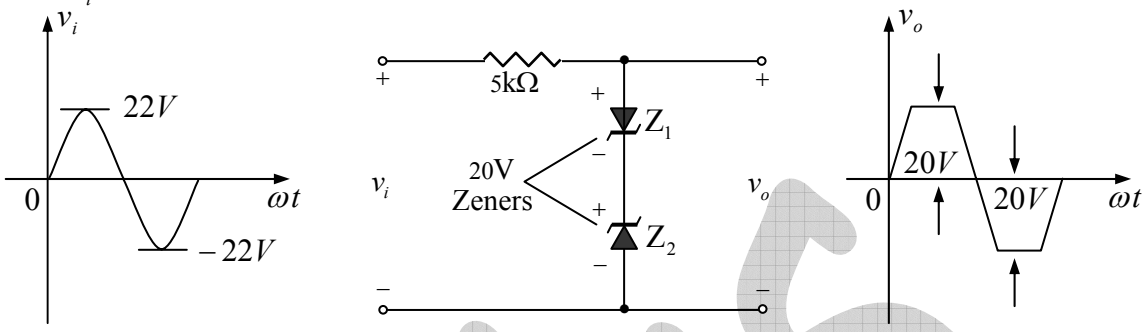


Figure 2.48: 40-V peak-to-peak sinusoidal ac regulator.

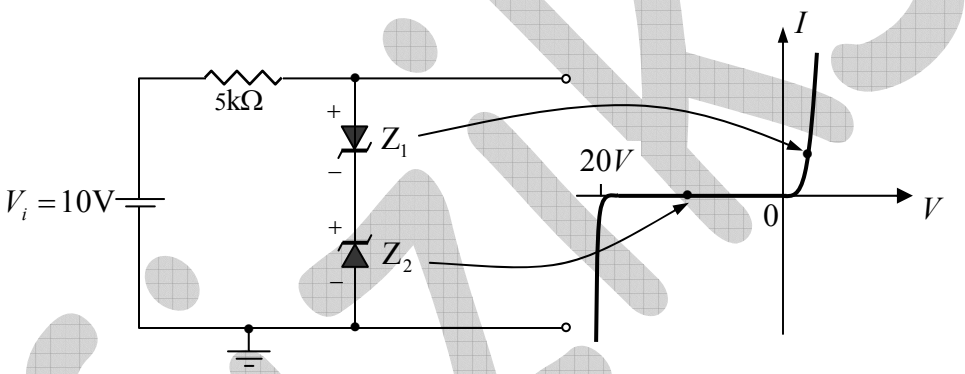


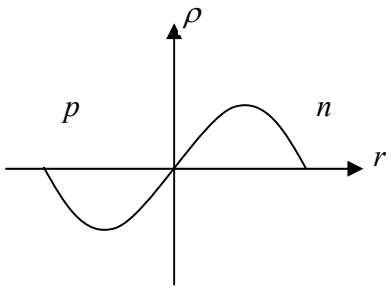
Figure 2.49: circuit operation at $v_i = 10\ V$.

The region of operation for each diode is indicated in the adjoining figure. Note that Z_1 is in a low-impedance region, while the impedance of Z_2 is quite large, corresponding with the open-circuit representation. The result is that $v_o = v_i$ when $v_i = 10\ V$. The input and output will continue to duplicate each other until v_i reaches 20 V. Z_2 will then "turn on" (as a Zener diode) while Z_1 will be in a region of conduction with a resistance level sufficiently small compared to the series 5 $k\Omega$ resistor to be considered a short circuit. The resulting output for the full range of v_i is provided in figure 2.48. Note that the waveform is not purely sinusoidal, but its rms value is lower than that associated with a full 22 V peak signal.

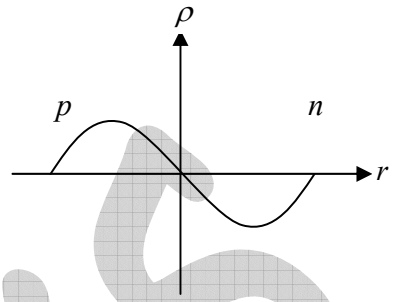
Multiple Choice Questions (MCQ)

Q1. Which graph represents the nature of the curve between charge density ρ and distance r near the depletion region of $p-n$ junction diode?

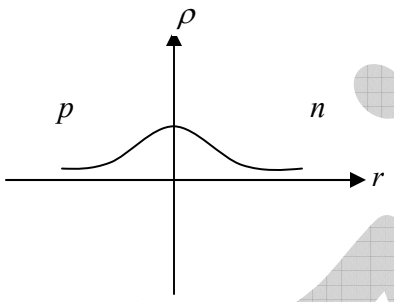
(a)



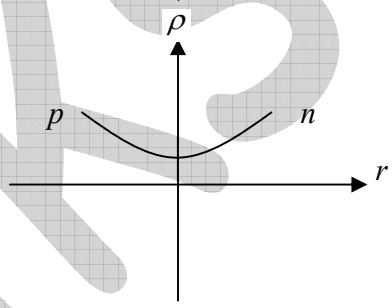
(b)



(c)



(d)



Q2. The junction capacitance of a $p-n$ junction depends on

(a) Doping concentration only

(b) Applied voltage only

(c) Both doping concentration and applied voltage.

(d) Barrier potential only.

Q3. Which of the following statements related to Zener diode is correct?

(a) Its depletion layer is large

(b) Zener breakdown involves collision

(c) High electric field breaks the covalent bonds

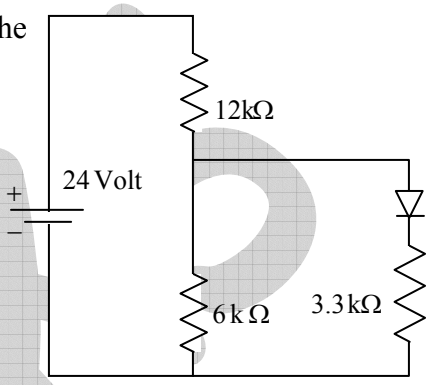
(d) On increasing temperature breakdown voltage increases

Q4. Consider a Ge diode with $N_D = N_A = 8\times10^{14}\,cm^{-3}$ and intrinsic carrier concentration $n_i = 2\times10^{13}\,cm^{-3}$. At room temperature of $300\,K$, the height of the potential barrier under open circuited conditions is

(Boltzmann constant $k = 1.38\times10^{-23}\,J/K$)

(a) $0.1\,V$ (b) $0.2\,V$ (c) $0.3\,V$ (d) $0.4\,V$

Q5. In the following circuit, the voltage drop across the ideal diode in forward bias condition is $0.7\,V$. The current passing through the $6k\Omega$ resistance is



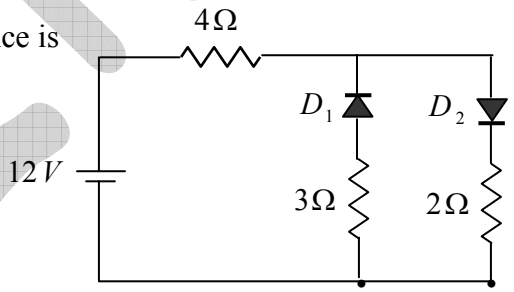
(a) $0.7\,mA$

(b) $1.5\,mA$

(c) $2.0\,mA$

(d) $2.5\,mA$

Q6. The circuit shown in figure has two oppositely connected ideal diodes in parallel, then the current flowing through 4Ω resistance is



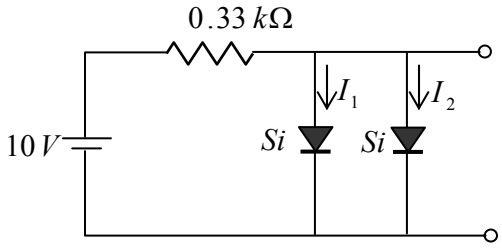
(a) $1.33\,A$

(b) $1.71\,A$

(c) $2.00\,A$

(d) $2.31\,A$

Q7. A parallel diode configuration is shown in figure given below, the value of the diode currents I_1 and I_2 are?



(a) $I_1 = 15\,mA,\ I_2 = 15\,mA,$

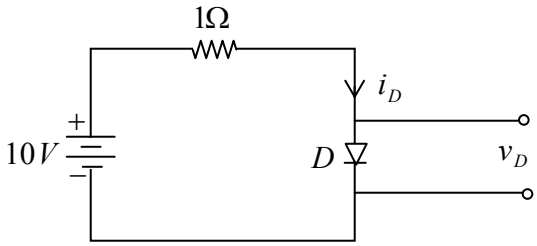
(b) $I_1 = 14\,mA,\ I_2 = 14\,mA,$

(c) $I_1 = 15\,mA,\ I_2 = 14\,mA,$

(d) $I_1 = 14\,mA,\ I_2 = 15\,mA,$

Q8. A diode D as shown in the circuit has an i-v relation that can be approximated by

$$i_D = \begin{cases} v_D^2 + 2v_D, & \text{for } v_D > 0 \\ 0, & \text{for } v_D \le 0 \end{cases}$$



The value of i_D in the circuit is

(a) $\left(-1+\sqrt{11}\right) A$ (b) $8\,A$

(c) $5\,A$ (d) $2\,A$

Q9. Which one of the following figures is the correct representation of a full-wave rectifier circuit consisting of two diodes, a load resistor and a centre-tapped transformer?

Figure 1

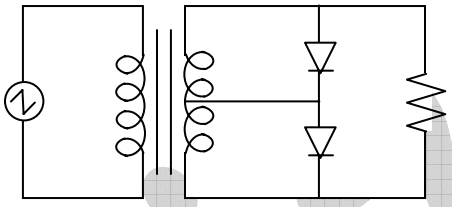


Figure 2

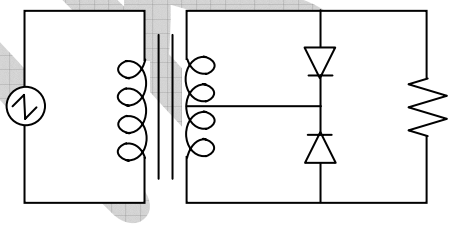


Figure 3

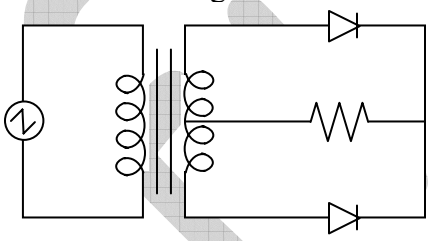
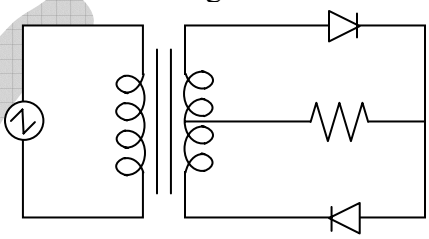


Figure 4



(a) Figure 1 (b) Figure 2 (c) Figure 3 (d) Figure 4

Ans: (c)

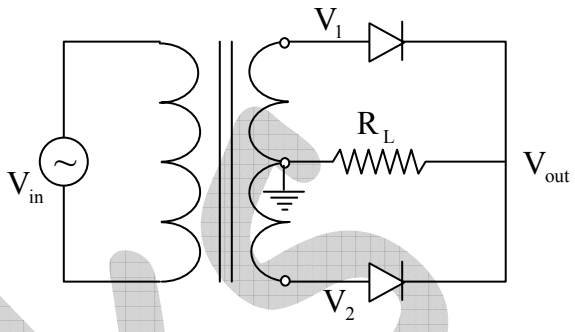
Q10. For the rectifier circuit shown in the figure, the sinusoidal voltage $(V_1$ or $V_2)$ at the output of the transformer has a maximum value of $5\ V$. The load resistance R_L is $1\ k\Omega$. If I_{av} is the average current through the resistor R_L the circuit corresponds to a

(a) Full wave rectifier with $I_{av} = \frac{20}{\pi}\ mA$

(b) Half wave rectifier with $I_{av} = \frac{20}{\pi}\ mA$

(c) Half wave rectifier with $I_{av} = \frac{10}{\pi}\ mA$

(d) Full wave rectifier with $I_{av} = \frac{10}{\pi}\ mA$



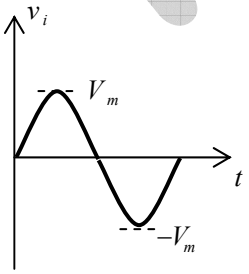
Q11. An a.c. supply of $220\ V$, $50\ Hz$ is applied to a half wave rectifier circuit through a transformer of turn ratio $10:1$. The forward resistance of the diode is $100\ \Omega$ and the load resistance is $2\ k\Omega$, then the output d.c. voltage and the peak inverse voltages are:

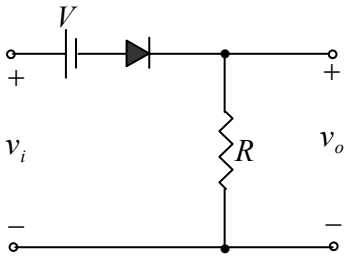
(a) $7V, 7.5V$ (b) $8V, 15.5V$ (c) $9V, 31V$ (d) $10V, 62V$

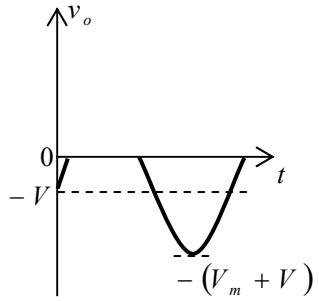
Q12. An a.c. voltage of $220\ V$, $50\ Hz$ is applied to the primary of a $5:1$ step down transformer. The secondary of the transformer is centre taped and connected to a full wave rectifier with a load resistance $500\ \Omega$. Forward resistance of the diode is $50\ \Omega$, then the output d.c. voltage and the peak inverse voltages are:

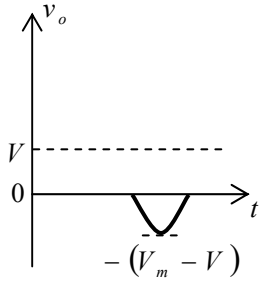
(a) $18V, 124V$ (b) $36V, 124V$ (c) $18V, 62V$ (d) $36V, 62V$

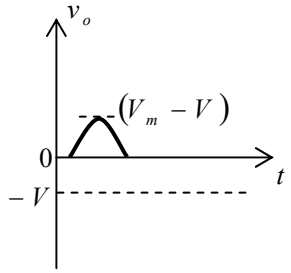
Q13. Identify the correct output waveforms for the circuit given below (assume diodes to be ideal)

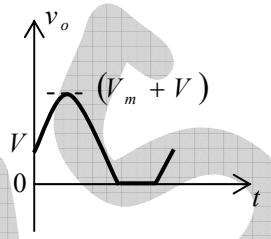




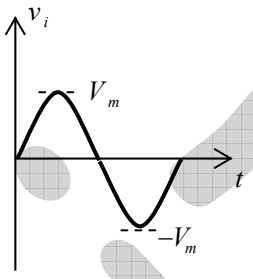
(a) 

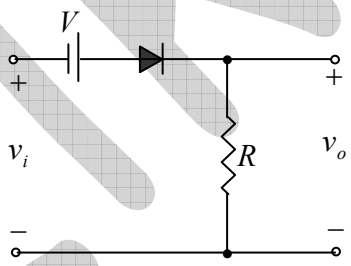
(b) 

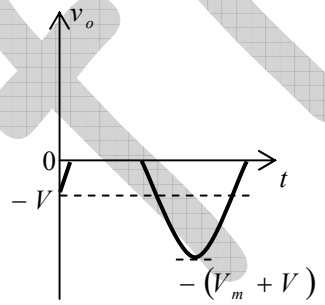
(c) 

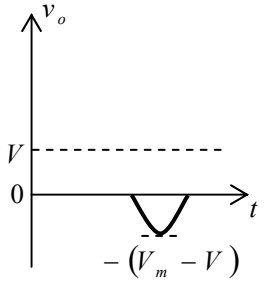
(d) 

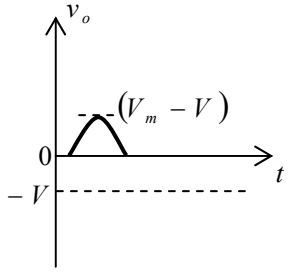
Q14. Identify the correct output waveforms for the circuit given below (assume diodes to be ideal)

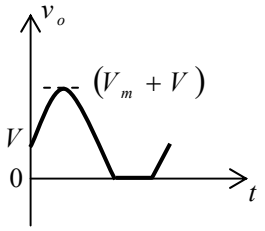




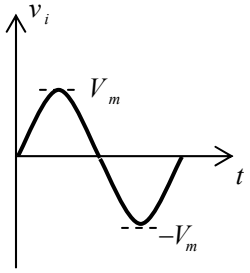
(a) 

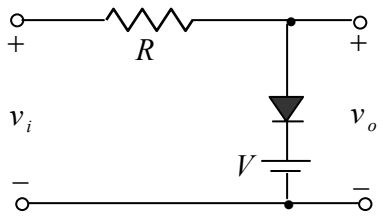
(b) 

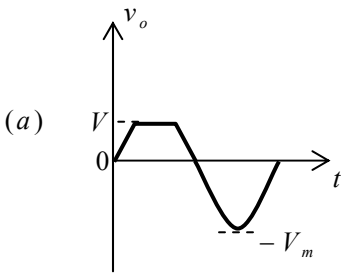
(c) 

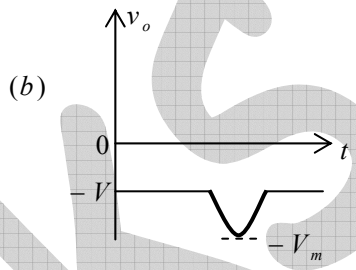
(d) 

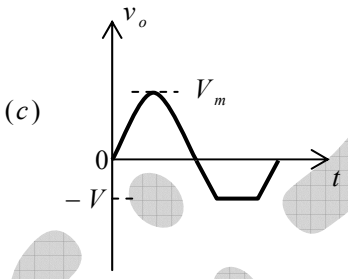
Q15. Identify the correct output waveforms for the circuit given below (assume diodes to be ideal)

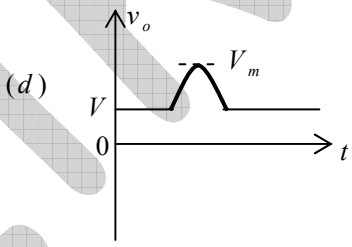




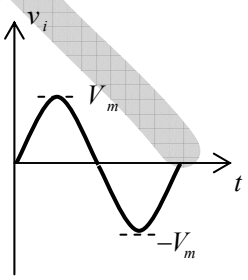


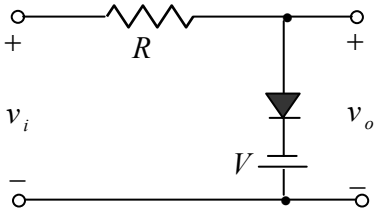


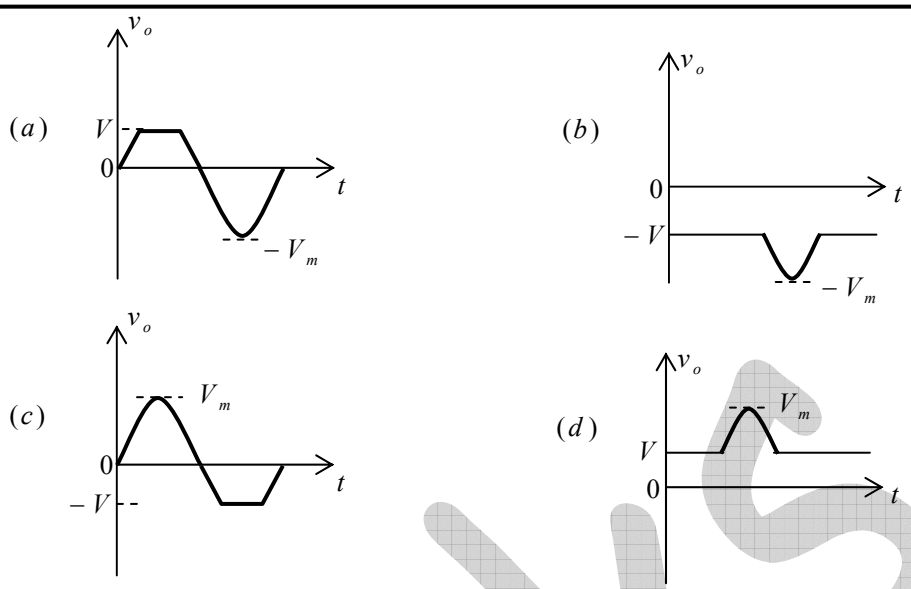




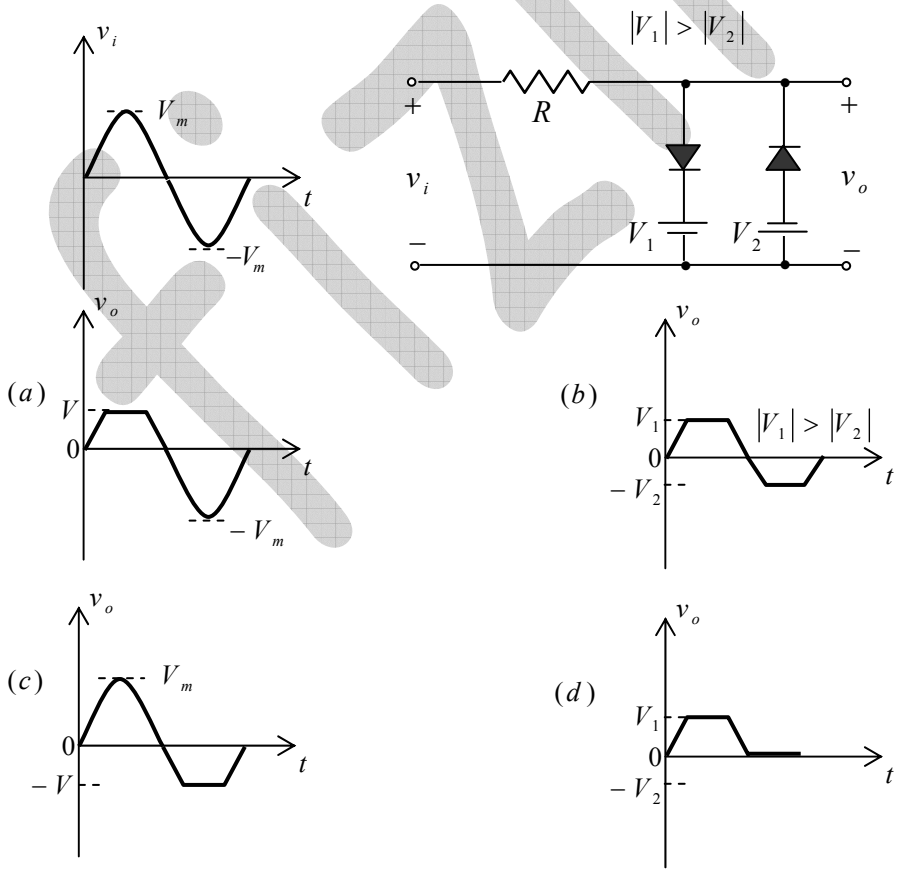
Q16. Identify the correct output waveforms for the circuit given below (assume diodes to be ideal)



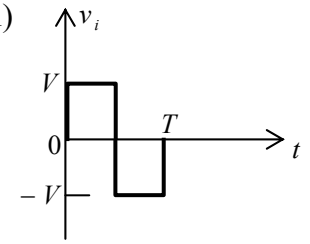


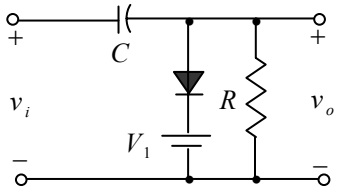


Q17. Identify the correct output waveforms for the circuit given below (assume diodes to be ideal)

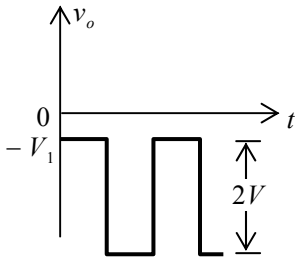


Q18. Identify the correct output waveforms for the circuit given below (assume diodes to be ideal)

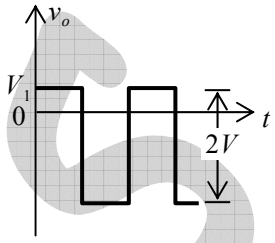




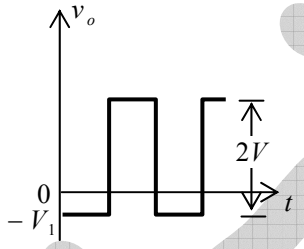
(a)



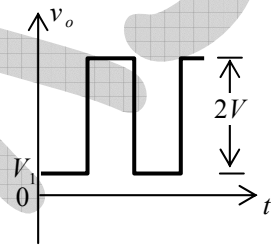
(b)



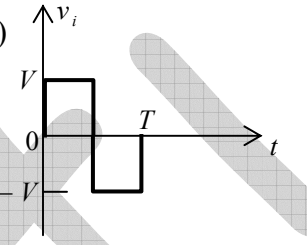
(c)

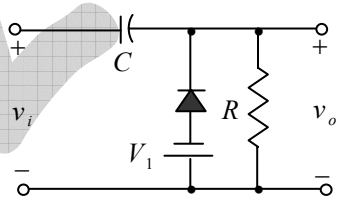


(d)

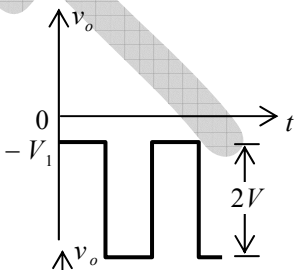


Q19. Identify the correct output waveforms for the circuit given below (assume diodes to be ideal)

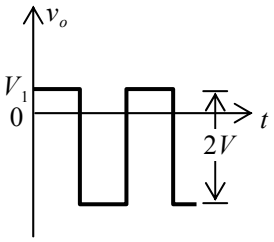




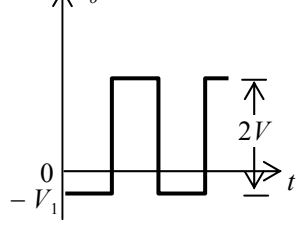
(a)



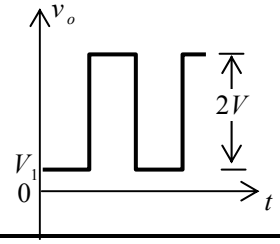
(b)



(c)



(d)



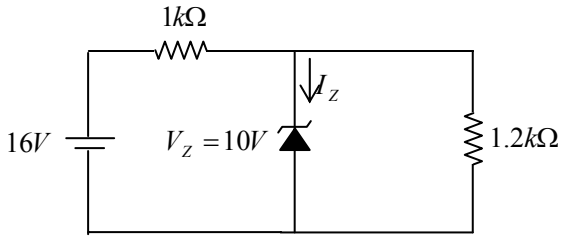
Q20. A zener diode network is shown in the figure given below. The power dissipated by the zener diode is:

(a) $20\ mW$

(b) $30\ mW$

(c) $40\ mW$

(d) Zero



Q21. For a Zener shunt regulator, Zener voltage is $10V$, series resistance is $1\,k\Omega$, load resistance is $2\,k\Omega$ and the input voltage varies from $20V$ to $40V$. Then the maximum and the minimum values of Zener currents are:

(a) $20\,mA$ and $5\,mA$

(b) $20\,mA$ and $10\,mA$

(c) $25\,mA$ and $5\,mA$

(d) $25\,mA$ and $10\,mA$

Q22. The range of values of V_i that will maintain the Zener diode in the "ON" state is

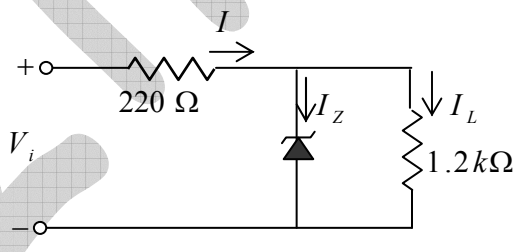
($V_Z = 20V$, $I_{ZM} = 60mA$)

(a) $24V - 37V$

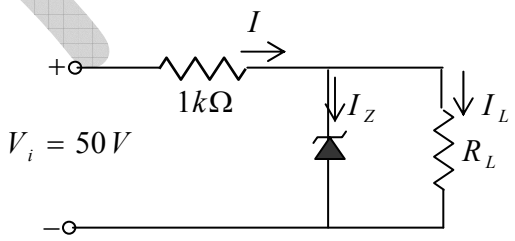
(b) $14V - 27V$

(c) $34V - 47V$

(d) $25V - 30V$



Q23. For the given zener diode network, the range of load resistance R_L that will maintain output voltage to $10V$ be: ($V_Z = 10V$, $I_{ZM} = 32mA$)



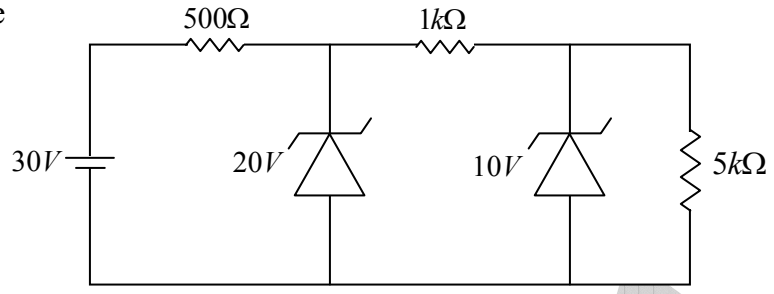
(a) $250\ \Omega - 1.25\,k\Omega$

(b) $350\ \Omega - 1.50\,k\Omega$

(c) $150\ \Omega - 1.20\,k\Omega$

(d) $250\ \Omega - 1.50\,k\Omega$

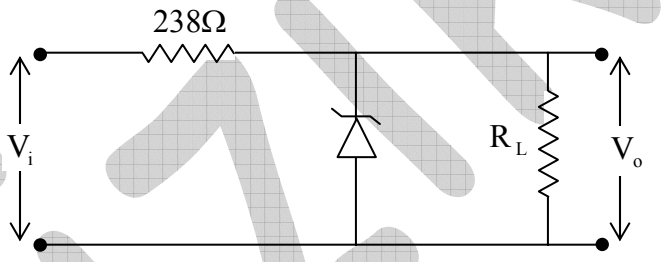
Q24. In the following circuit, the voltage across and the current through the $5\,k\Omega$ resistance are



(a) $20V,\ 4mA$ (b) $20V,\ 5mA$

(c) $10V,\ 2mA$ (d) $10V,\ 5mA$

Q25. The voltage regulator circuit shown in the figure has been made with a Zener diode rated at $15V,\ 200mW$. It is required that the circuit should dissipate $150mW$ power across the fixed load resistor R_L.



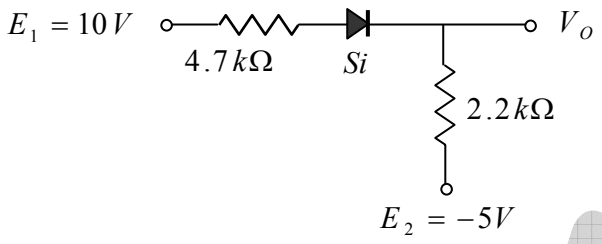
For stable operation of this circuit, the input voltage V_i must have a range

(a) $17.5\,V - 20.5\,V$ (b) $15.5\,V - 20.5\,V$

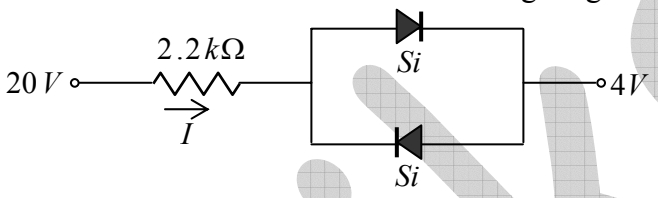
(c) $15.5\,V - 22.5\,V$ (d) $17.5\,V - 22.5\,V$

Numerical Answer Type (NAT)

Q26. The magnitude of output voltage V_o for the circuit as given below is…… *Volts*

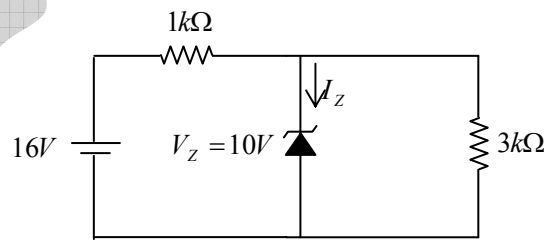


Q27. The current I for the network shown in the figure given below is .....*mA*

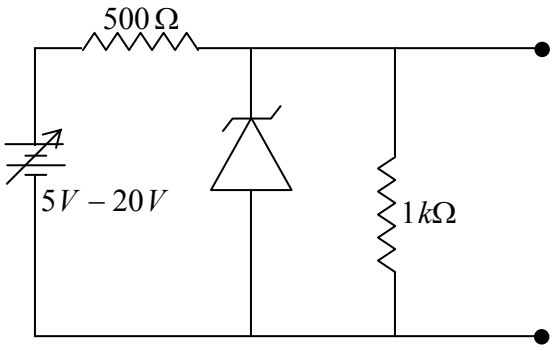


Q28. An a.c. voltage of $220\,V$, $50\,Hz$ is applied to the primary of a $5:1$ step down transformer. The secondary of the transformer is bridge type and connected to a full wave rectifier with a load resistance $500\,\Omega$. Forward resistance of the diode is $50\,\Omega$, then the output d.c. voltage is……*Volts*

Q29. A zener diode network is shown in the figure given below. The power dissipated by the zener diode is.....*mW*



Q30. A variable power supply ($5\,V - 20\,V$) is connected to a Zener diode specified by a breakdown voltage of $10\,V$ (see figure). The ratio of the maximum power to the minimum power dissipated across the load resistor is ……



Multiple Select Questions (MSQ)

Q31. Consider the following statements regarding the magnitude of barrier potential of a $p-n$ junction. Which of the statements given below are correct?

(a) It is independent of temperature

(b) It depends on difference between Fermi levels on two sides of junction

(c) It depends on forbidden energy gap on two types of semiconductors

(d) It depends on impurity concentration in p and n type semiconductors

Q32. Consider the following statements about Avalanche diodes

(a) Multiplication occurs due to carrier collision with the atoms

(b) Multiplication occurs due to direct breaking of covalent bonds

(c) It is lightly doped $p-n$ junction diode

(d) On increasing temperature breakdown voltage increases

Which of the statements given above are correct?

Q33. Which of the following statements are not correct?

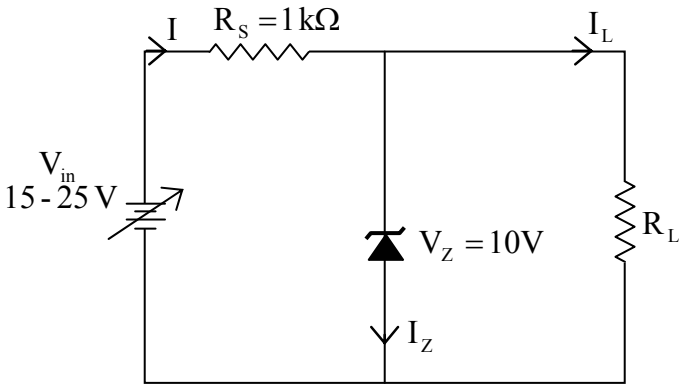
(a) Barrier potential increases by $2.5\ mV$ per degree rise in temperature

(b) Reverse current in Si diode is 1000 times more than in Ge diode

(c) Reverse saturation current is independent of magnitude of reverse bias

(d) Reverse saturation current increases with increase in temperature

Q34. Pick the correct statements based on the circuit shown below:



(a) The maximum Zener current $I_{Z,\max}$, when $R_L = 10k\Omega$ is $14mA$.

(b) The minimum Zener current $I_{Z,\min}$, when $R_L = 10k\Omega$ is $4mA$.

(c) With $V_{in} = 20V$, $I_L = I_Z$, when $R_L = 2k\Omega$.

(d)The power dissipated across the Zener when $R_L = 10k\Omega$ and $V_{in} = 20V$ is $100mW$.

Solution
MCQ

Ans.1: (a)

Ans.2: (c)

Ans.3: (c)

Ans.4: (b)

Ans.5: (a)

Let current through $12k\Omega$ is I and through diode is I_D

Then $0.7 + I_D \times 3.3 = (I - I_D) \times 6$ (1)

and $-24 + I \times 12 + (I - I_D) \times 6 = 0$ (2)

From (1) and (2) $I_D \approx 1mA.$, $I \approx \frac{5}{3}mA. \Rightarrow I - I_D \approx \frac{2}{3}mA \approx 0.7mA$.

Ans.6: (c)

Diode D_1 is OFF and diode D_2 is ON. Thus $I = \frac{12}{4+2} = 2.00A$

Ans.7: (b)

Total current $I = \frac{10 - 0.7}{0.33} = 28mA \Rightarrow I_{D_1} = I_{D_2} = \frac{I}{2} = 14mA$

Ans.8: (b) $-10 + (v_D^2 + 2v_D) \times 1 + v_D = 0 \Rightarrow v_D = 2V \Rightarrow i_D = 2\ A$

Ans.9: (c)

Ans.10: (d) $I_{av} = \frac{2V_m / R_L}{\pi} = \frac{10}{\pi}mA$

Ans.11: (c)

$$V_{P_{rms}} = 220\,V \Rightarrow V_{P_{\max}} = 220\sqrt{2}\ V; \frac{V_{P_{\max}}}{V_{S_{\max}}} = \frac{N_P}{N_S} = \frac{10}{1} \Rightarrow V_{S_{\max}} = 22\sqrt{2}\ V$$

$$I_{dc} = \frac{I_{\max}}{\pi} = \frac{V_{S_{\max}}}{\pi(R_L + r_f)} = 4.7\,mA \Rightarrow V_{dc} = I_{dc}R_L = 9.4\,V$$

$$PIV = V_{S_{\max}} = 22\sqrt{2}\ V = 31V$$

Ans.12: (c)

$$V_{P_{rms}} = 220\,V \Rightarrow V_{P_{\max}} = 220\sqrt{2}\ V; \frac{V_{P_{\max}}}{V_{S_{\max}}} = \frac{N_P}{N_S} = \frac{5}{1} \Rightarrow V_{S_{\max}} = 44\sqrt{2}\ V$$

$$I_{dc} = \frac{2I_{\max}}{\pi} = \frac{2V_{\max}}{\pi\left(R_L + r_f\right)} = \frac{2\left(V_{S_{\max}}/2\right)}{\pi\left(R_L + r_f\right)} = \frac{62}{\pi\left(500+50\right)} = 0.036\,mA$$

$$\Rightarrow V_{dc} = I_{dc}R_L = .036\times 500 = 18\,V \quad \text{and} \quad PIV = 2V_{\max} = V_{S_{\max}} = 44\sqrt{2}\ V = 62\,V$$

Ans.13: (c)

Ans.14: (d)

Ans.15: (a)

Ans.16: (b)

Ans.17: (b)

Ans.18: (b)

Ans.19: (c)

Ans.20: (d)

Open circuit voltage $V_i = \frac{1.2\times 16}{1+1.2} = 8.7V < V_Z$. So zener diode is OFF and $P_Z = 0$.

Ans.21: (c)

$$I_{Z,\max} = I_{\max} - I_L = \frac{V_{\max} - V_Z}{R} - \frac{V_L}{R_L} = \frac{40-10}{1} - \frac{10}{2} = 25\ mA$$

$$I_{Z,\min} = I_{\min} - I_L = \frac{V_{\min} - V_Z}{R} + \frac{V_L}{R_L} = \frac{20-10}{1} - \frac{10}{2} = 5\ mA$$

Ans.22: (a)

$$V_{i_{\min}} = \frac{\left(R + R_L\right)V_Z}{R_L} = \frac{\left(220+1200\right)\times 20}{1200} = 23.6V$$

$$I_L = \frac{V_Z}{R_L} = \frac{20}{1200} = 16.6V,\ I_{R_{\max}} = I_{ZM} + I_L = 60 + 16.6 = 76.6\,mA.$$

$$V_{i_{\max}} = I_{R_{\max}}R + V_Z = 76.6\times 0.22 + 20 = 37\,V.$$

Ans.23: (a)

$$R_{L_{\min}} = \frac{R_{L_{\min}} \times 50}{1k\Omega + R_{L_{\min}}} = 10 \Rightarrow R_{L_{\min}} = 250\ \Omega .$$

$$R_{L_{\max}} = \frac{10}{I_{L_{\min}}} \text{ where } I_{L_{\min}} = I_R _ I_{ZM} = \frac{50-10}{1} - 32 = 8mA \Rightarrow R_{L_{\max}} = \frac{10}{8} = 1.25\,k\Omega .$$

Ans.24: (c)

Ans.25: (a)

$$\text{Since } V_Z = 15V \text{ and } P_{ZM} = 200mW \Rightarrow I_{ZM} = \frac{40}{3} mA .$$

$$P_L = 150mW \Rightarrow 150mW = \frac{V_Z^2}{R_L} \Rightarrow R_L = \frac{225}{150} = 1.5k\Omega$$

$$V_{i_{\min}} = \frac{(R + R_L)V_Z}{R_L} = \frac{(238 + 1500) \times 15}{1500} = 17.3\ V$$

$$I_L = \frac{V_Z}{R_L} = \frac{15}{1500} = 10\,mA,\ I_{R_{\max}} = I_{ZM} + I_L = 23.33\,mA, V_{i_{\max}} = I_{R_{\max}} R + V_Z = 20.5\,V .$$

NAT

Ans.26: 0.45

Under given biasing condition diode is ON. Let current through $4.7k\Omega$ resistance is I.

Apply KVL in the input section,

$$-10+I\times 4.7k+0.7+I\times 2.2k-5=0\Rightarrow I=2.07mA$$

And thus $V_0=2.07\times 2.2-5=-0.45V$

Ans.27: 7

Diode D_1 is ON and diode D_2 is OFF.

Apply KVL, $-20+I\times 2.2k+0.7+4=0\Rightarrow I=7mA$

Ans.28: 36

$$V_{P_{rms}}=220\,V\Rightarrow V_{P_{\max}}=220\sqrt{2}\ V;\ \frac{V_{P_{\max}}}{V_{S_{\max}}}=\frac{N_P}{N_S}=\frac{5}{1}\Rightarrow V_{S_{\max}}=44\sqrt{2}\ V$$

$$I_{dc}=\frac{2I_{\max}}{\pi}=\frac{2V_{\max}}{\pi\left(R_L+r_f\right)}=\frac{2V_{S_{\max}}}{\pi\left(R_L+r_f\right)}=\frac{124}{\pi\left(500+50\right)}=0.072\,mA$$

$$\Rightarrow V_{dc}=I_{dc}R_L=.072\times 500=36\ V$$

Ans.29: 27

Open circuit voltage $V_i=\frac{3\times 16}{1+3}=12V>V_Z$.

So zener diode is ON $\Rightarrow V_L=V_Z=10V$ and $V_R=6V$.

$$I_L=\frac{V_L}{R_L}=3.3mA,\ I_R=6mA\Rightarrow I_Z=6-3.3=2.7mA\Rightarrow P_Z=V_ZI_Z=27\,mW$$

Ans.30: 9.2

When $V = 5V \Rightarrow$ open circuit voltage $V_i = \frac{1000}{1500} \times 5 = 3.33 < V_Z = 10V$

$\Rightarrow V_L = V_i = 3.33V \Rightarrow P_{L,\min} = \frac{V_i^2}{R_L}$.

When $V = 20V \Rightarrow$ open circuit voltage $V_i = \frac{1000}{1500} \times 20 = 13.33 > V_Z = 10V$

$\Rightarrow V_L = V_z = 10V \Rightarrow P_{L,\max} = \frac{V_Z^2}{R_L}$

$\Rightarrow \frac{P_{L,\max}}{P_{L,\min}} = \frac{V_Z^2}{V_i^2} = \left(\frac{10}{3.33}\right)^2 = 9.2$

MSQ

Ans.31: (b), (c) and (d)

Ans.32: (a), (c) and (d)

Ans.33: (a), (b) and (d)

Ans.34: (a), (b) and (c)

(a) $I_{Z,\max} = I_{\max} - I_L = \frac{V_{\max} - V_Z}{R} - \frac{V_L}{R_L} = \frac{25-10}{1} - \frac{10}{10} = 14mA$

(b) $I_{Z,\min} = I_{\min} - I_L = \frac{V_{\min} - V_Z}{R} - \frac{V_L}{R_L} = \frac{15-10}{1} - \frac{10}{10} = 4mA$

(c) $V_{in} = 20V$, $I_L = \frac{10}{2} = 5mA$, $I_Z = \frac{20-10}{1} - 5 = 5mA \Rightarrow I_L = I_z$

(d) $V_{in} = 20V$, $I_L = \frac{10}{10} = 1mA$, $I_Z = \frac{20-10}{1} - 1 = 9mA \Rightarrow P_Z = V_Z I_Z = 10 \times 9 = 90mW$

3. Bipolar Junction Transistors

3.1 Transistor Construction

Transistor is a three-layer semiconductor device consisting of either two *n*- and one *p*-type layer of material or two *p*- and one *n*-type layers of material. The former is called *npn* transistor, while latter is called an *pnp* transistor. Both are shown in figure 3.1 with proper biasing.

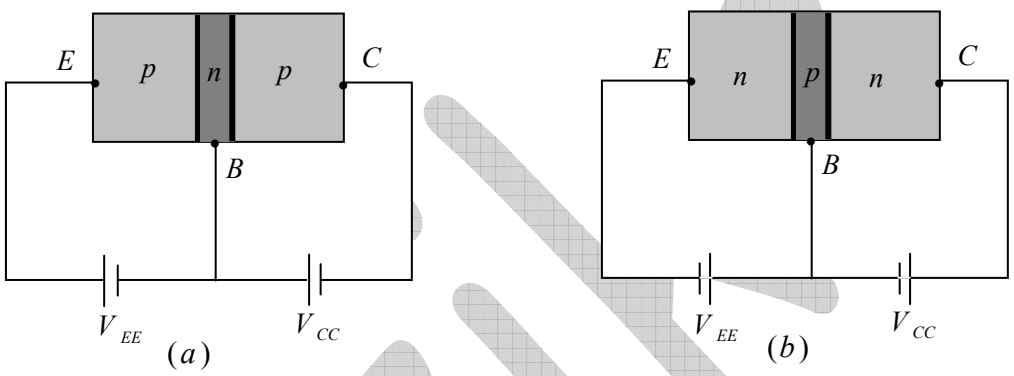


Figure 3.1: Types of transistors: (a) *pnp* (b) *npn*.

The emitter layer is heavily doped, the base lightly doped, and the collector only lightly doped. The outer layers have widths much greater than the sandwiched *p*- or *n*-type material. The ratio of the total width to that of the center layer is 150:1. The doping of the sandwiched layer is also considerably less than that of the outer layer (typically, 10:1 or less).This lower doping level decreases the conductivity (increases the resistance) of this material by limiting the number of "free" carriers.

The terminals have been indicated by the capital letters *E* for *emitter*, *C* for *collector*, and *B* for *base*. The term bipolar junction transistor (BJT) reflects the fact that holes and electrons participate in the injection process into the oppositely polarized material.

3.2 Transistor Operation

The basic operation of the transistor is described using *pnp* transistor as shown in figure 3.1(a). The operation of the *npn* transistor is exactly the same if the roles played by the electron and holes are interchanged. In figure 3.2 the *pnp* transistor has been redrawn without the base-to-collector bias (similar to *forward-biased* diode). The depletion region has been reduced in width due to applied bias, resulting in a heavy flow of majority carriers from *p*- to the *n*-type material.

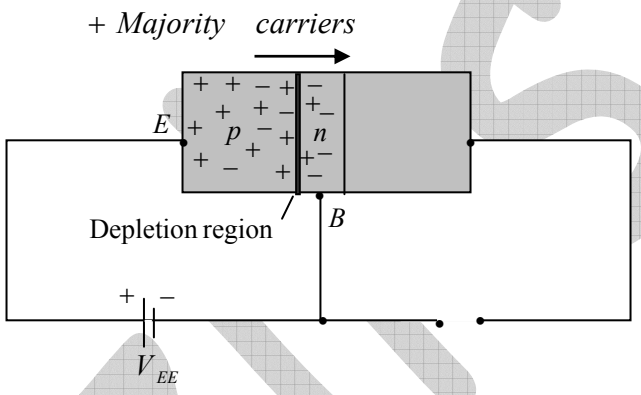


Figure 3.2: Forward-biased junction of a *pnp* transistor.

Let us now remove the base-to-emitter bias of the *pnp* transistor of figure 3.1(a) as shown in figure 3.3 (similar to *reverse-biased* diode). Recall that the flow of majority carriers is zero, resulting in only a minority-carrier flow. Thus

"One p-n junction of a transistor is reversed biased, while the other is forward biased."

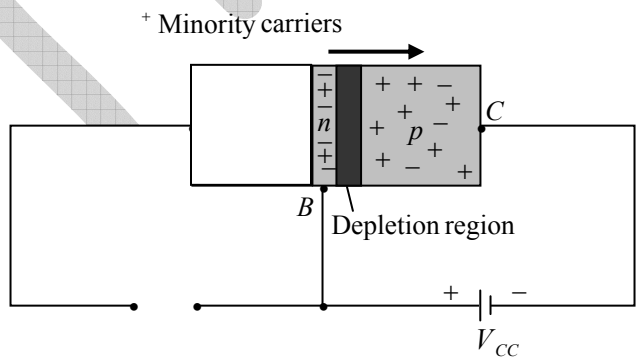


Figure 3.3: Reverse based junction of a *pnp* transistor.

In figure 3.4 both biasing potentials have been applied to a *pnp* transistor, with the resulting majority and minority-carrier flow indicated. The widths of the depletion regions, indicating clearly which junction is forward-biased and which is reversed-biased. A large number of majority carriers will diffuse across the forward-biased *p*-*n* junction into the *n*-type material. Since *n*-type material is very thin and has low conductivity, a very small number of these carriers will take this path of high resistance to the base terminal. The larger number of these majority carriers will diffuse across the reverse-biased junction into the *p*-type material connected to the collector terminal. Thus there has been an *injection* of minority carriers into the *n*-type base region material. Combining this with the fact that all the minority carriers in the depletion region will cross the reversed-biased junction of a diode accounts for the flow indicated in the figure 3.4.

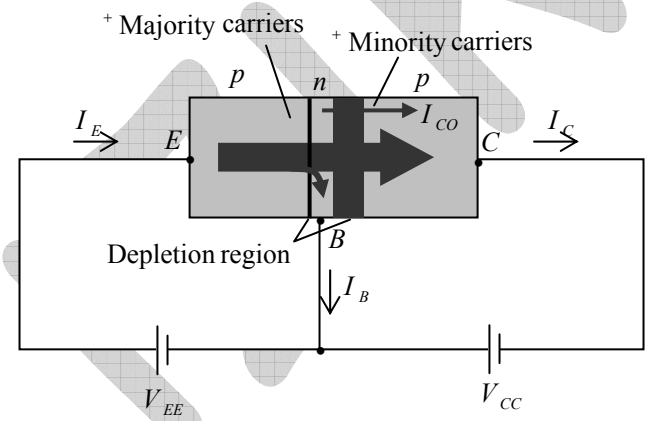


Figure 3.4: Majority and minority carrier flow of a *pnp* transistor.

Applying Kirchhoff's current law to the transistor of figure 3.4 as if it were a single node, we obtain

$$I_E = I_C + I_B$$

The minority current component is called the leakage current and is given by the symbol I_{CO} (collector current with emitter terminal open). The collector current, therefore is:

$$I_C = I_{C_{\text{majority}}} + I_{CO_{\text{minority}}}$$

3.3 Transistor Configurations

3.3.1 Common-Base Configuration

The common-base configuration with *pnp* and *npn* transistors are shown in figure 3.5. The common-base terminology is derived from the fact that the base is common to both the input and output sides of the configurations.

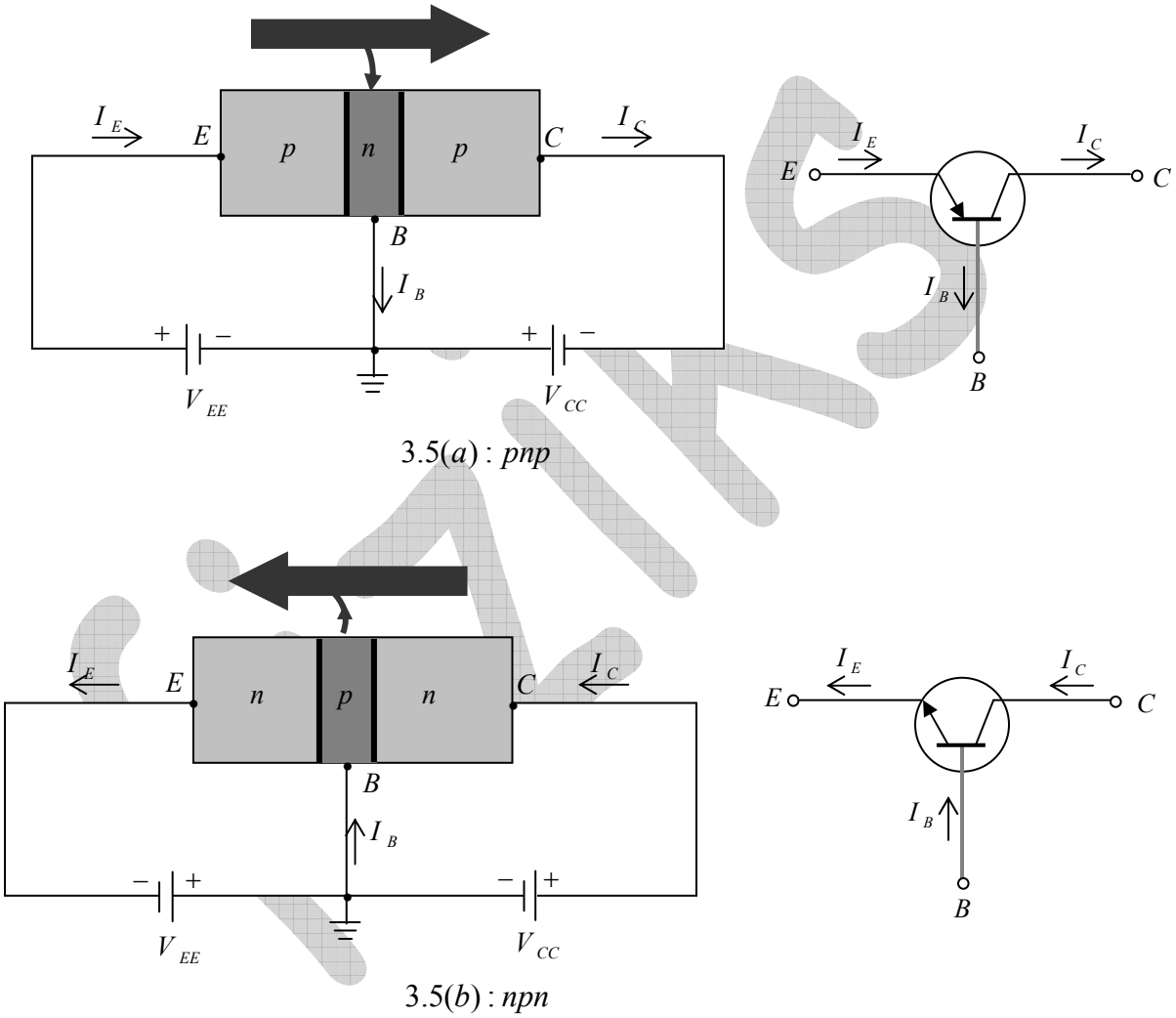


Figure 3.5: Notation and symbols used with the common base-configuration: (a) *pnp* transistor; (b) *npn* transistor.

To fully describe the behavior of a three terminal device such as common base amplifiers requires two set of characteristics- one for the *driving point* or *input* parameters and the other for the *output* side.

Input Characteristics

The input set for the common base amplifiers as shown in figure 3.6 will relate an input current (I_E) to an input voltage (V_{BE}) for various levels of output voltage (V_{CB}).

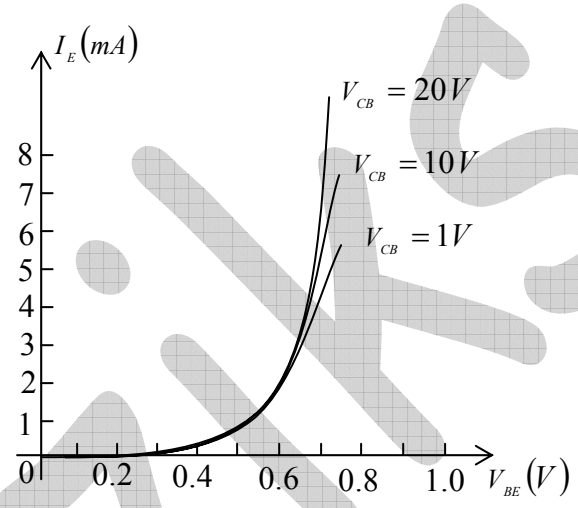


Figure 3.6: Input or driving point characteristics for a common-base silicon transistor.

Output Characteristics

The output set will relate an output current (I_C) to an output voltage (V_{CB}) for various levels of input current (I_E). The output characteristics have three basic regions of interest: the ***active***, ***cutoff***, and ***saturation*** regions. The active region is the region normally employed for linear (undistorted) amplifiers.

In the active region the collector-base junction is reversed-biased, while the base-emitter junction is forward biased.

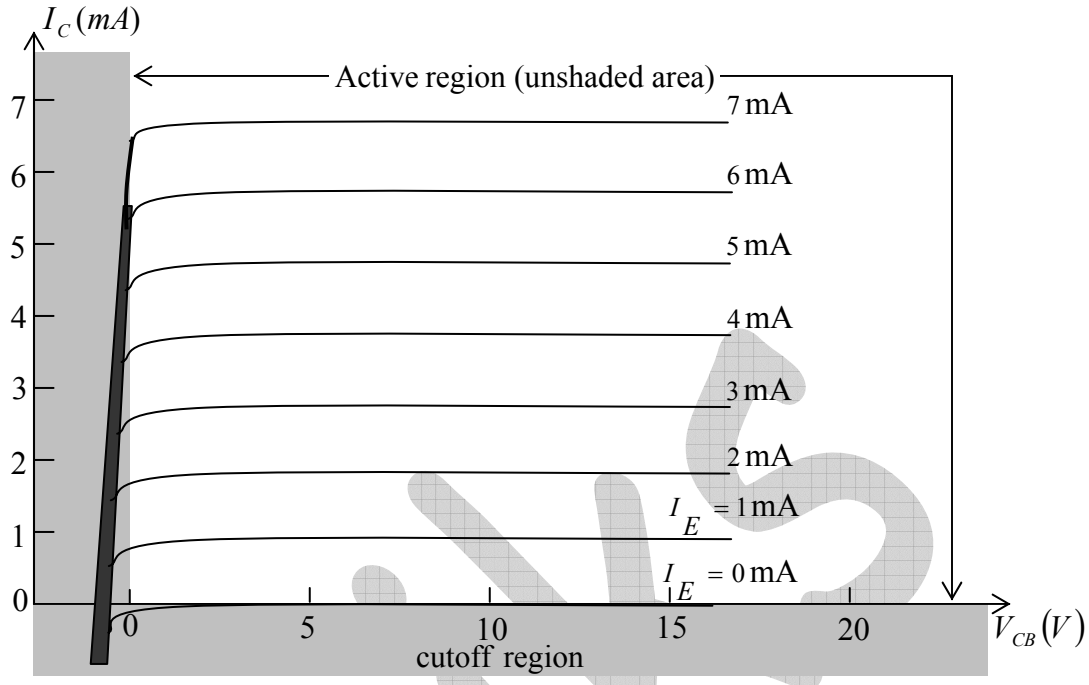


Figure 3.7: Output or collector characteristics for a common-base transistor amplifier.

The circuit condition that exists when $I_E = 0$ for common base configuration is shown in figure 3.8. Note that I_{CBO} is temperature dependent and increases so rapidly with temperature.

In the output characteristics as the emitter current increases above zero, the collector current increases to a magnitude essentially equal to that of the emitter current as determined by the basic transistor current relations. Note also the almost negligible effect of V_{CB} on the collector current for the active region.

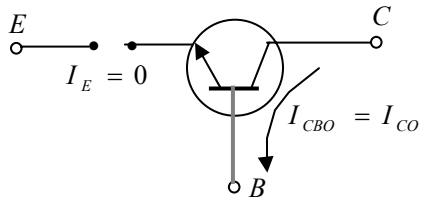


Figure 3.8: Reverse Saturation current.

In the cutoff region the collector-base and base-emitter junctions are both reversed-biased.

In the saturation region the collector-base and base-emitter junctions are both forward-biased.

Alpha (α)

In the dc mode the levels of I_C and I_E due to majority carriers are related by a quantity called *alpha* and defined by the following equations:

$\alpha_{dc} = \dfrac{I_C}{I_E}$ where I_C and I_E are the levels of current at the point of operation.

Thus $I_C = I_{C_{\text{majority}}} + I_{CO_{\text{minority}}} \Rightarrow I_C = \alpha I_E + I_{CBO}$

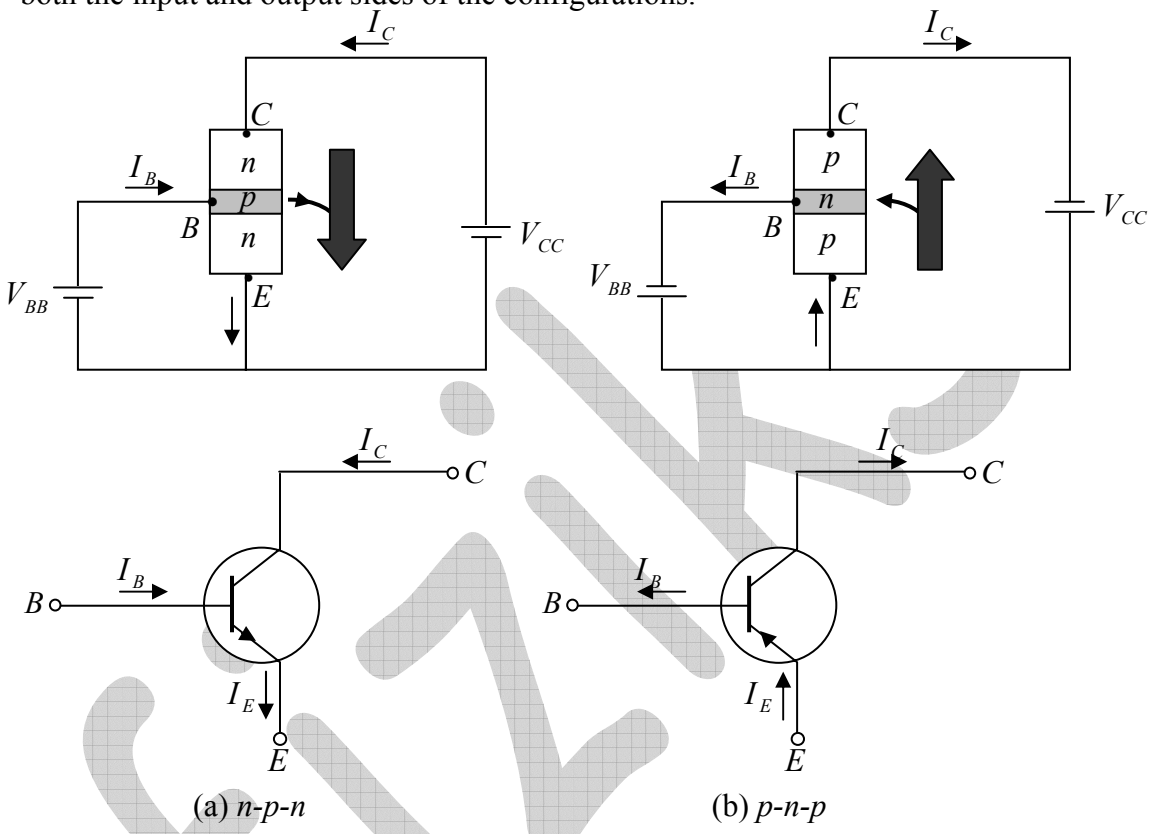
For ac situations where the point of operation moves on the characteristics curve, an ac *alpha* is defined by $\alpha_{ac} = \left.\dfrac{\Delta I_C}{\Delta I_E}\right|_{V_{CB}=\text{constant}}$.

The ac alpha is formally called the *common-base, short-circuit, amplification factor*.

The typical values of *voltage amplification* $\left(V_o / V_i\right)$ for the common-base configuration vary from 50 to 300. The *current amplification* $\left(I_C / I_E\right)$ is always less than 1 for the common-base configuration.

3.3.2 Common Emitter Configuration

The common-emitter configuration with *pnp* and *npn* transistors are shown in figure 3.9. The common-emitter terminology is derived from the fact that the emitter is common to both the input and output sides of the configurations.



(a) *n-p-n* (b) *p-n-p*

Figure 3.9: Notation and symbols used with the common-emitter configuration.

To fully describe the behavior of a three terminal device such as common emitter amplifier requires two set of characteristics- one for the *input* or *base-emitter* circuit and one for the *output* or *collector-emitter* circuit.

The output characteristics will relate an output current (I_C) to an output voltage (V_{CE}) for various levels of input current (I_B). The input characteristics for the common emitter amplifiers will relate an input current (I_B) to an input voltage (V_{BE}) for various levels of output voltage (V_{CE}).

In the active region the collector-base junction is reversed-biased, while the base-emitter junction is forward biased.

In the cutoff region the collector-base and base-emitter junctions are both reversed-biased.

In the saturation region the collector-base and base-emitter junctions are both forward-biased.

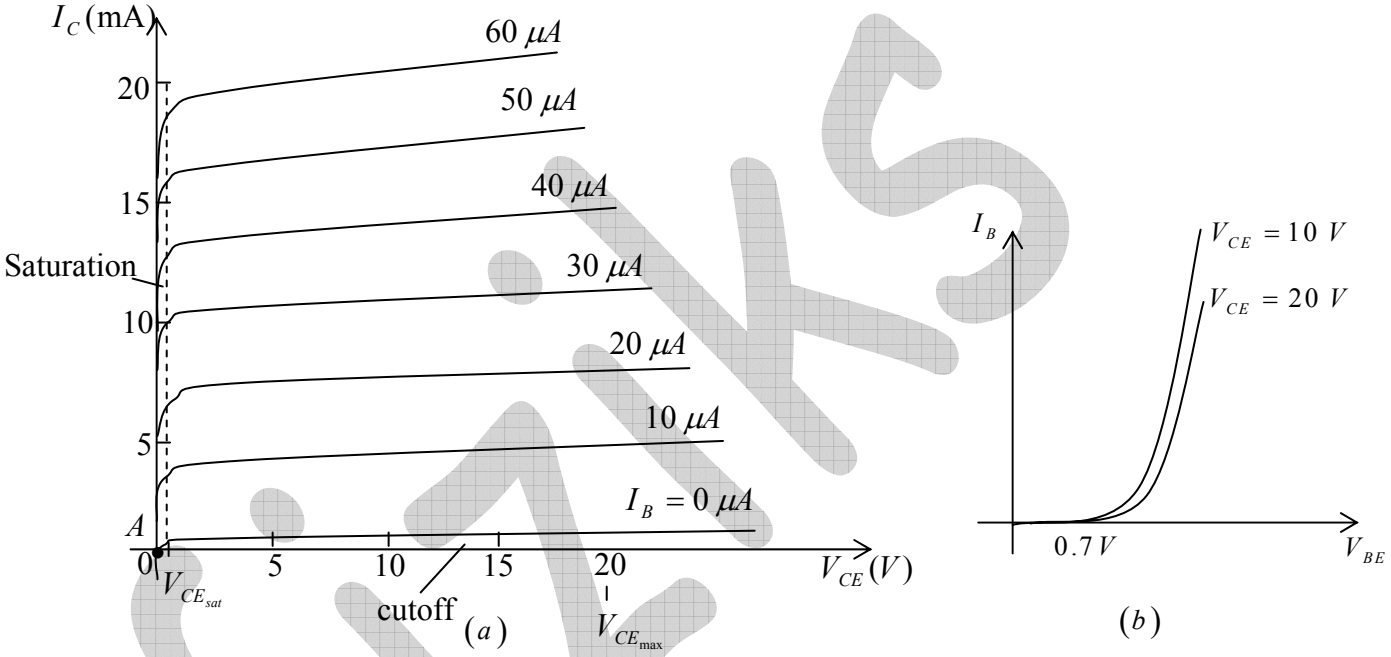


Figure 3.10: Characteristics of a silicon transistor in the common emitter configuration: (a) Collector characteristics; (b) base characteristics.

Since $I_C = \alpha I_E + I_{CBO} = \alpha(I_C + I_B) + I_{CBO} \Rightarrow I_C = \dfrac{\alpha I_B}{1-\alpha} + \dfrac{I_{CBO}}{1-\alpha}$

or $I_C = \beta I_B + I_{CEO}$ where $I_{CEO} = \left.\dfrac{I_{CBO}}{1-\alpha}\right|_{I_B=0}$ and $\beta = \dfrac{\alpha}{1-\alpha}$.

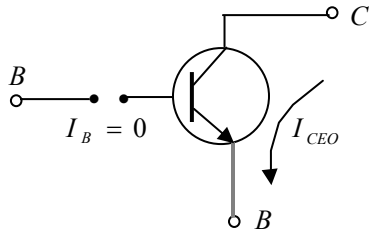


Figure 3.11: Circuit condition related to I_{CEO}.

Beta (β)

In the dc mode the levels of I_C and I_B are related by a quantity called *beta* and defined by the following equations: $\beta_{dc} = \frac{I_C}{I_B}$ where I_C and I_B are the levels of current at the point of operation. For ac situations where the point of operation moves on the characteristics curve, an ac beta is defined by $\beta_{ac} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE}=\text{constant}}$. The formal name for β_{ac} is *common emitter forward-current amplification factor*.

3.3.3 Common-Collector Configuration

The third and final transistor configuration is the common–collector configuration, shown in figure 3.12 with the proper current directions and voltage notation. The common-collector configuration is used primarily for impedance-matching purposes since it has a high input impedance and low output impedance, opposite to that of the common-base and common-emitter configurations.

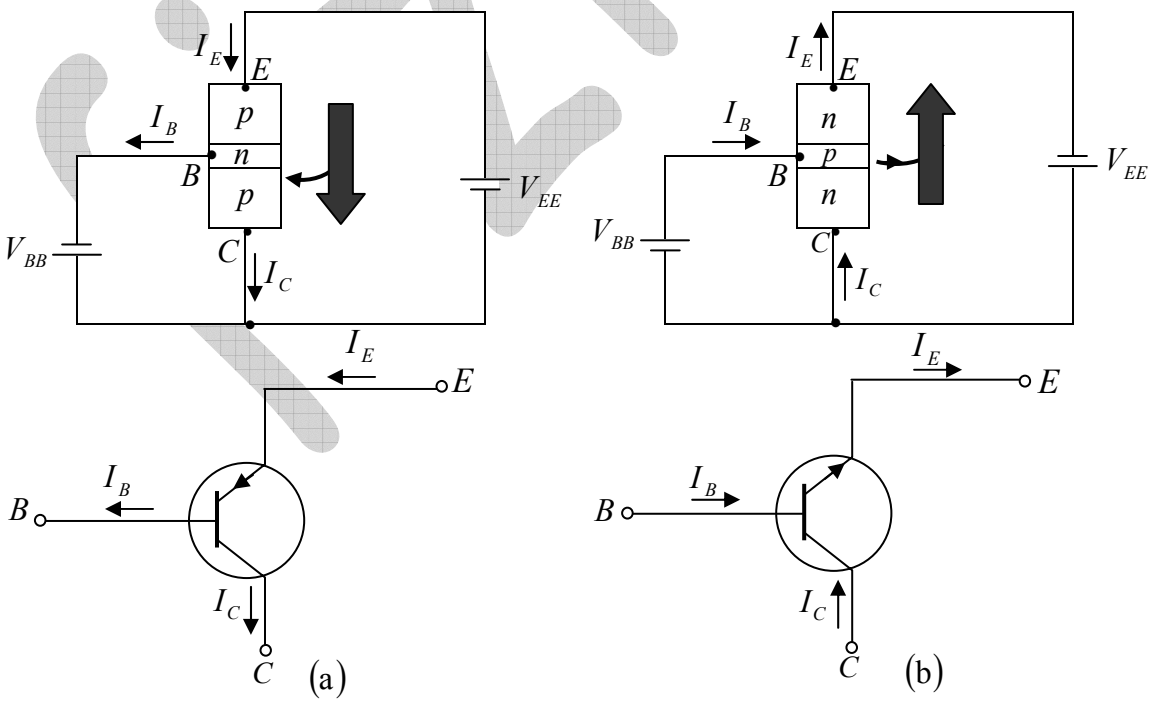


Figure 3.12: Notation and symbols used with the common-collector configuration.

A common collector circuit configuration is provided in figure 3.13 with the load resistor connected from emitter to ground. Note that the collector is tied to ground even though the transistor is connected in a manner similar to the common emitter configuration. For all practical purposes, the output characteristics of the CC configuration are same as for the CE configuration.

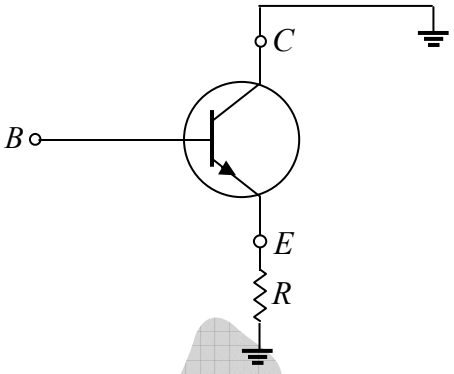


Figure 3.13: Common-collector configuration.

3.4 DC Biasing-BJTs

3.4.1 Introduction

The analysis or design of a transistor amplifier requires knowledge of both the dc and ac response of the system. The improved output ac power level is the result of a transfer of energy from the applied dc supplies. The analysis or design of any electronic amplifier, therefore, has two components: the *dc portion* and the *ac portion*. Fortunately, the superposition theorem is applicable and the investigation of the dc conditions can be totally separated from the ac response. However, one must keep in mind that during the design stage the choice of parameters for the required dc levels will affect the ac response and vice-versa.

The dc level of operation of a transistor is controlled by a number of factors, including the range of possible operating points on the device characteristics. Each design will also determine the stability of the system, that is, how sensitive the system is to temperature variations.

Although a number of networks will be analyzed, there is an underlying similarly between the analysis of each configuration due to the recurring use of the following important basic relationships for a transistor:

$$V_{BE} = 0.7V,\ I_E = (\beta + 1) I_B \cong I_C \text{ and } I_C = \beta I_B$$

3.4.2 Operating Point

Since the operating point is a fixed point on the characteristics it is also called the quiescent point (abbreviated *Q-point*). By definition, quiescent means quiet, still, inactive. Figure 3.14 shows a general output device characteristic with four operating points indicated. The biasing circuit can be designed to set the device operation at any of these points or others within the active region. The maximum ratings are indicated on the characteristics by a horizontal line for the maximum collector current $I_{C_{\max}}$ and a vertical line at the maximum collector-to-emitter voltage $V_{CE_{\max}}$. The maximum power constraint is defined by the curve $P_{C_{\max}}$ in the same figure. At the lower end of the scales are the *cutoff regions*, defined by $I_B \le 0\ \mu A$ and the *saturation region*, defined by $V_{CE} \le V_{CE_{sat}}$.

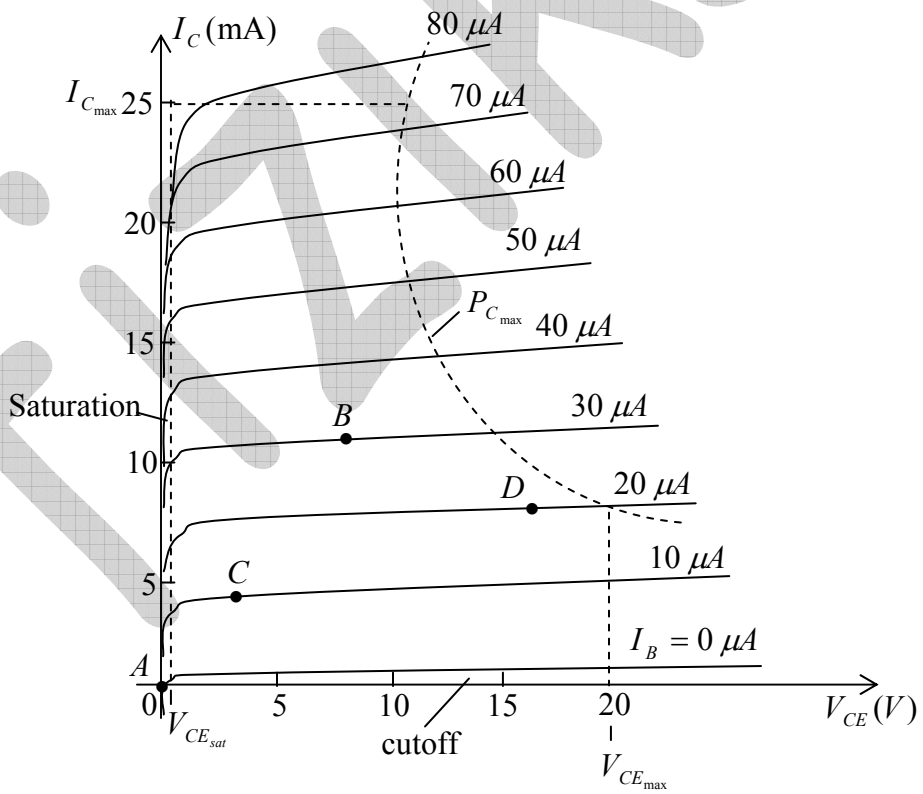


Figure 3.14: Various operating points within the limits of operation of a transistor.

If no bias were used, the device would initially be completely off, resulting in a *Q*-point at **A**-namely zero current through the device (and zero voltage across it). Since it is necessary to bias a device so that it can respond to the entire range of an input signal *point A* would not be suitable. For ***point B*** if a signal is applied to the circuit, the device will vary in current and voltage from operating point, allowing the device to react to both the positive and negative excursion of the input signal. If the input signal is properly chosen, the voltage and current of the device will vary but not enough to drive the device into cutoff or saturation. ***Point C*** would allow some positive and negative variation of the output signal but the peak-to-peak value would be limited by the proximity of $V_{CE} = 0\ V$, $I_C = 0\ mA$. Operating at point C also raise some concern about the nonlinearities introduced by the fact that the spacing between I_B curves is rapidly changing in this region.

In general, it is preferable to operate where the gain of the device is fairly constant (or linear) to ensure that the amplification over the entire swing of input signal is the same. ***Point D*** sets the device operating point near the maximum voltage and power level. The output voltage swing in the positive direction is thus limited if the maximum voltage is not to be exceeded. ***Point B*** is a region of more linear spacing and therefore, seems the best operating point in terms of linear gain and largest possible voltage and current swing. This is usually the desired condition for small-signal amplifiers but not the case necessarily for power amplifiers. In this discussion, we will be concentrating primarily on biasing the transistor for *small-signal amplification operation*.

Having selected and biased the BJT at a desired operating point, the *effect of temperature* must also be taken into account. Temperature causes the device parameters such as the transistor current gain (β_{ac}) and the transistor leakage current (I_{CEO}) to change. Higher temperatures result in increased leakage currents in the device, thereby changing the operating condition set by the biasing network. The result is that the network design must also provide a degree of temperature stability so that temperature changes result in minimum changes in the operating point. This maintenance of the operating point can be

specified by a stability factor, S, which indicates the degree of change in operating point due to a temperature variation. A highly stable circuit is desirable and the stability of a few basic bias circuits will be compared.

For the BJT to be biased in its linear or active operating region the following must be true:

1. The base-emitter junction must be forward-biased (p-region voltage more positive) with a resulting forward-bias voltage of about 0.6 to 0.7 V.
2. The base-collector junction must be reverse-biased (n-region more positive), with the reverse-bias voltage being any value within the maximum limits of the device.

Operation in the cutoff, saturation and linear regions of the BJT characteristic are provided as follows:

1. Linear-region operation:

   Base-emitter junction forward biased

   Base-collector junction reversed biased

2. Cutoff-region operation:

   Base-emitter junction reverse biased

   Base-collector junction reversed biased

3. Saturation-region operation:

   Base-emitter junction forward biased

   Base-collector junction forward biased

3.5 Fixed-Bias Circuit

The fixed-bias circuit of figure 3.15 provides a relatively straightforward and simple introduction to transistor dc bias analysis. Even though the network employs an *npn* transistor, the equations and calculations apply equally well to a *pnp* transistor configuration merely by changing all current directions and the voltage polarities.

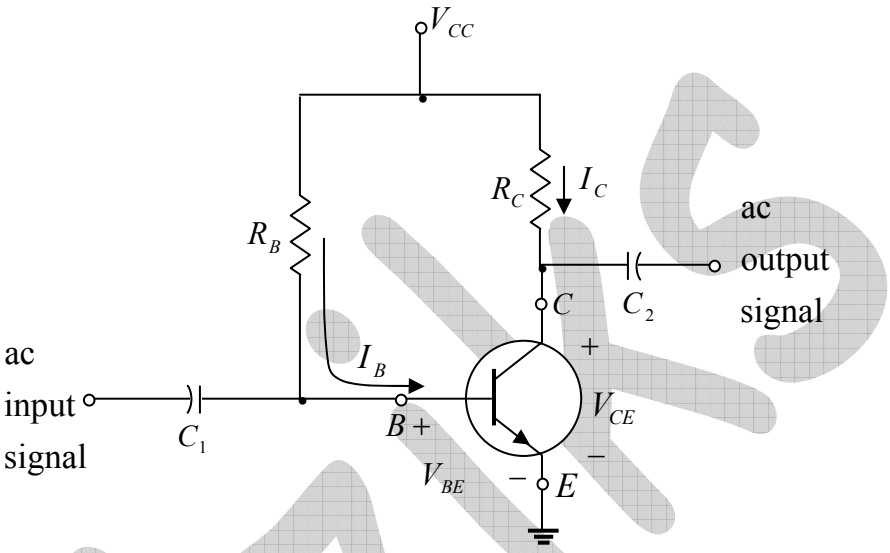


Figure 3.15: Fixed-bias circuit

For the dc analysis the network can be isolated from the indicated ac levels by replacing the capacitors with an open-circuit equivalent. In addition, the dc supply V_{CC} can be separated into two supplies (for analysis purposes only) as shown in figure 3.16 to permit a separation of input and output circuits. It also reduces the linkage between the two to the base current I_B.

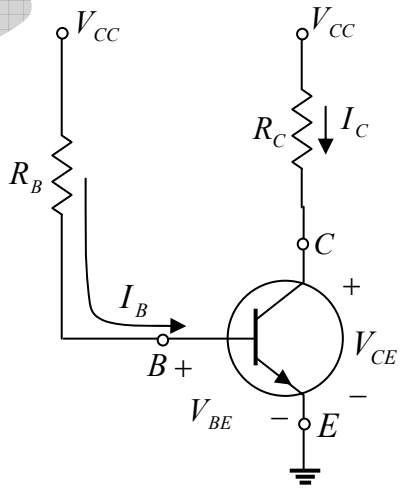


Figure 3.16: DC equivalent of figure 3.15.

3.5.1 Q-Point

Forward Bias of Base-Emitter

Consider first the base-emitter circuit loop of figure shown 3.17. Writing Kirchhoff's voltage equation in the clockwise direction for the loop, we obtain $-V_{CC} + I_B R_B + V_{BE} = 0$

Note the polarity of the voltage drop across R_B as established by the indicated direction of I_B. Solving the equation for the current I_B will result in the following: $I_B = \dfrac{V_{CC} - V_{BE}}{R_B}$

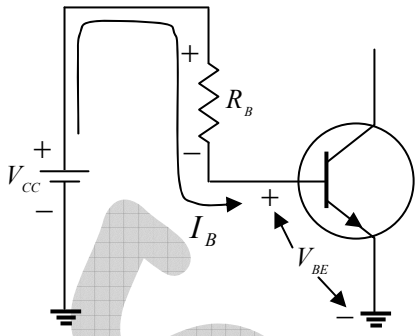


Figure 3.17: Base-emitter loop.

Since the supply voltage V_{CC} and the base-emitter voltage V_{BE} are constants, the selection of a base resistor, R_B sets the level of base current for the operating point.

Collector-Emitter Loop

The collector-emitter section of the network appears in figure 3.18 with the indicated direction of current I_C and the resulting polarity across R_C.

The magnitude of the collector current is related directly to I_B through

$$I_C = \beta I_B$$

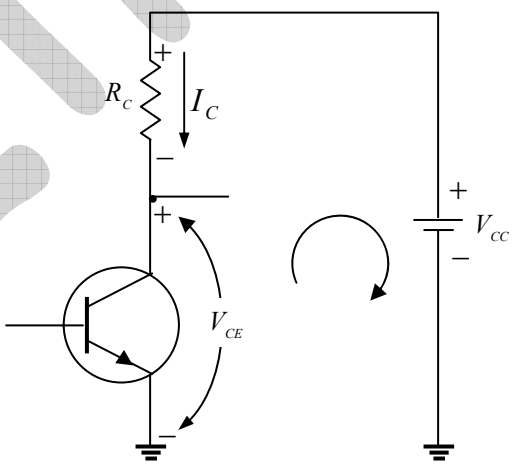


Figure 3.18: Collector-emitter loop.

It is interesting to note that since the base current is controlled by the level of R_B and I_C is related to I_B by a constant β the magnitude of I_C is not a function of the resistance R_C. Change R_C to any level and it will not affect the level of I_B or I_C as long as we remain in the active region of the device. However, as we shall see, the level of R_C will determine the magnitude of V_{CE}, which is an important parameter.

Applying Kirchhoff's voltage law in the clockwise direction around the indicated closed-loop of figure 3.18 will result in the following:

$$-V_{CC} + I_C R_C + V_{CE} = 0 \Rightarrow V_{CE} = V_{CC} - I_C R_C$$

As a brief review of single and double subscript notation recall that

$$V_{CE} = V_C - V_E$$

where V_{CE} is the voltage from collector to emitter and V_C and V_E are the voltages from collector and emitter to ground respectively. But in this case since $V_E = 0\ V$, we have

$$V_{CE} = V_C .$$

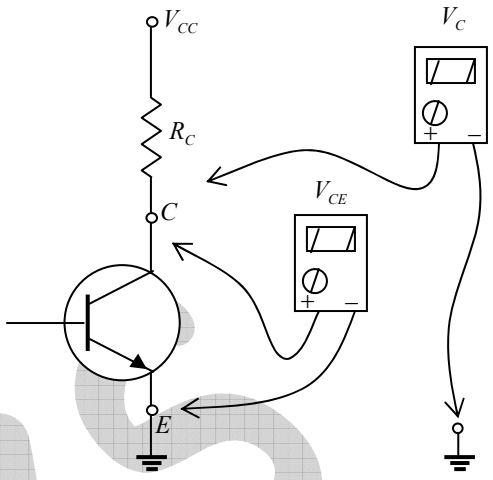


Figure 3.19: Measuring V_{CE} and V_C.

In addition, since $V_{BE} = V_B - V_E$ and $V_E = 0\ V$ then $V_{BE} = V_B$.

Keep in mind that voltage levels such as V_{CE} are determined by placing the positive lead of the voltmeter at the collector terminal with the negative lead at the emitter terminal as shown in figure 3.19. V_C is the voltage from collector to ground and is measured as shown in the same figure. In this case the two readings are identical, but in the networks to follow the two can be quite different.

3.5.2 Transistor Saturation

The term saturation is applied to any system where levels have reached their maximum values. For a transistor operating in the saturation region the current is a maximum value for the particular design. Change the design and the corresponding saturation level may rise or drop.

Saturation conditions are normally avoided because the base-collector junction is no longer reverse-biased and the output amplified signal will be distorted. An operating point in the saturation region is depicted in figure 3.20. Note that it is in a region where the characteristic curves join and the collector-to-emitter voltage is at or below $V_{CE_{sat}}$. In addition, the collector current is relatively high on the characteristics.

If we approximate the curves of figure 3.20(a) by those appearing in figure 3.20(b), a quick direct method for determining the saturation level becomes apparent. In figure 3.20(b) the current is relatively high and the voltage V_{CE} is assumed to be zero volts. Applying Ohm's law the resistance between collector and emitter terminals can be determined as follows: $R_{CE} = \frac{V_{CE}}{I_C} = \frac{0V}{I_{C_{sat}}} = 0\Omega$

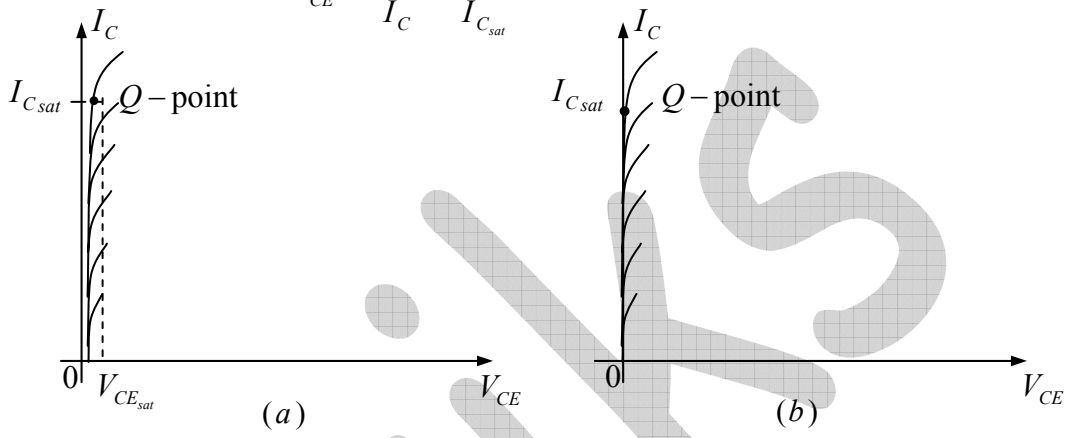


Figure 3.20: Saturation region (a) actual (b) approximate.

For saturation current set $V_{CE} = 0\,V$ and find $I_{C_{sat}}$. For the fixed-bias configuration of figure 3.22 the short circuit has been applied, causing the voltage across R_C to be the applied voltage V_{CC}. The resulting saturation current for the fixed-bias configuration is

$$I_{C_{sat}} = \frac{V_{CC}}{R_C}$$

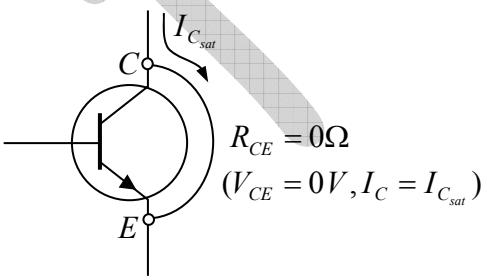


Figure 3.21: Determining $I_{C_{sat}}$

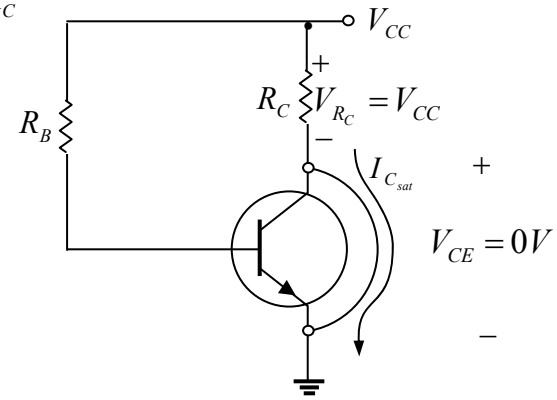


Figure 3.22: $I_{C_{sat}}$ for the fixed-bias configuration.

3.5.3 Load-Line Analysis

We will now investigate how the network parameters define the possible range of Q-points and how the actual Q-point is determined.

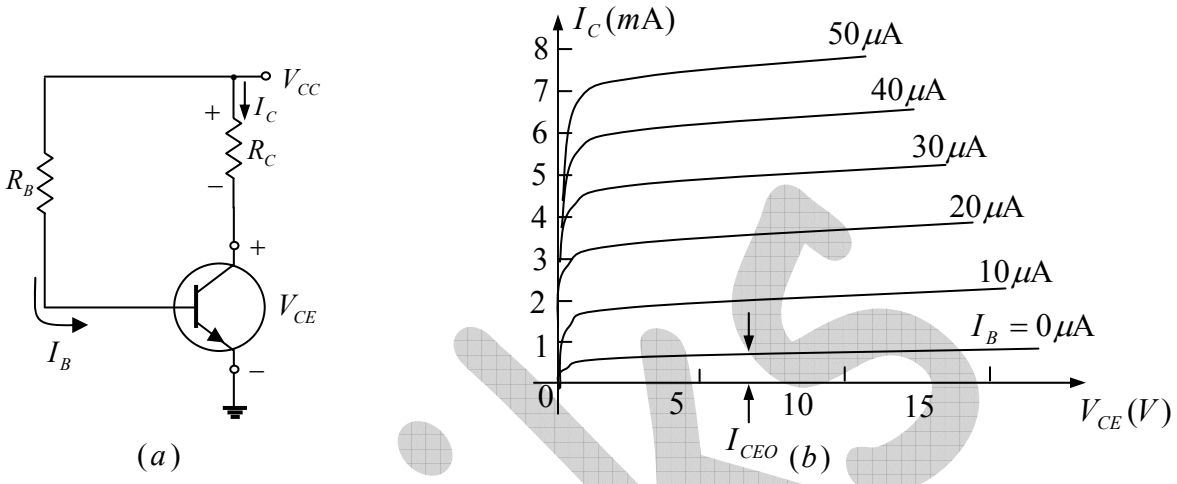


Figure 3.23: Load-line analysis (a) the network (b) the device characteristics.

The network of figure 3.23(a) establishes an output equation that relates the variables I_C and V_{CE} in the following manner: $V_{CE} = V_{CC} - I_C R_C$

The output characteristics of the transistor also relate the same two variables I_C and V_{CE} in figure 3.23(b).

We must now superimpose the straight line defined by equation $V_{CE} = V_{CC} - I_C R_C$ on the characteristics. If we choose I_C to be $0\ mA$, we are specifying the horizontal axis as the line on which one point is located. By substituting $I_C = 0\ mA$, we find that $V_{CE} = V_{CC}\big|_{I_C = 0\ mA}$ defining one point for the straight line as shown in figure 3.24.

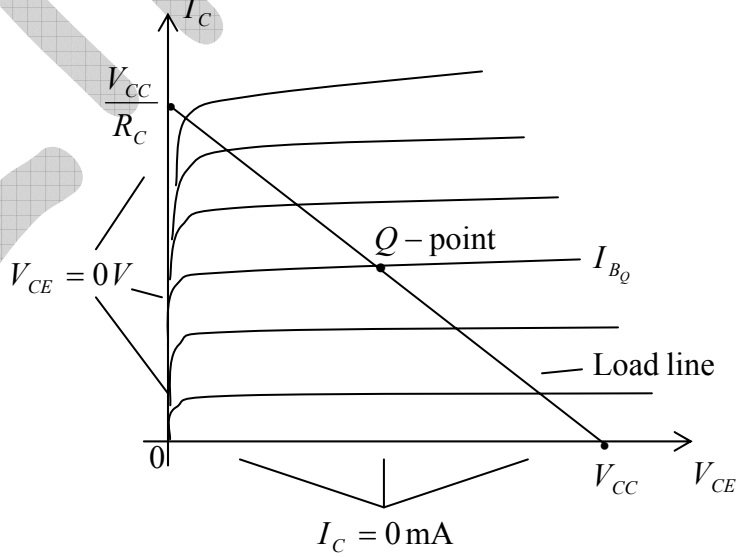


Figure 3.24: Fixed-bias load.

If we now choose V_{CE} to be $0\,V$, which establishes the vertical axis as the line on which the second point will be defined, we find that I_C is determined by the following equation:

$$0 = V_{CC} - I_C R_C \quad \text{and} \quad I_C = \left.\frac{V_{CC}}{R_C}\right|_{V_{CE}=0V}$$ as appearing on figure 3.24.

By joining the two points defined by equation $V_{CE} = V_{CC}\big|_{I_C=0mA}$ and $I_C = \left.\frac{V_{CC}}{R_C}\right|_{V_{CE}=0V}$ the straight line established by equation $V_{CE} = V_{CC} - I_C R_C$ can be drawn. The resulting line on the graph of figure 3.24 is called the ***load line*** since it is defined by the load resistor R_C. By solving for the resulting level of I_B the actual Q-point can be established.

If the level of I_B is changed by varying the value of R_B the Q-point moves up or down the load line as shown in figure 3.25.

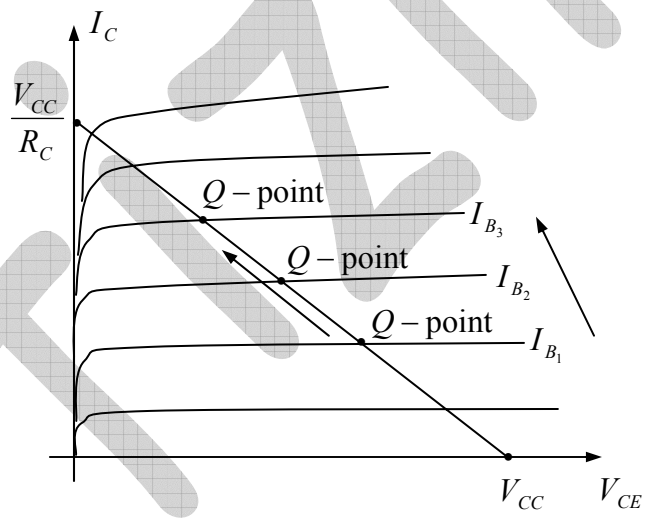


Figure 3.25: Movement of Q-point with increasing levels of I_B.

NOTE: For Fixed R_B if temperature of the device increases the Q-point will moves towards the saturation region as shown in figure 3.25. Since $I_C = \beta I_B + I_{CEO}$ with increase in temperature reverse current $I_{CEO} = (\beta +) I_{CBO}$ increases.

If V_{CC} is held fixed and R_C changed, the load line will shift as shown in figure 3.26. If I_B is held fixed, the Q-point will move as shown in the same figure.

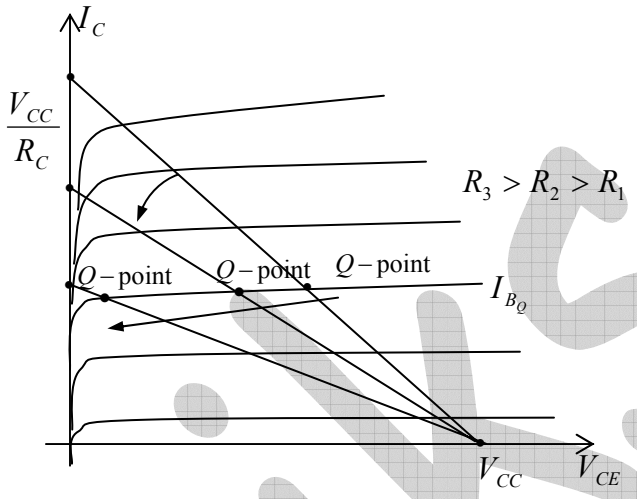


Figure 3.26: Effect of increasing levels of R_C on the load line and Q-point.

If R_C is fixed and V_{CC} varied, the load line shifts as shown in figure 3.27.

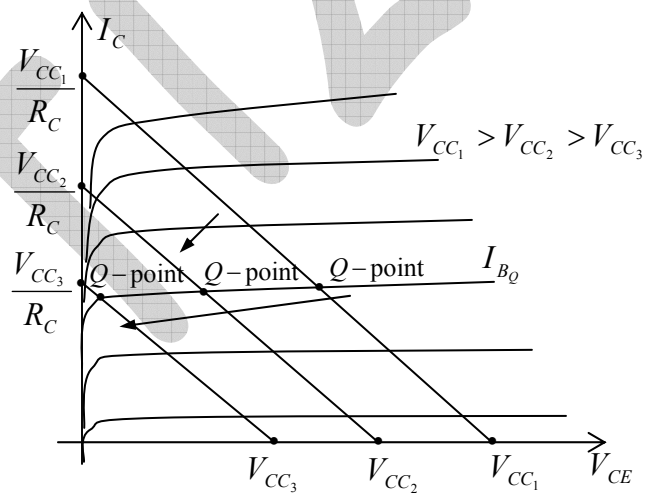
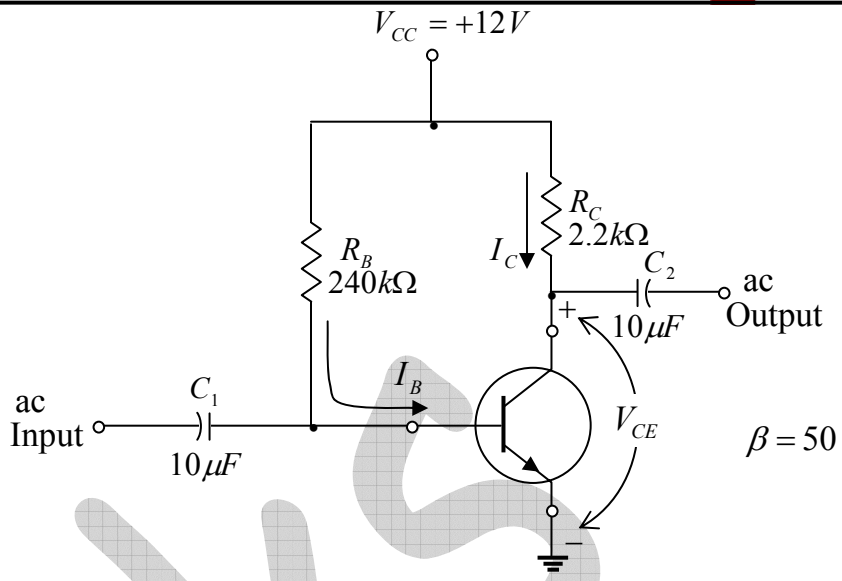


Figure 3.27: Effect of lower values of V_{CC} on the load line and Q-point.

Example: Determine the following for the fixed-bias configuration of figure shown below.



(a) I_{B_Q} and I_{C_Q}

(b) V_{CE_Q}

(c) V_B and V_C

(d) V_{BC}

(e) Saturation level.

Solution:

(a) $I_{B_Q} = \dfrac{V_{CC} - V_{BE}}{R_B} = \dfrac{12V - 0.7V}{240\,k\Omega} = 47.08\,\mu A$

$I_{C_Q} = \beta I_{B_Q} = (50)(47.08\,\mu\text{A}) = 2.35\,m\text{A}$

(b) $V_{CE_Q} = V_{CC} - I_C R_C = (12)(2.35\,mA)(2.2\,k\Omega) = 6.83\,V$

(c) $V_B = V_{BE} = 0.7\,V$

$V_C = V_{CE} = 6.83\,V$

(d) Using double-subscript notation yields

$V_{BC} = V_B - V_C = 0.7\,V - 6.83\,V = -6.13\,V$

With the negative sign revealing that the junction is reversed-biased, as it should be for linear amplification.

(e) $I_{c_{sat}} = \dfrac{V_{CC}}{R_C} = \dfrac{12V}{2.2\,k\Omega} = 5.45\,m\text{A}$

$I_{C_Q} = 2.35\,m\text{A}$, which is far from the saturation level and about one-half the maximum value for the design.

3.6 Emitter-Stabilized Bias Circuit

The dc bias network of figure 3.28 contains an emitter resistor to improve the stability level over that of the fixed-bias configuration.

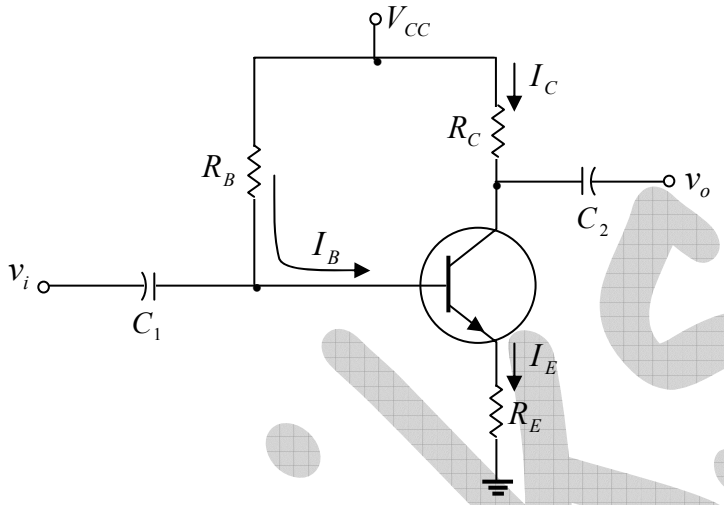


Figure 3.28: BJT circuit with emitter resistor.

3.6.1 *Q*-Point

The analysis will be performed by first examining the base-emitter loop and then using the results to investigate the collector-emitter loop.

Base–Emitter Loop

The base-emitter loop of the network can be redrawn as shown in 3.29. Writing Kirchhoff's voltage law around the indicated loop in the clockwise direction will result in the following equation:

$$-V_{CC} + I_B R_B + V_{BE} + I_E R_E = 0$$

$$V_{CC} - I_B R_B - V_{BE} - (\beta + 1) I_B R_E = 0 \quad \because I_E = (\beta + 1) I_B$$

Grouping terms will then provide the following:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1) R_E}.$$

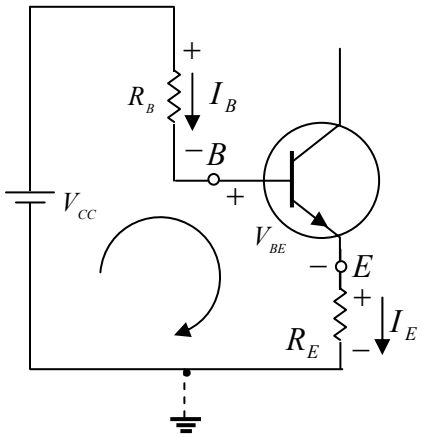


Figure 3.29: Base-emitter loop.

Note that the only difference between this equation for I_B and that obtained for the fixed-bias configuration is the term $(\beta + 1) R_E$.

There is an interesting result that can be derived from equation $I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E}$ if the equation is used to sketch a series network that would result in the same equation. Such is the case for the network of figure 3.30. Solving for the current I_B will result in the same equation obtained above. Note that aside from the base-to-emitter voltage V_{BE} the resistor R_E is reflected back to the input base circuit by a factor $(\beta + 1)$. In other words, the emitter resistor, which is part of the collector–emitter loop, "appears as" $(\beta + 1)R_E$ in the base-emitter loop. Since β is typically 50 or more, the emitter resistor appears to be a great deal larger in the base circuit.

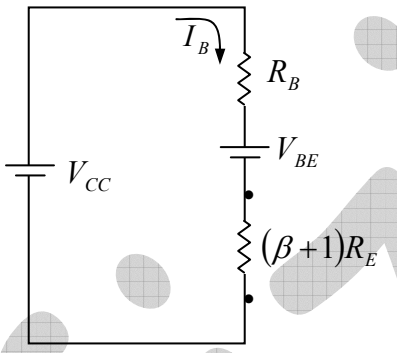


Figure 3.30: Network derived from I_B.

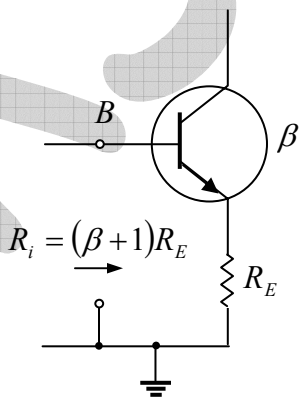


Figure 3.31: Reflected impedance level of R_E.

In general, therefore, for the configuration of figure 3.31,

$$R_i = (\beta + 1)R_E$$

This equation is one that will prove useful in the analysis to follow. In fact, it provides a fairly easy way to remember equation $I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E}$. Using Ohm's law, we know that the current through a system is the voltage divided by the resistance of the circuit. For the base-emitter circuit the net voltage is $V_{CC} - V_{BE}$. The resistance levels are R_B plus R_E reflected by $(\beta + 1)$.

Collector–Emitter Loop

The collector-emitter loop is redrawn in figure 3.32.

Writing Kirchhoff's voltage law for the indicated loop in the clockwise direction will result in

$$I_E R_E + V_{CE} + I_C R_C - V_{CC} = 0$$

Substituting $I_C \approx I_E$ and grouping terms gives

$$V_{CE} = V_{CC} - I_C (R_C + R_E)$$

The single-subscript voltage V_E is the voltage from emitter to ground and is determined by

$$V_E = I_E R_E$$

while the voltage from collector to ground can be determined from

$$V_{CE} = V_C - V_E \quad \text{or} \quad V_C = V_{CE} + V_E \quad \text{and} \quad V_C = V_{CC} - I_C R_C$$

The voltage at the base with respect to ground can be determined from

$$V_B = V_{CC} - I_B R_B \quad \text{or} \quad V_B = V_{BE} + V_E$$

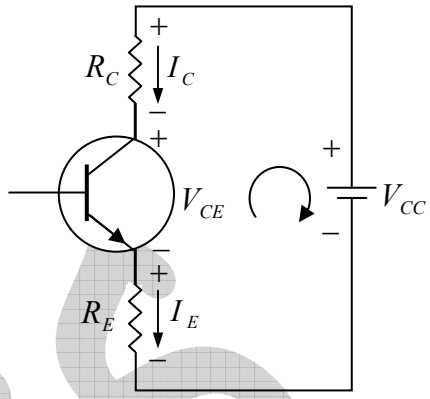


Figure 3.32: Collector-emitter loop.

3.6.2 Saturation Level

The collector saturation level or maximum collector current for an emitter-bias design can be determined using the same approach applied to the fixed-bias configuration:

$$I_{C_{sat}} = \frac{V_{CC}}{R_C + R_E}$$

The addition of the emitter resistor reduces the collector saturation level below that obtained with a fixed-bias configuration using the same collector resistor.

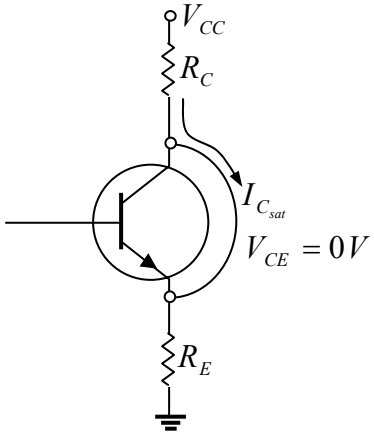


Figure 3.33: Determining $I_{C_{sat}}$ for the emitter-stabilized bias circuit.

3.6.3 Load-Line Analysis

The load-line analysis of the emitter-bias network is only slightly different from that encountered for the fixed-bias configuration. The level of I_B as determined by equation $I_B = \dfrac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E}$ defines the level of I_B on the characteristics of figure 3.34 (denoted I_{B_Q}).

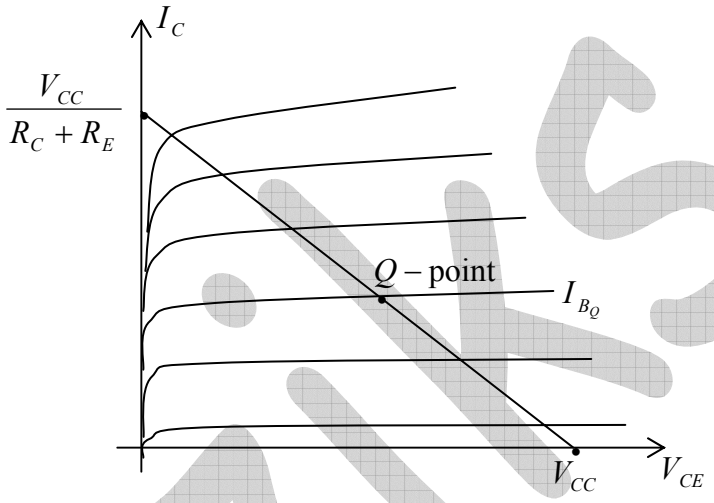


Figure 3.34: Load-Line for the emitter-bias configuration.

The collector-emitter loop equation that defines the load line is the following:

$$V_{CE} = V_{CC} - I_C\left(R_C + R_E\right)$$

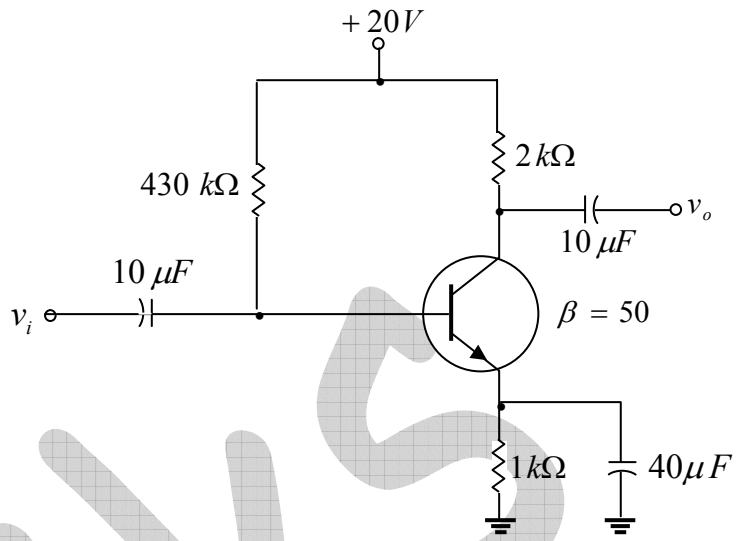
$$V_{CE} = V_{CC}\big|_{I_C = 0V}$$

as obtained for the fixed-bias configuration. Choosing $V_{CE} = 0\ V$ gives

$$I_C = \left.\frac{V_{CC}}{R_C + R_E}\right|_{V_{CE} = 0V}$$

as shown in figure 3.34. Different levels of I_{B_Q} will, of course, move the Q-point up or down the load line.

Example: For the emitter bias network of figure shown below, determine:



(a) I_B

(b) I_C

(c) V_{CE}

(d) V_C

(e) V_E

(f) V_B

(g) V_{BC}

(h) $I_{C_{sat}}$

Solution:

(a) $I_B = \dfrac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E} = \dfrac{20V - 0.7V}{430k\Omega + (51)(1k\Omega)} = \dfrac{19.3\ \ V}{481\ \ k\Omega} = 40.1\ \ \mu A$

(b) $I_C = \beta I_B = (50)(40.1\ \ \mu A) = 2.01\ mA$

(c) $V_{CE} = V_{CC} - I_C(R_C + R_E) = (20) - (2.01mA)(2k\Omega + 1k\Omega) = 13.97\ V$

(d) $V_C = V_{CC} - I_C R_C = (20) - (2.01mA)(2k\Omega) = 15.98\ V$

(e) $V_E = V_C - V_{CE} = 15.98\ V - 13.97V = 2.01\ V$

(f) $V_B = V_{BE} + V_E = 0.7\ V + 2.01V = 2.71\ V$

(g) $V_{BC} = V_B - V_C = 2.71\ V - 15.98V = -13.27\ V$ (reverse-biased as required)

(h) $I_{C_{sat}} = \dfrac{V_{CC}}{R_C + R_E} = \dfrac{20\ \ V}{2k\Omega\ \ +\ \ 1\ \ k\Omega} = \dfrac{20\ \ V}{3\ \ k\Omega} = 6.67\ mA$

which is about twice the level of I_{C_Q}.

3.7 Voltage-Divider Bias

In the previous bias configurations the bias current I_{C_Q} and voltage V_{CE_Q} were a function of the current gain (β) of the transistor. However, since β is temperature sensitive, specially for silicon transistors, and the actual value of beta is usually not well defined, it would be desirable to develop a bias circuit that is less dependent, or in fact, independent of the transistor beta.

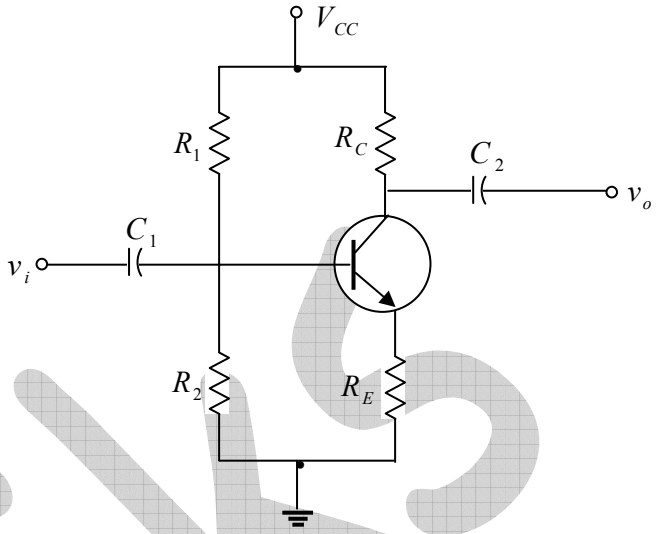


Figure 3.35: Voltage-divider bias configuration.

The voltage-divider bias configuration of figure 3.35 is such a network. If analyzed on an exact basis the sensitivity to changes in beta is quite small.

3.7.1 *Q*-point

The input side of the network can be redrawn as shown in figure 3.36 for the dc analysis. The Thevenin equivalent network for the network to the left of the base terminal can then be found in the following manner.

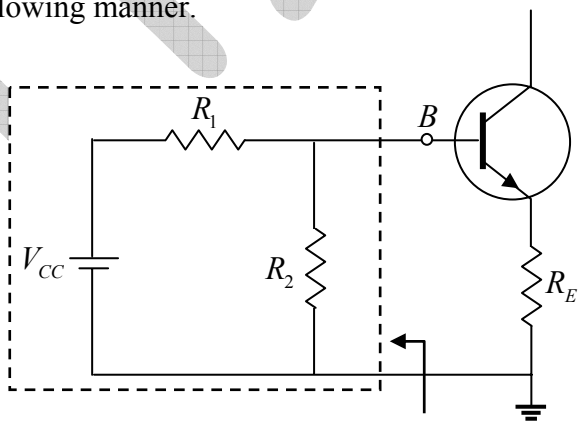


Figure 3.36: Redrawing the input side of the network of figure 3.35.

Thevenin's Resistance (R_{Th}):

The voltage source is replaced by a short-circuit equivalent as shown in figure 3.37.

$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2}$$

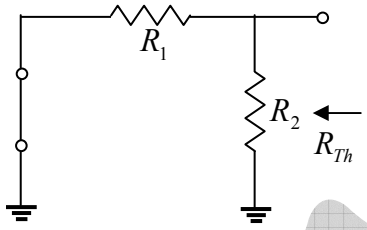


Figure 3.37: Determining R_{TH}.

Thevenin's Voltage (E_{Th}):

The voltage source V_{CC} is returned to the network and the open-circuit Thevenin voltage of figure 3.38 determined as follows:

Applying the voltage-divider rule:

$$E_{Th} = V_{R_2} = \frac{R_2 V_{CC}}{R_1 + R_2}$$

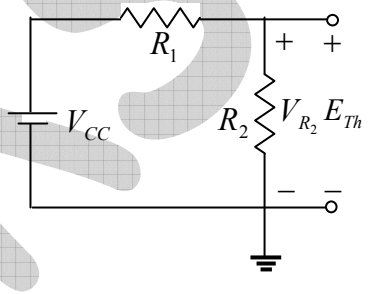


Figure 3.38: Determining E_{TH}.

The Thevenin network is then redrawn as shown in figure 3.39 and I_{B_Q} can be determined by first applying Kirchhoff's voltage law in the clockwise direction for the loop indicated: $-E_{Th} + I_B R_{Th} + V_{BE} + I_E R_E = 0$

$$I_B = \frac{E_{Th} - V_{BE}}{R_{Th} + (\beta + 1) R_E} \qquad \because I_E = (\beta +) I_B$$

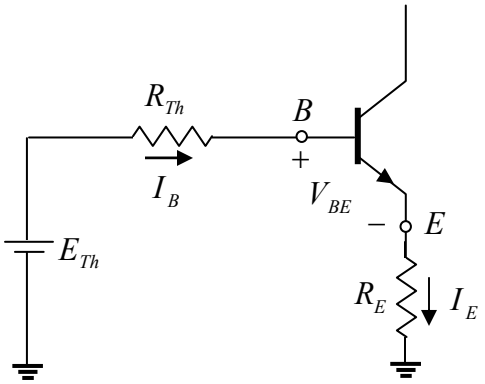


Figure 3.39: Thevenin equivalent circuit.

Once I_B is known the remaining quantities of the network can be found in the same manner as developed for the emitter-bias configuration. That is $V_{CE} = V_{CC} - I_C (R_C + R_E)$.

The remaining equations for V_E, V_C and V_B are also the same as obtained for the emitter-bias configuration.

3.8.2 Transistor Saturation

The output collector-emitter circuit for the voltage-divider configuration has the same appearance as the emitter-biased circuit. The resulting equation for the saturation current (when V_{CE} is set to zero volts on the schematic) is therefore the same as obtained for the emitter-biased configuration. That is, $I_{C_{sat}} = I_{C_{\max}} = \dfrac{V_{CC}}{R_C + R_E}$.

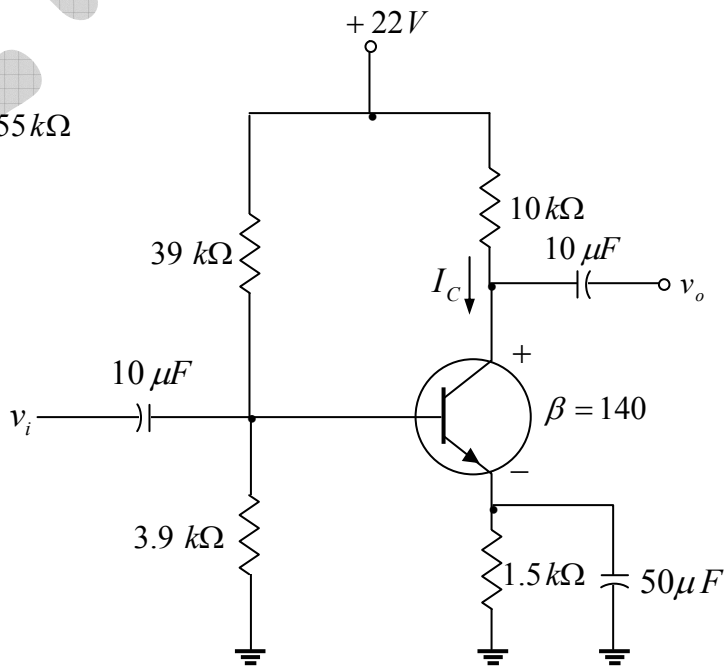
3.8.3 Load-Line Analysis

The similarities with the output circuit of the emitter-biased configuration result in the same intersections for the load line of the voltage-divider configuration. The load line will therefore have the same appearance as that of figure 3.24, with

$$I_{C_{sat}} = \left.\frac{V_{CC}}{R_C + R_E}\right|_{V_{CE}=0V} \quad \text{and} \quad V_{CE} = V_{CC}\big|_{I_C=0mA}$$

The level of I_B is of course determined by a different equation for the voltage-divider bias and the emitter-bias configurations.

Example: Determine the dc bias voltage V_{CE} and the current I_C for the voltage-divider configuration of figure shown below.



Solution: $R_{TH} = R_1 \,||\, R_2 = \dfrac{(39k\Omega)(3.9k\Omega)}{39k\Omega + 3.9k\Omega} = 3.55\,k\Omega$

$$E_{Th} = \frac{R_2 V_{CC}}{R_1 + R_2} = \frac{(3.9\,k\Omega)(22\,V)}{39\,k\Omega + 3.9\,k\Omega} = 2V$$

$$I_B = \frac{E_{Th} - V_{BE}}{R_{Th} + (\beta+1)R_E} = \frac{2V - 0.7V}{3.55k\Omega + (141)(1.5k\Omega)}$$

$$\Rightarrow I_B = \frac{1.3V}{3.55\ \ k\Omega + 211.5\ \ k\Omega} = 6.05\ \mu A$$

$$\Rightarrow I_C = \beta I_B = (140)(6.05\ \mu A) = 0.85\ mA$$

$$\because V_{CE} = V_{CC} - I_C(R_C + R_E)$$

$$\Rightarrow V_{CE} = 22 - (0.85\,mA)(10k\Omega + 1.5k\Omega) = 12.22\ V$$

3.8 DC Bias with Voltage Feedback

An improved level of stability can also be obtained by introducing a feedback path from collector to base as shown in figure 3.40.

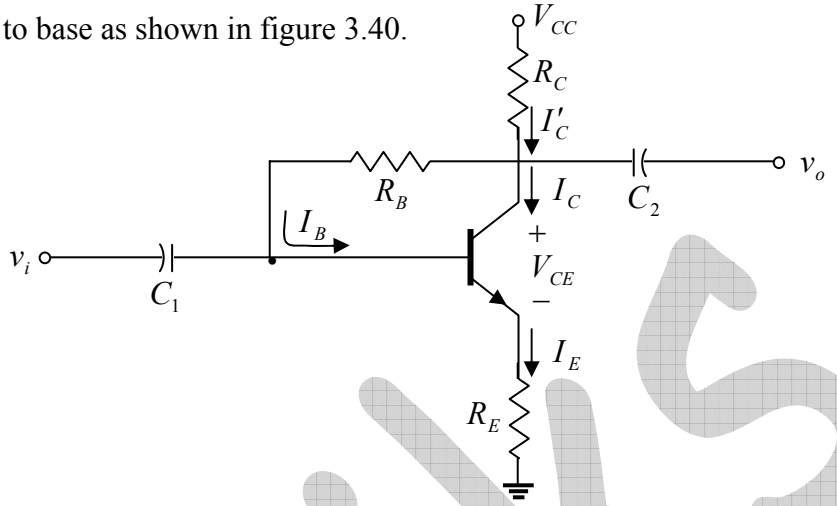


Figure 3.40: DC Bias with voltage feedback.

3.8.1 *Q*-point (Base-Emitter Loop)

Figure 3.41 shows the base-emitter loop for the voltage feedback configuration.

Writing Kirchhoff's voltage law around the indicated loop in the clockwise direction will result in

$$-V_{CC} + I'_C R_C + I_B R_B + V_{BE} + I_E R_E = 0$$

It is important to note that the current through R_C is not I_C but I'_C (where $I'_C = I_C + I_B \approx I_C$). Thus

$$V_{CC} - \beta I_B R_C - I_B R_B - V_{BE} - \beta I_B R_E = 0$$

$$\Rightarrow I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta\left(R_C + R_E\right)}.$$

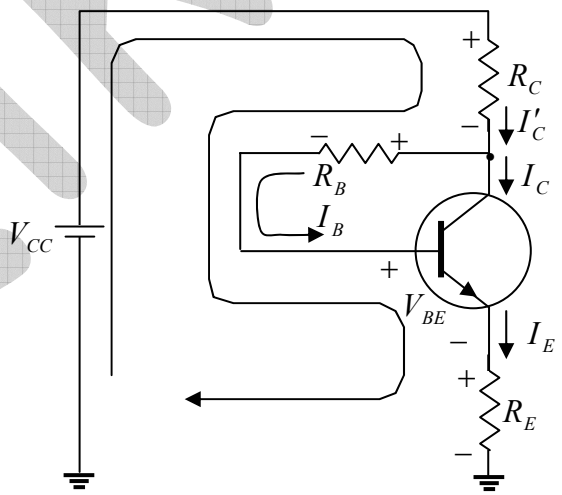


Figure 3.41: Base-emitter loop for the network of Figure 3.40.

In general, therefore, the feedback path results in a reflection of the resistance R_C back to the input circuit, much like the reflection of R_E.

In general, the equation for I_B had the following format: $I_B = \dfrac{V'}{R_B + \beta R'}$ with the absence of R' for the fixed-bias configuration, $R' = R_E$ for the emitter-bias setup (with $(\beta + 1) \cong \beta$), and $R' = R_C + R_E$ for the collector-feedback arrangement. The voltage V' is the difference between two voltage levels.

$$\Rightarrow I_{C_Q} = \frac{\beta V'}{R_B + \beta R'} \qquad \because I_C = \beta I_B$$

Collector-Emitter Loop

The collector-emitter loop for the network is provided in figure 3.42. Applying Kirchhoff's voltage law around the indicated loop in the clockwise direction will result in

$$-V_{CC} + I'_C R_C + V_{CE} + I_E R_E = 0$$

$$-V_{CC} + I_C (R_C + R_E) + V_{CE} = 0$$

$\because I'_C = I_C$ and $I_E = I_C$

$$\Rightarrow V_{CE} = V_{CC} - I_C (R_C + R_E)$$

which is exactly as obtained for the emitter-bias and voltage-divider bias configurations.

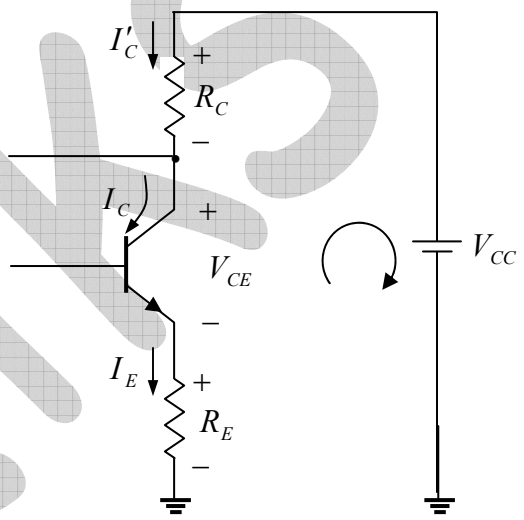


Figure 3.42: Collector-emitter loop for the network of figure 3.40.

3.8.2 Saturation Conditions

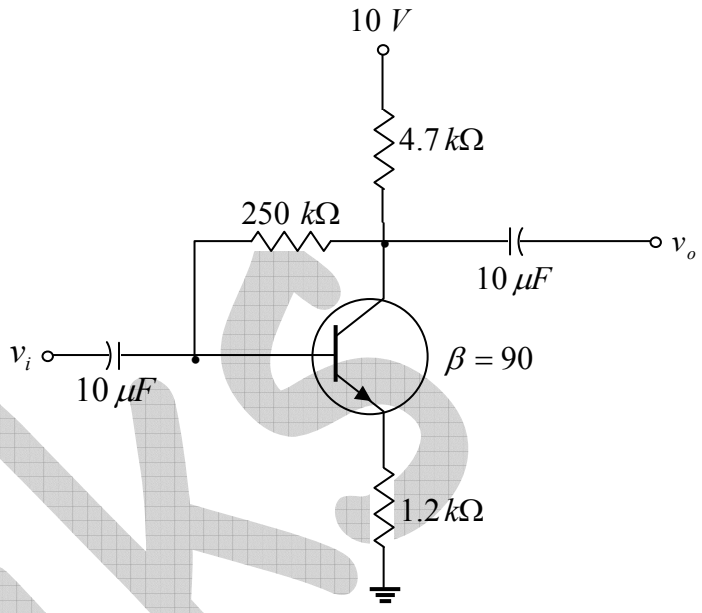
Using the approximation $I'_C = I_C$ the equation for the saturation current is the same as obtained for the voltage-divider and emitter-bias configurations. That is,

$$I_{C_{sat}} = I_{C_{\max}} = \frac{V_{CC}}{R_C + R_E}$$

3.8.3 Load-Line Analysis

Continuing with the approximation $I'_C = I_C$ will result in the same load line defined for the voltage-divider and emitter-biased configurations. The level of I_{B_Q} will be defined by the chosen bias configuration.

Example: Determine the quiescent levels of I_{C_Q} and V_{CE_Q} for the network of figure shown below.



Solution:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta(R_C + R_E)}$$

$$I_B = \frac{10V - 0.7V}{250\ \ k\Omega + (90)(4.7\ \ k\Omega + 1.2\ \ k\Omega)}$$

$$I_B = \frac{9.3V}{25.\ \ k\Omega + 531\ \ k\Omega} = \frac{9.3V}{781\ \ k\Omega}$$

$$I_B = 11.91\ \mu A$$

$$I_{C_Q} = \beta I_B = (90)(11.91\mu\text{A}) = 1.07\ mA$$

$$\because V_{CE_Q} = V_{CC} - I_C(R_C + R_E)$$

$$\Rightarrow V_{CE_Q} = 10 - (1.07mA)(4.7k\Omega + 1.2k\Omega) = 3.69\ V$$

Multiple Choice Questions (MCQ)

Q1. The correct sequence of transistor configuration for input resistance is

(a) $CB < CE < CC$ (b) $CC < CE < CB$

(c) $CB \approx CE < CC$ (d) $CB \approx CE \approx CC$

Q2. The correct sequence transistor configuration for output resistance is

(a) $CB < CE < CC$ (b) $CC < CE < CB$

(c) $CB \approx CE < CC$ (d) $CB \approx CE \approx CC$

Q3. Which of the following is true regarding common emitter transistor amplifier?

(a) It is used for impedance matching

(b) It is also known as emitter follower

(c) Output voltage is 180^o out of phase with respect to input voltage.

(d) It is used in high frequency application.

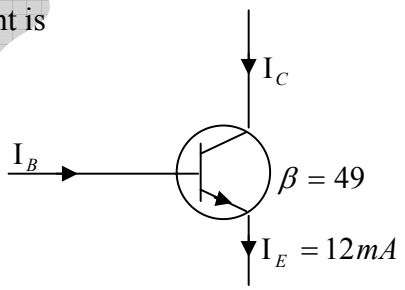
Q4. A transistor circuit in common emitter configuration is shown in the figure with given parameters. The value of collector current is

(a) 10.8 mA

(b) 11.52 mA

(c) 11.76 mA

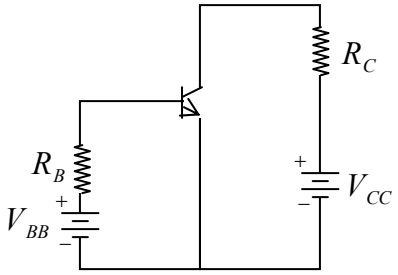
(d) 11.88 mA

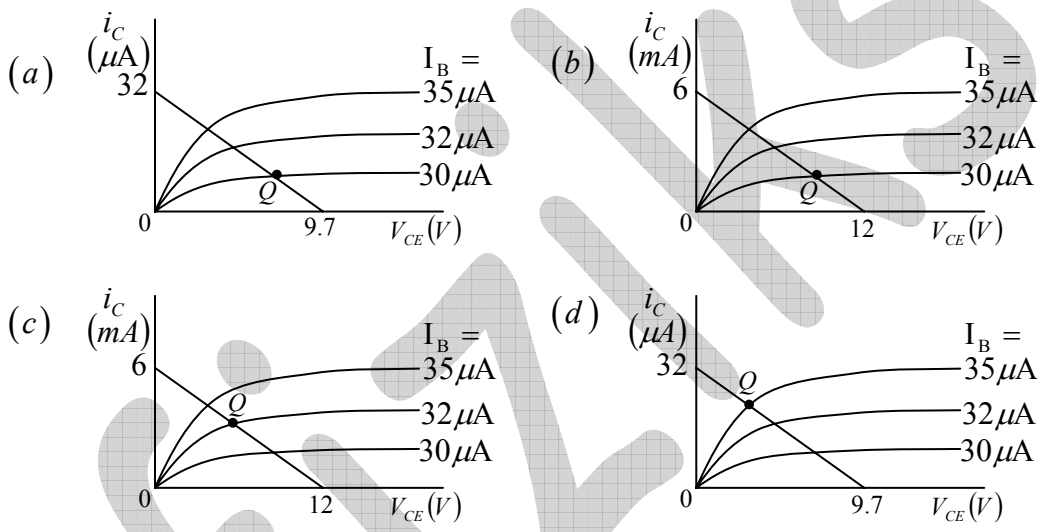


Q5. A transistor have $\alpha_{dc} = 0.98$, collector to base leakage current $I_{CBO} = 4\,\mu A$ and the base current is $50\,\mu A$. Then the value of emitter current is

(a) 1.0 mA (b) 2.5 mA (c) 4.0 mA (d) 5.0 mA

Q6. A silicon transistor with built-in voltage $0.7V$ is used in the circuit shown, with $V_{BB} = 9.7V$, $R_B = 281k\Omega$, $V_{CC} = 12V$ and $R_C = 2k\Omega$. Which of the following figures correctly represents the load line and quiescent Q-point?





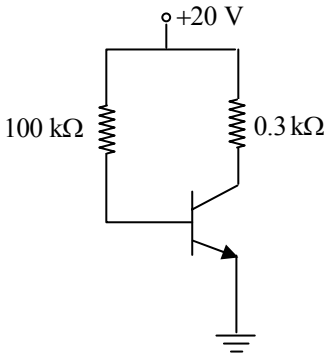
Q7. Consider the following circuit in which the current gain β_{dc} of the transistor is 100 and $V_{BE} = 0.7V$. Which one of the following correctly represents the collector current I_C and collector-emitter voltage V_{CE}?

(a) $23.7mA$, $8.9V$

(b) $18.7mA$, $12.9V$

(c) $18.7mA$, $8.9V$

(d) $23.7mA$, $12.9V$



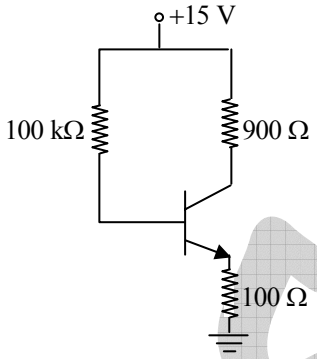
Q8. Consider the following circuit in which the current gain β_{dc} of the transistor is 100 and $V_{BE} = 0.7V$. Which one of the following correctly represents the collector current I_C and collector-emitter voltage V_{CE}?

(a) $12mA,\ 0.5V$

(b) $14mA,\ 1V$

(c) $16mA,\ 2V$

(d) $18mA,\ 3V$



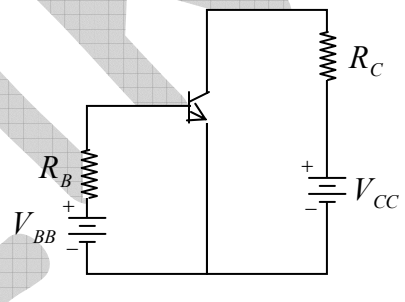
Q9. A silicon transistor with built-in voltage $0.7V$ is used in the circuit shown below, with $V_{BB} = 9.7V$, $R_B = 300k\Omega$, $V_{CC} = 12V$, $\beta = 100$ and $R_C = 2k\Omega$. Which of the following correctly represent the quiescent Q-point?

(a) $(3mA, 6V)$

(b) $(6mA, 6V)$

(c) $(3mA, 3V)$

(d) $(6mA, 3V)$



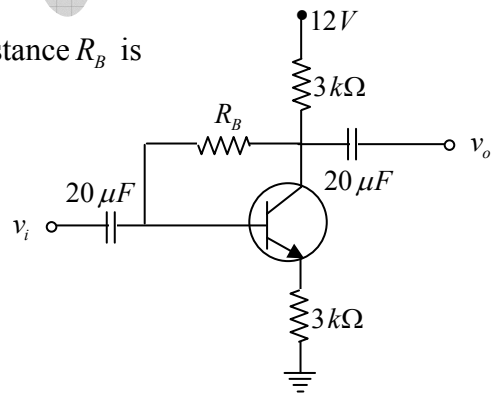
Q10. The current gain of the transistor in the following circuit is $\beta_{dc} = 100$ and $I_C = 1.6\ mA$. The value of base resistance R_B is

(a) 150 $k\Omega$

(b) 200 $k\Omega$

(c) 250 $k\Omega$

(d) 300 $k\Omega$



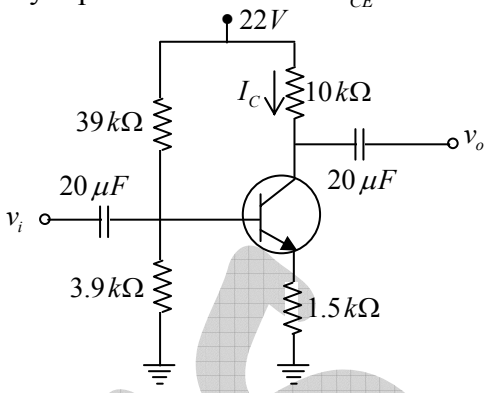
Q11. A silicon transistor with built-in voltage $0.7V$ is used in the circuit shown below, with $\beta = 140$. Which of the following correctly represent the value of V_{CE}?

(a) $10\,V$

(b) $12\,V$

(c) $14\,V$

(d) $16\,V$



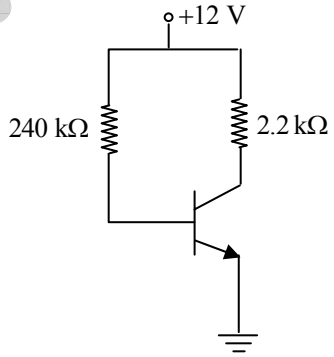
Numerical Answer Type Questions (NAT)

Q12. A power amplifier has gain of $20\ dB$. If the the output power is $150\ W$ then the input power to the amplifier is .....*Watt*

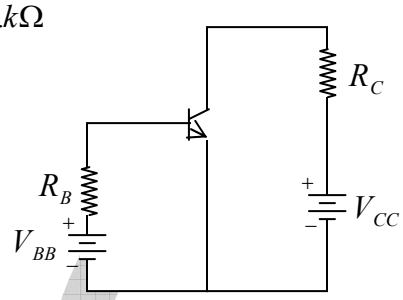
Q13. In a transistor, the change in base current from $100\,\mu A$ to $125\,\mu A$, causes a change in collector current from $5\,mA$ to $7.5\,mA$ keeping collector–to–emitter voltage constant at $10\,V$. Base to emitter voltage is $0.7V$. Then the current gain (β) of the transistor is……

Q14. The leakage current of a transistor are $I_{CBO} = 5\,\mu A$ and $I_{CEO} = 0.4\,mA$. The base current is $30\,\mu A$ and $V_{BE} = 0.7V$. Then β for the transistor is……...

Q15. Consider the following circuit in which the current gain β_{dc} of the transistor is 500 and $V_{BE} = 0.7V$. Then the collector current I_C is .....*mA*



Q16. A silicon transistor with built-in voltage $0.7V$ is used in the circuit shown below, with $V_{BB} = 9.7V$, $V_{CC} = 12V$, $\beta = 100$, $R_B = 300k\Omega$ and $V_{CE} = 6V$. Which of the following correctly represent the value of R_C is .....$k\Omega$



Multiple Select Type Questions (MSQ)

Q17. Which of the following statements are correct?

(a) When transistor is operating in active region input section is forward biased and output section reverse biased.

(b) When transistor is operating in saturation region both sections are forward biased.

(c) When transistor is operating in cut off region both sections are reversed biased.

(d) Two $p-n$ junction diodes connected back to back can be used as a transistor.

Q18. Which of the following statements are correct?

(a) The correct relation between α and β is $\beta = \dfrac{\alpha}{1-\alpha}$.

(b) Collector current in C.B. configuration can be expressed by $I_C = \alpha I_E + I_{CBO}$.

(c) Collector current is C.E. configuration can be expressed by $I_C = \beta I_B + I_{CEO}$.

(d) I_{CEO} is less than I_{CBO} and does not depend on temperature.

Q19. Which of the following are the characteristics of a common collector transistor amplifier?

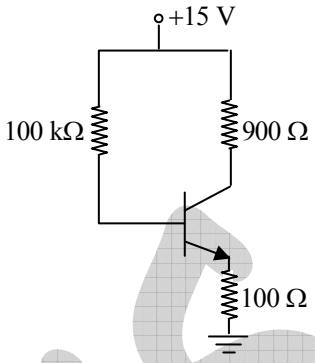
(a) Low voltage gain [≈ 1]

(b) High current gain

(c) High input impedance

(d) High output impedance

Q20. Consider the following circuit in which the current gain β_{dc} of the transistor is 100 and $V_{BE} = 0.7V$. Which one of the following correctly represents the collector current I_C and collector-emitter voltage V_{CE}?



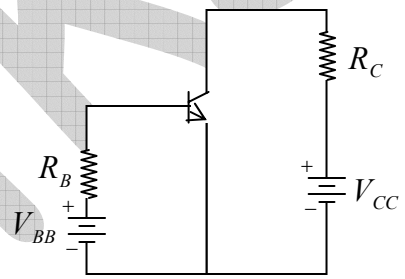
(a) $7\ mA$

(b) $14\ mA$

(c) $1\ V$

(d) $2\ V$

Q21. A silicon transistor with built-in voltage $0.7V$ is used in the circuit shown below, with $V_{BB} = 9.7V$, $V_{CC} = 12V$, $\beta = 100$, $R_C = 2k\Omega$ and $I_C = 3mA$. Which of the following are correct?



(a) $I_B = 30\ \mu A$

(b) $I_B = 40\ \mu A$

(c) $R_B = 300k\Omega$

(d) $R_B = 400k\Omega$

Solution

(MCQ)

Ans.1: (a)

Ans.2: (b)

Ans.3: (c)

Ans.4: (c)

$$I_E = I_B + I_C = \frac{I_C}{\beta} + I_C \Rightarrow I_C = \frac{\beta I_E}{\beta + 1} = 11.76\,mA$$

Ans.5: (b)

$$I_E \approx I_C = \frac{\alpha I_B}{1-\alpha} + \frac{I_{CBO}}{1-\alpha} = \frac{0.98 \times 50 \times 10^{-3}}{0.02} + \frac{4 \times 10^{-3}}{0.02} = 2.45\,mA$$

Ans.6: (c)

$$V_{CE} = V_{CC} - I_C R_C \text{ and } I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{9.7 - 0.7}{281 \times 10^3} = 32\,\mu A$$

On V_{CE}-axis $I_C = 0$ thus $V_{CE} = V_{CC} = 12\,V$ and on I_C-axis $V_{CE} = 0$ thus $I_C = \frac{V_{CC}}{R_C}$.

$$\Rightarrow I_C = \frac{12}{2} = 6\ mA.$$

Ans.7: (d)

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{24 - 0.7}{100}\,mA = 23.7 \times 10^{-2}\,mA$$

$$I_C \approx \beta I_B = 23.7mA\,, V_{CE} = V_{CC} - I_C R_C = 20 - 23.7 \times 0.3 = 12.9\,V$$

Ans.8: (b)

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + R_E} = \frac{15 - 0.7}{100 + 0.1}\,mA = \frac{14.3}{100.1}\,mA \approx 14\,mA$$

$$I_C \approx \beta I_B = 14mA\,, V_{CE} = V_{CC} - I_C\left(R_C + R_E\right) = 15 - \left(900 + 100\right) \times 14 \times 10^{-3} = 1.0V.$$

Ans.9: (a)

$$I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{9.7 - 0.7}{300 \times 10^3} = 30\mu A \Rightarrow I_C = \beta I_B = 100 \times 30 \times 10^{-6} = 3mA$$

and $V_{CE} = V_{CC} - I_C R_C = 12 - 3 \times 2 = 6V$

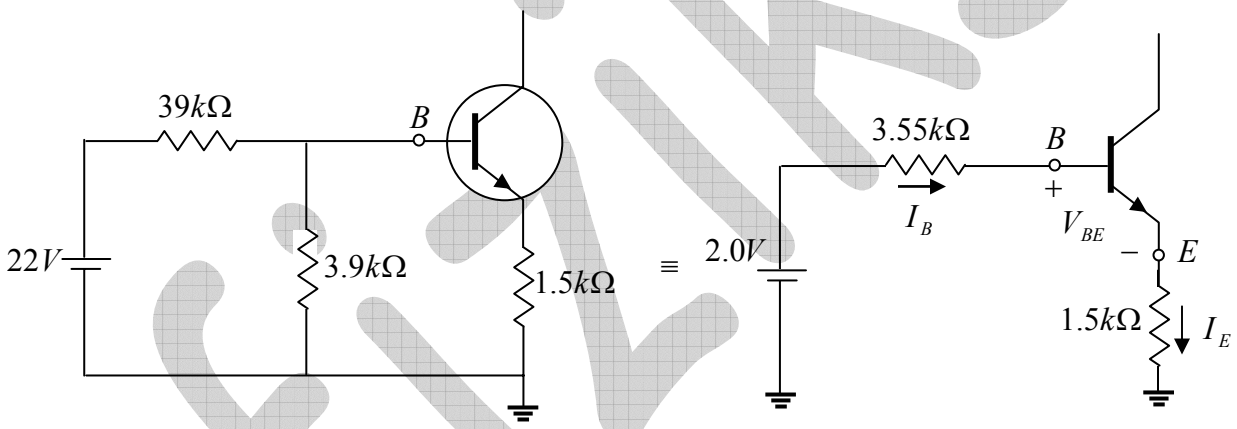
Ans.10: (a)

$$\Rightarrow I_C = 1.6\, mA = \beta I_B \Rightarrow I_B = 0.016\ mA$$

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta(R_C + R_E)} = \frac{12 - 0}{R_B + 100(3+3)} = 0.016\, mA \Rightarrow R_B = 150\, k\Omega$$

Ans.11: (b)

Drawing Thevenin's equivalent of input circuit



$$E_{Th} = \frac{3.9}{39 + 3.9} \times 22 = 2.0V\ ,\ R_{Th} = \frac{39 \times 3.9}{39 \times 3.9} = 3.55k\Omega$$

Applying KVL to input section,

$$-2.0 + 3.55 \times I_B + 0.7 + 1.5 \times I_E = 0 \Rightarrow -2.0 + 3.55 \times I_B + 0.7 + 1.5 \times \beta I_B = 0 \quad \because I_E \approx \beta I_B$$

$$I_B = \frac{2\,V - 0.7V}{3.55k + 140 \times 1.5k} = 6\,\mu A \Rightarrow I_C = \beta I_B = 0.85\,mA$$

$$V_{CE} = V_{CC} - I_C(R_C + R_E) = 22 - 0.85(10 + 1.5) = 22 - 9.78 = 12.22\ V$$

(NAT)

Ans.12: **1.5**

$$Power\ gain\ in\ dB = 10\log_{10}\frac{P_{out}}{P_{in}} \Rightarrow 20 = 10\log_{10}\frac{150}{P_{in}} \Rightarrow P_{in} = 1.5W$$

Ans.13: **100** $$\beta = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE}} = \frac{2.5mA}{25\mu A} = 100$$

Ans.14: **79**

$$I_{CEO} = (\beta+1)I_{CBO} \Rightarrow 0.4\,mA = (\beta+1)5\times10^{-3}\,mA \Rightarrow \beta = \frac{0.4}{5\times10^{-3}} - 1 = \frac{400}{5} - 1 = 79$$

Ans.15: 2.35

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12-0.7}{240}mA = 47\mu A,\quad I_C \approx \beta I_B = 50\times47\times10^{-3}\,mA = 2.35\,mA$$

Ans.16: **2**

$$I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{9.7-0.7}{300\times10^3} = 30\mu A \Rightarrow I_C = \beta I_B = 100\times30\times10^{-6} = 3mA$$

and $$V_{CE} = V_{CC} - I_C R_C \Rightarrow R_C = \frac{V_{CC} - V_{CE}}{I_C} = \frac{12-6}{3}k\Omega = 2k\Omega$$

MSQ

Ans.17: (a), (b) and (c)

Ans.18: (a), (b) and (c)

Ans.19: (a), (b) and (c)

Ans.20: (b), (c) $$I_B = \frac{V_{CC} - V_{BE}}{R_B + R_E} = \frac{15-0.7}{100+0.1}mA = \frac{14.3}{100.1}mA \approx 14mA$$

$$I_C \approx \beta I_B = 14mA, V_{CE} = V_{CC} - I_C(R_C + R_E) = 15 - (900+100)\times14\times10^{-3} = 1.0V.$$

Ans.21: (a), (c) $$\because I_C = \beta I_B \Rightarrow I_B = \frac{I_C}{\beta} = \frac{3}{100}mA = 30\mu A$$

$$I_B = \frac{V_{BB} - V_{BE}}{R_B} \Rightarrow \frac{9.7-0.7}{R_B} = 30\mu A \Rightarrow R_B = 300k\Omega$$

4. Operational Amplifier

4.1 Characteristics of an Op-Amp

4.1.1 Input Offset Voltage

Input offset voltage is the voltage that must be applied between the two input terminals of an op-amp to null the output, as shown in figure 4.1. In the figure V_{dc1} and V_{dc2} are dc voltages and R_s represent the source resistance.

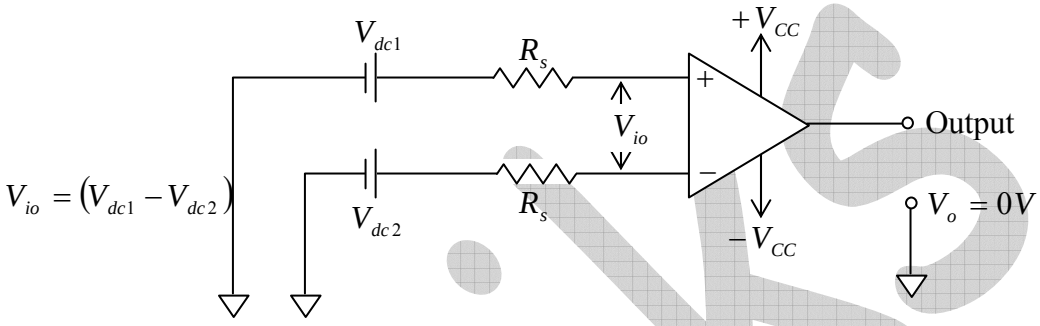


Figure 4.1: Defining input offset voltage.

4.1.2 Input Offset Current

The algebraic difference between the currents into the inverting and non-inverting terminal is referred to as *input offset current* I_{io}. Thus $I_{io} = |I_{B1} - I_{B2}|$ where I_{B1} is the current into the non-inverting input and I_{B2} is the current into the inverting input.

As the matching between two input terminals is improved, the difference between I_{B1} and I_{B2} becomes smaller; that is the I_{io} value decreases further.

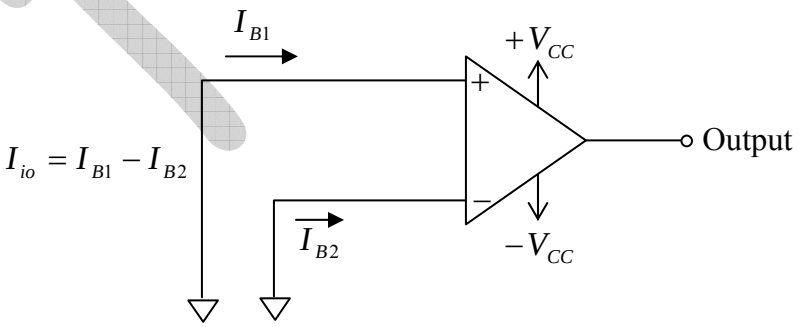


Figure 4.2: Defining input offset current.

4.1.3 Input Bias Current

Input bias current I_B is the average of the currents that flow into the inverting and non-inverting input terminals of the op-amp. In equation form $I_B = \frac{I_{B1} + I_{B2}}{2}$

4.1.4 Differential Input Resistance

Differential input resistance R_i (often referred to as input resistance) is the equivalent resistance that can be measured at either the inverting or non-inverting input terminal with the other terminal connected to ground.

4.1.5 Common-mode Rejection Ratio (*CMRR*)

The common-mode rejection ratio (*CMRR*) is defined as the ratio of the differential voltage gain A_d to the common-mode voltage gain A_{cm}; that is, $CMRR = \frac{A_d}{A_{cm}}$.

The differential voltage gain A_d is the same as the large-signal voltage gain A, which is specified on the data sheets; however, the common-mode voltage gain can be determined from the circuit shown in figure 4.3 and using the equation $A_{cm} = \frac{V_{ocm}}{V_{cm}}$ where

V_{ocm} = output common-mode voltage, V_{cm} = input common-mode voltage,

A_{cm} = common-mode voltage gain.

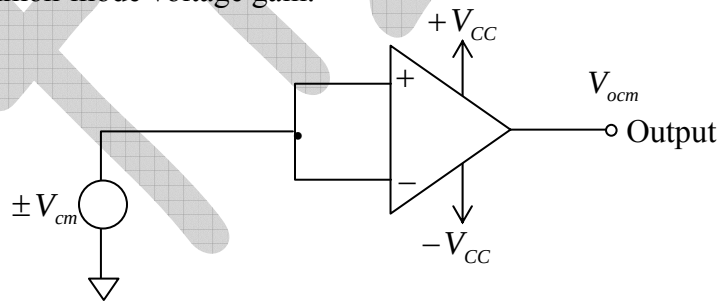


Figure 4.3: Common-mode configuration.

The *CMRR* can also be expressed as the ratio of the change in input offset voltage to the total change in common-mode voltage. Thus $CMRR = \frac{V_{io}}{v_{cm}}$

Thus $CMRR = \dfrac{A_d}{A_{cm}} = \dfrac{A_d}{v_{ocm}/v_{cm}} = \dfrac{A_d v_{cm}}{v_{ocm}} \Rightarrow v_{0cm} = \dfrac{A_d v_{cm}}{CMRR}$

The *CMRR* value is very large and is therefore usually specified in decibels (*dB*).

Thus $CMRR(dB) = 20\log\left(\dfrac{A_d}{A_{cm}}\right)$ or $CMRR(dB) = 20\log\left(\dfrac{V_{io}}{v_{cm}}\right)$.

Generally A_{cm} is very small and $A_d = A$ is very large; therefore, the *CMRR* is very large. The higher the value of *CMRR*, the better is the matching between two input terminals and the smaller is the output common-mode voltage.

4.1.6 Supply Voltage Rejection Ratio

The change in an op-amp's input offset voltage V_{io} caused by variations in supply voltages is called the *supply voltage rejection ratio* (*SVRR*). A variety of terms equivalent to *SVRR* are used by different manufacturers. Such as the *power supply rejection ratio* (*PSRR*) and the power supply sensitivity (*PSS*). These terms are expressed either in microvolt per volt or in decibels. If we denote the change in supply voltages by ΔV, *SVRR* can be defined as follows: $SVRR = \dfrac{\Delta V_{io}}{\Delta V}$ or $SVRR = 20\log\left(\dfrac{\Delta V}{\Delta V_{io}}\right)$.

This means that the lower the value of *SVRR* in microvolt/volt, the better for op-amp performance.

4.1.7 Large-signal Voltage Gain

Since the op-amp amplifies difference voltage between two input terminals, the voltage gain of the amplifier is defined as:

$$\text{Voltage Gain} = \frac{\text{Output Voltage}}{\text{Differential Input Voltage}} \Rightarrow A = \frac{V_o}{V_{id}}.$$

Because output signal amplitude is much larger than the input signal, the voltage gain is commonly called large-signal voltage gain.

4.1.8 Output Voltage Swing

The output voltage swing $V_{o\max}$ of the Op-Amp is guaranteed to be between $+V_{CC}$ and $-V_{CC}$. In fact, the output voltage swing indicates the values of positive and negative saturation voltages of the op-amp. The output voltage never exceeds these limits for given supply voltages $+V_{CC}$ and $-V_{CC}$.

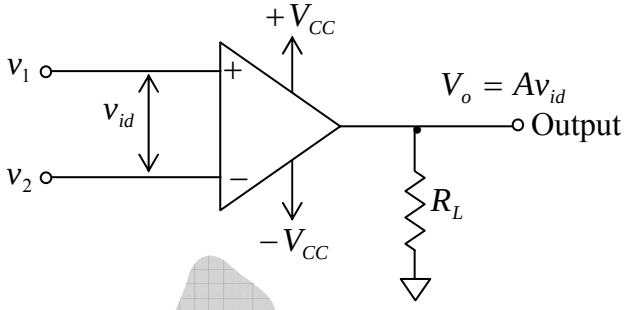


Figure 4.4: Determining voltage gain.

4.1.9 Output Resistance

Output resistance R_o is the equivalent resistance that can be measured between the output terminal of the op-amp and the ground.

4.1.10 Transient Response

The response of any practically useful network to a given input is composed of two parts: the *transient* and *steady-state* response. The transient response is that portion of the complete response before the output attains some fixed value. Once reached, this fixed value remains at that level and is, therefore, referred to as a steady-state value. The response of the network after it attains a fixed value is independent of time and is called the steady-state response. Unlike the steady-state response, the transient response is time variant. The rise time and the percent of overshoot are the characteristics of the transient response. The time required by the output to go from 10% to 90% of its final value is called the *rise time*. Conversely, overshoot is the maximum amount by which the output deviates from the steady-state value. Overshoot is generally expressed as a percentage. Smaller the value of rise time larger is the bandwidth.

4.1.11 Slew Rate

Sew rate (*SR*) is defined as the maximum rate of change of output voltage per unit of time and is expressed in volts per microseconds. $SR = \left.\frac{dV_0}{dt}\right|_{\text{maximum}} \quad V/\mu s$

Slew rate indicates how rapidly the output of an op-amp can change in response to changes in the input frequency. The slew rate changes with change in voltage gain and is normally specified at unity (+1) gain. The slew rate of an op-amp is fixed; therefore, if the slope requirements of the output signal are greater than the slew rate, then distortion occurs. Thus slew rate is one of the important factors in selecting the op-amp for ac applications, particularly at relatively high frequencies.

4.1.12 Gain-Bandwidth Product

The gain-bandwidth product (*GB*) is the bandwidth of the op-amp when the voltage gain is 1. Equivalent terms for gain-bandwidth product are closed-loop bandwidth, unity gain bandwidth, and small-signal bandwidth.

4.1.13 The Ideal Op-Amp

An ideal op-amp would exhibit the following electrical characteristics:

1. Infinite voltage gain A.
2. Infinite input resistance R_i so that almost any signal source can drive it and there is no loading of the preceding stage.
3. Zero output resistance R_o so that output can drive an infinite number of other devices.
4. Zero output voltage when input voltage is zero.
5. Infinite bandwidth so that any frequency signal from 0 to ∞ *Hz* can be amplified without attenuation.
6. Infinite common-mode rejection ratio so that the output common-mode noise voltage is zero.
7. Infinite slew rate so that output voltage changes occur simultaneously with input voltage changes.

There are practical op-amps that can be made to approximate some of these characteristics using a negative feedback arrangement. In particular, the input resistance, output resistance, and bandwidth can be brought close to ideal values by this method.

4.1.14 Equivalent Circuit of an Op-Amp

Figure 4.5 shows an equivalent circuit of an op-amp. This circuit includes important values from the data sheets: A, R_i and R_o. Note that Av_{id} is an equivalent Thevenin voltage source, and R_o is the Thevenin equivalent resistance looking back into the output terminal of an op-amp.

The equivalent circuit is useful in analyzing the basic operating principles of op-amps and in observing the effects of feedback arrangements. For the circuit shown, the output voltage is: $v_0 = Av_{id} = A(v_1 - v_2)$

where A = large-signal voltage gain, v_{id} = difference input voltage

v_1 = voltage at the non-inverting input terminal with respect to ground

v_2 = voltage at the inverting terminal with respect to ground.

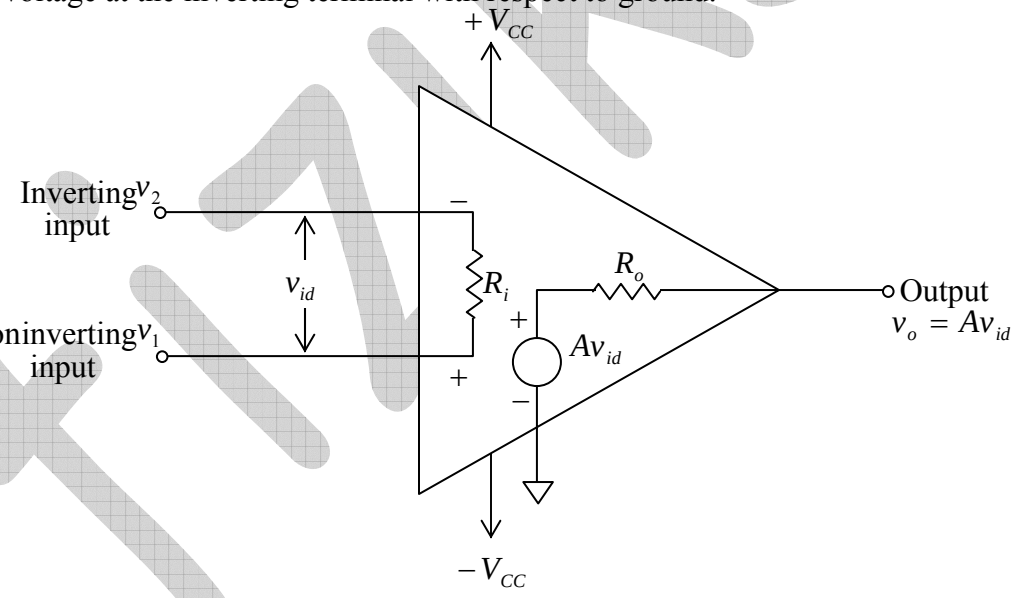


Figure 4.5: Equivalent circuit of an op-amp.

Equation $v_0 = Av_{id} = A(v_1 - v_2)$ indicates that the output voltage v_0 is directly proportional to the algebraic difference between the two input voltages. In other words, the op-amp amplifies the difference between the two input voltages; it does not amplify the input voltages themselves. For this reason the polarity of the output voltage depends on the polarity of the difference voltage.

4.1.15 Ideal Voltage Transfer Curve

Equation $v_0 = Av_{id} = A(v_1 - v_2)$ is the basic op-amp equation, in which the output offset voltage is assumed to be zero. This equation is useful in studying the op-amp's characteristics and in analyzing different circuit configurations that employ feedback. The graphic representation of this equation is shown in figure 4.6, where the output voltage v_0 is plotted against input difference voltage v_{id} keeping gain A constant.

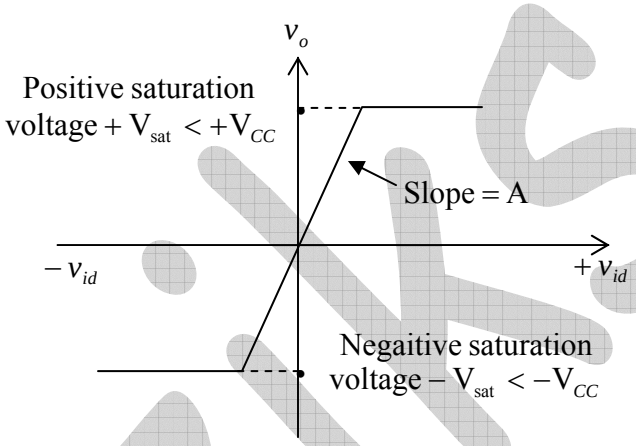


Figure 4.6: Ideal voltage transfer curve.

4.2 Open-Loop Op-Amp Configurations

In the case of amplifiers the term *open loop* indicates that no connection, either direct or via another network, exists between the output and input terminals. That is, the output signal is not fed back in any form as part of the input signal, and the loop that would have been formed with feedback is open.

When connected in open-loop configuration, the op-amp simply functions as a high-gain amplifier. There are three open-loop op-amp configurations:

1. Differential amplifier
2. Inverting amplifier
3. Non-inverting amplifier

These configurations are classed according to the number of inputs used and the terminal to which the input is applied when a single input is used.

4.2.1 The Differential Amplifier

Figure 4.7 shows the open-loop differential amplifier in which input signals v_{in1} and v_{in2} are applied to the positive and negative input terminals. Since the op-amp amplifies the difference between the two input signals, this configuration is called the *differential amplifier*. The op-amp is a versatile device because it amplifies both ac and dc voltages. The source resistances R_{in1} and R_{in2} are normally negligible compared to the input resistance R_i. Therefore, the voltage drops across these resistors can be assumed to be zero, which then implies that $v_1 = v_{in1}$ and $v_2 = v_{in2}$ thus $v_0 = A(v_{in1} - v_{in2})$.

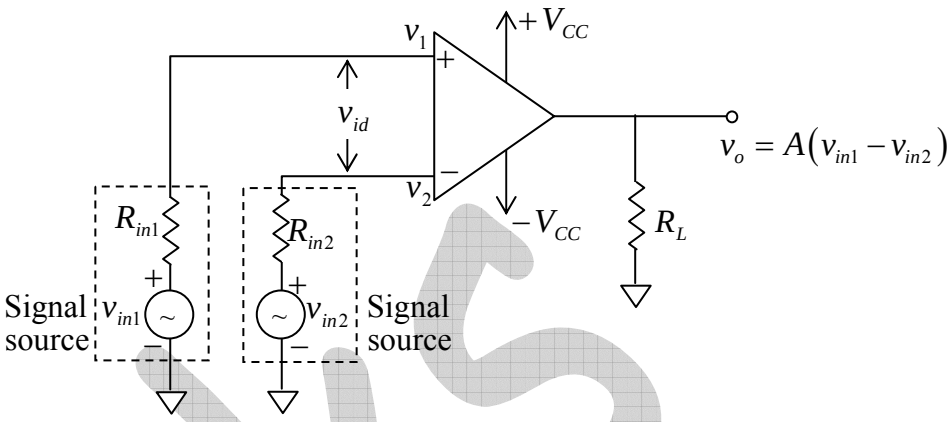


Figure 4.7: Differential amplifier.

4.2.2 The inverting Amplifier

In the inverting amplifier only one input is applied and that is to the inverting input terminal. The non-inverting input terminal is grounded as shown in figure 4.8.

$\because v_1 = 0\,V$ and $v_2 = v_{in}$ thus $v_o = -Av_{in}$.

The negative sign indicates that the output voltage is out of phase with respect to input by 180^0 or is of opposite polarity. Thus in the inverting amplifier the input signal is amplified by gain *A* and is also inverted at the output.

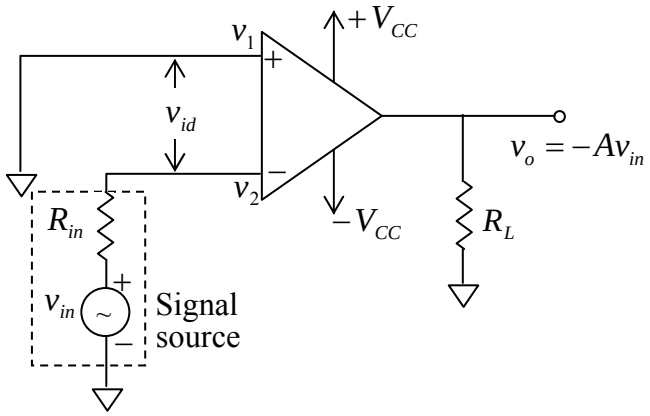


Figure 4.8: Inverting amplifier.

4.2.3 The Non-inverting Amplifier

Figure 4.9 shows the open-loop non-inverting amplifier. In this configuration the input is applied to the non-inverting input terminal, and the inverting terminal is connected to ground.

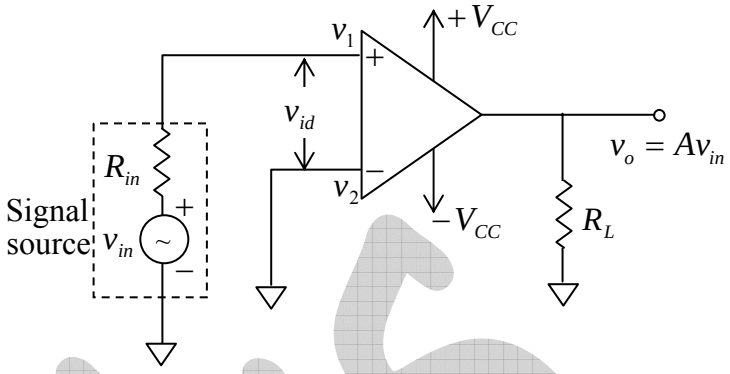


Figure 4.9: Non-inverting amplifier.

In the circuit, $v_1 = v_{in}$ and $v_2 = 0\,V$, thus $v_o = Av_{in}$. This means that the output voltage is larger than the input voltage by gain A and is in phase with the input signal. In all three open-loop configurations any input signal (differential or single) that is only slightly greater than zero drives the output to saturation level. This results from the very high gain (A) of the op-amp.

NOTE: *Thus, when operated open-loop, the output of the op-amp is either negative or positive saturation or switches between positive and negative saturation levels. For this reason, open-loop op-amp configurations are not used in linear applications.*

4.3 An Op-Amp with Negative Feedback

Since the open-loop gain of the op-amp is very high, only the smaller signals (of the order of microvolt or less) having very low frequency may be amplified accurately without distortion. However, these small signals are very susceptible to noise and almost impossible to obtain in the laboratory.

Besides being large, the open-loop voltage gain of the op-amp is not a constant. The voltage gain varies with changes in temperature and power supply in open-loop op-amps, which makes the *open-loop op-amp* unsuitable for many linear applications. In most linear applications the output is proportional to the input and is of the same type.

In addition, the *bandwidth* (band of frequencies for which the gain remains constant) of most open-loop op-amps is negligibly small-almost zero. For this reason the open-loop op-amp is impractical in ac applications.

We can select as well as control the gain of the op-amp if we introduce a modification in the basic circuit. This modification involves the use of ***feedback***; that is, an output signal is fed back to the input either directly or via another network. If the signal fed back is of opposite polarity or out of phase by 180^o (or odd integer multiples of 180^o) with respect to the input signal, the feedback is called ***negative feedback***. An amplifier with negative feedback has a self-correcting ability against any change in output voltage caused by changes in environmental conditions.

On the other hand, if the signal fed back is in phase with the input signal, the feedback is called ***positive feedback***. In positive feedback the feedback signal aids the input signal. For this reason it is also referred to as regenerative feedback. Positive feedback is necessary in ***oscillator circuits***.

When used in amplifiers, negative feedback stabilizes the gain, increases the bandwidth, and changes the input and output resistances. Of course, the price paid for these improvements is reduced voltage gain. Other benefits of negative feedback include a decrease in harmonic or nonlinear distortion and reduction in the effect of input offset voltage at the output. Negative feedback also reduces the effect of variations in temperature and supply voltages on the output of the op-amp.

4.3.1 Block Diagram Representation of Feedback Configurations

An op-amp that uses feedback is called a feedback amplifier. A feedback amplifier is sometimes referred to as a closed-loop amplifier because the feedback forms a closed loop between the input and the output. A feedback amplifier essentially consists of two parts: an op-amp and a feedback circuit. The feedback circuit can take any form whatsoever depending on the intended application of the amplifier. This means that the feedback circuit may be made up of passive components, active components, or combinations of both. Here, in order to develop the basic feedback concepts, we use only purely ***resistive feedback circuits***.

A closed-loop amplifier can be represented by using two blocks, one for an op-amp and another for a feedback circuit. There are four ways to connect these two blocks. These connections are classified according to whether the voltage or current is fed back to the input in series or in parallel, as follows:

1. Voltage-series feedback
2. Voltage-shunt feedback
3. Current-series feedback
4. Current-shunt feedback

The four types of configurations are illustrated in figure 4.10. In figure 4.10 (a) and (b) the voltage across load resistor R_L is the input voltage to the feedback circuit. The feedback quantity (either voltage or current) is the output of the feedback circuit and is proportional to the output voltage. On the other hand, in the current-series and current-shunt feedback circuits of figure 4.10 (c) and (d) the load current i_L flows into the feedback circuit. The output of the feedback circuit (either voltage or current) is proportional to the load current i_L.

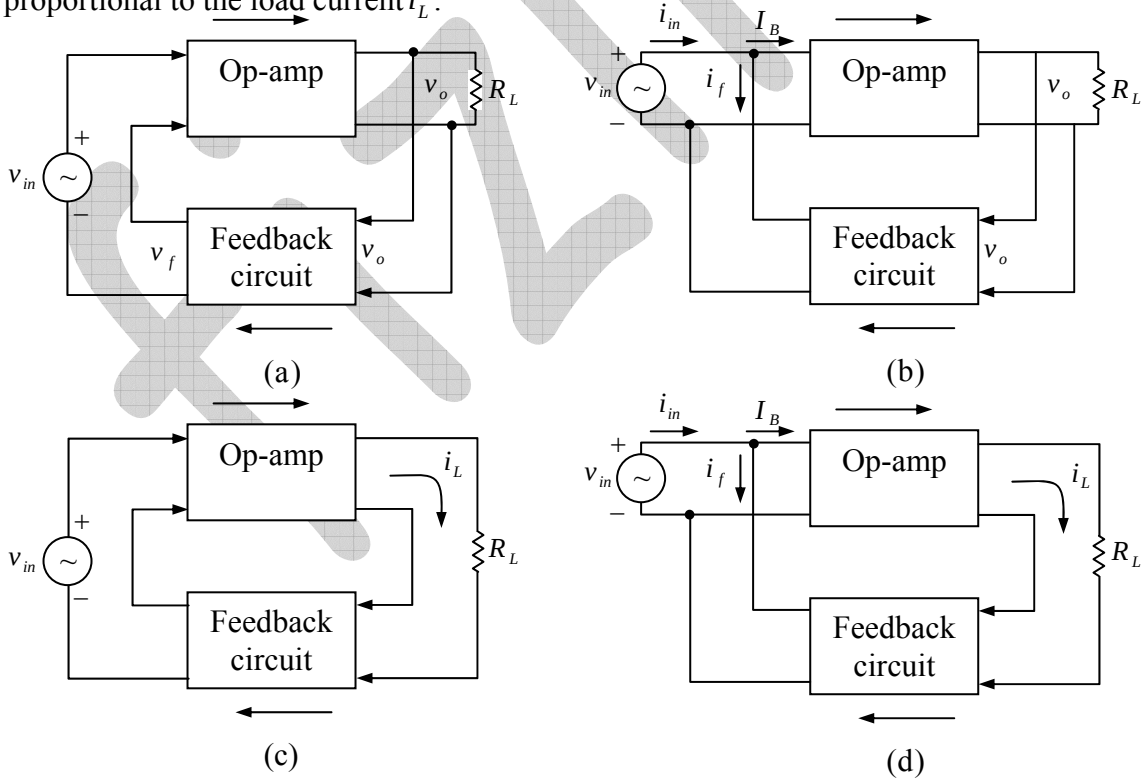


Figure 4.10: Feedback configurations. (a) Voltage-series. (b) Voltage-shunt. (c) Current-series. (d) Current-shunt.

The ***voltage-series*** and ***voltage-shunt feedback*** configurations are important because they are most commonly used. An in-depth analysis of these two configurations is presented here, computing voltage gain, input resistance, output resistance and bandwidth for each.

4.4 Voltage-Series Feedback Amplifier (Non-inverting feedback Amp)

The schematic diagram of the voltage-series feedback amplifier is shown in figure 4.11. The op-amp is represented by its schematic symbol, including its large-signal voltage gain A and the feedback of two resistors R_1 and R_F.

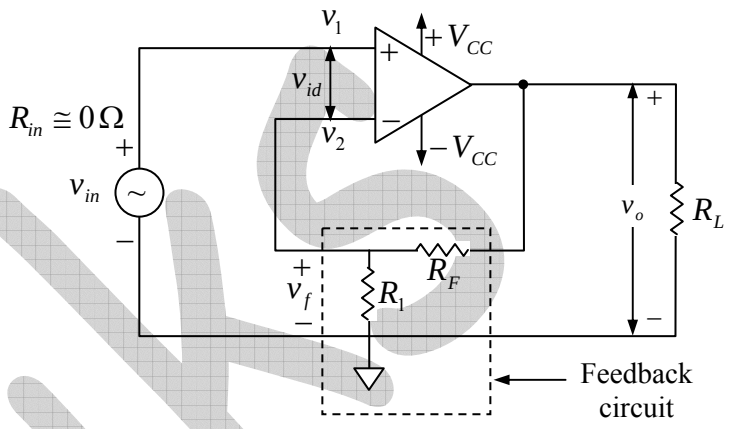


Figure 4.11: Voltage-series feedback amplifier.

The voltage gain for the op-amp with and without feedback, and the gain of the feedback circuit are defined as follows:

Open-loop voltage gain (or gain without feedback) $A = \dfrac{v_o}{v_{id}}$

Closed-loop voltage gain (or gain with feedback) $A_F = \dfrac{v_o}{v_{in}}$

Gain of the feedback circuit $B = \dfrac{v_f}{v_o}$

4.4.1 Negative Feedback

Referring to the circuit of figure 4.11, Kirchhoff's voltage equation for the input loop is: $v_{id} = v_{in} - v_f$ where v_{in} = input voltage, v_f = feedback voltage, v_{id} = difference input voltage.

From equation $v_{id} = v_{in} - v_f$, this difference voltage is equal to the input voltage v_{in} minus the feedback voltage v_f. In other words, the feedback voltage always opposes the input voltage (or is out of phase by 180^o with respect to the input voltage); hence the feedback is said to be negative.

4.4.2 Closed-Loop Voltage Gain

$\because v_o = A(v_1 - v_2)$ and $v_1 = v_{in}$, $v_2 = v_f = \dfrac{R_1 v_o}{R_1 + R_F}$

$$\Rightarrow v_o = A\left(v_{in} - \frac{R_1 v_o}{R_1 + R_F}\right) \Rightarrow v_o = \frac{A(R_1 + R_F)v_{in}}{R_1 + R_F + AR_1} \Rightarrow A_F = \frac{v_o}{v_{in}} = \frac{A(R_1 + R_F)}{R_1 + R_F + AR_1} \text{ (Exact)}$$

$$\Rightarrow \boxed{A_F = \frac{v_o}{v_{in}} = 1 + \frac{R_F}{R_1}} \text{ (Ideal)} \qquad \because AR_1 >> R_1 + R_F$$

The above equation is important because it shows that the gain of the voltage-series feedback amplifier is determined by the ratio of two resistors, R_1 and R_F.

As defined previously, the gain of the feedback circuit (B) is the ratio of v_f and v_o.

Thus $B = \dfrac{v_f}{v_o} = \dfrac{R_1}{R_1 + R_F}$, we can conclude that $A_F = \dfrac{1}{B}$ (ideal)

This means that the gain of the feedback circuit is the reciprocal of the closed-loop voltage gain. In other words for given R_1 and R_F the values of A_F and B are fixed. Finally, the closed-loop voltage gain A_F can be expressed in terms of open-loop gain A and feedback circuit gain B as follows. Thus $A_F = \dfrac{A\left(\dfrac{R_1 + R_F}{R_1 + R_F}\right)}{\dfrac{R_1 + R_F}{R_1 + R_F} + \dfrac{AR_1}{R_1 + R_F}} \Rightarrow \boxed{A_F = \dfrac{A}{1 + AE}}$

where AB = loop gain.

4.4.3 Difference Input Voltage Ideally Zero

$\because v_{id} = \dfrac{v_o}{A} \Rightarrow v_{id} \cong 0 \Rightarrow v_1 \cong v_2$ (Since A is very large (ideally infinite))

That is the voltage at the noninverting input terminal of an op-amp is approximately equal to that at the inverting input terminal provided that A is very large. This concept is useful in the analysis of closed-loop op-amp circuits.

For example, ideal closed-loop voltage gain can be obtained using the preceding results as follows. In the circuit of voltage series feedback amplifier,

$$v_1 = v_{in},\ v_2 = v_f = \frac{R_1 v_o}{R_1 + R_F} \Rightarrow v_{in} = \frac{R_1 v_o}{R_1 + R_F} \Rightarrow A_F = \frac{v_o}{v_{in}} = 1 + \frac{R_F}{R_1}.$$

4.4.4 Input Resistance with Feedback

Let R_i is the input resistance (open-loop) of the op-amp, and R_{iF} is the input resistance of the amplifier with feedback. Then $\boxed{R_{iF} = R_i(1+AB)}$

This means that the input resistance of the op-amp with feedback is $(1+AB)$ times that without feedback.

4.4.5 Output Resistance with Feedback

Output resistance is the resistance determined looking back into the feedback amplifier from the output terminal. Thus $\boxed{R_{oF} = \frac{R_o}{1+AB}}$

The result shows that the output resistance of the voltage-series feedback amplifier is $1/(1+AB)$ times the output resistance R_o of the op-amp. That is, the output resistance of the op-amp with feedback is much smaller than the output resistance without feedback.

4.4.6 Bandwidth with Feedback

The bandwidth of an amplifier is defined as the band (range) of frequencies for which the gain remains constant. $\boxed{f_F = f_0(1+AB)}$

Above equation indicate[illegible] of the noninverting amplifier with feedback, f_F is equal to its bandwidth without feedback, f_0 times $(1+AB)$.

4.4.7 Total Output Offset Voltage with Feedback

In an open-loop op-amp the total output offset voltage is equal to either the positive or negative saturation voltage. Since with feedback the gain of the non-inverting amplifier changes from A to $A/(1+AB)$ the total output offset voltage with feedback must also be $1/(1+AB)$ times the voltage without feedback. That is

$$\begin{pmatrix}\text{total output offset}\\ \text{voltage with feedback}\end{pmatrix} = \frac{\text{total ouput offset voltage without feedback}}{1+AB} \Rightarrow V_{ooT} = \frac{\pm V_{sat}}{1+AB}$$

From this analysis it is clear that the non-inverting amplifier with feedback exhibits the characteristics of the perfect voltage amplifier. That is, it has very high input resistance, very low output resistance, stable voltage gain, large bandwidth, and very little (ideally zero) output offset voltage.

4.4.8 Voltage Follower

The lowest gain that can be obtained from a non-inverting amplifier with feedback is 1. When the non-inverting amplifier is configured for unity gain, it is called a voltage follower because the output voltage is equal to and in phase with the input. In other words, in the voltage follower the output follows the input.

Although it is similar to the discrete emitter follower, the voltage follower is preferred because it has much higher input resistance, and the output amplitude is exactly equal to the input.

To obtain the voltage follower from the non-inverting amplifier simply open R_1 and short R_F. The resulting circuit is shown in figure 4.12. In this figure all the output voltage is fed back into the inverting terminal of the op-amp; consequently, the gain of the feedback circuit is $1\left(B = A_F = 1\right)$.

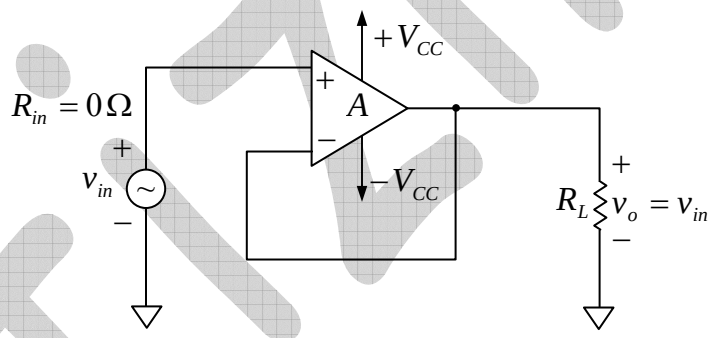


Figure 4.12: Voltage follower.

Thus $R_{iF} = AR_i$, $R_{oF} = \dfrac{R_o}{A}$, $f_F = Af_0$, $V_{ooT} = \dfrac{\pm V_{sat}}{A}$ $\quad \because (1+A) \cong A$

The voltage follower is also called a *non-inverting buffer* because, when placed between two networks, it removes the loading on the first network.

4.5 Voltage-Shunt Feedback Amplifier (Inverting Amplifier with Feedback)

Figure 4.13 shows the voltage shunt feedback amplifier using an op-amp. Note that the non-inverting terminal is grounded, and the feedback circuit has only one resistor R_F. However, an extra resistor R_1 is connected in series with the input signal source v_{in}.

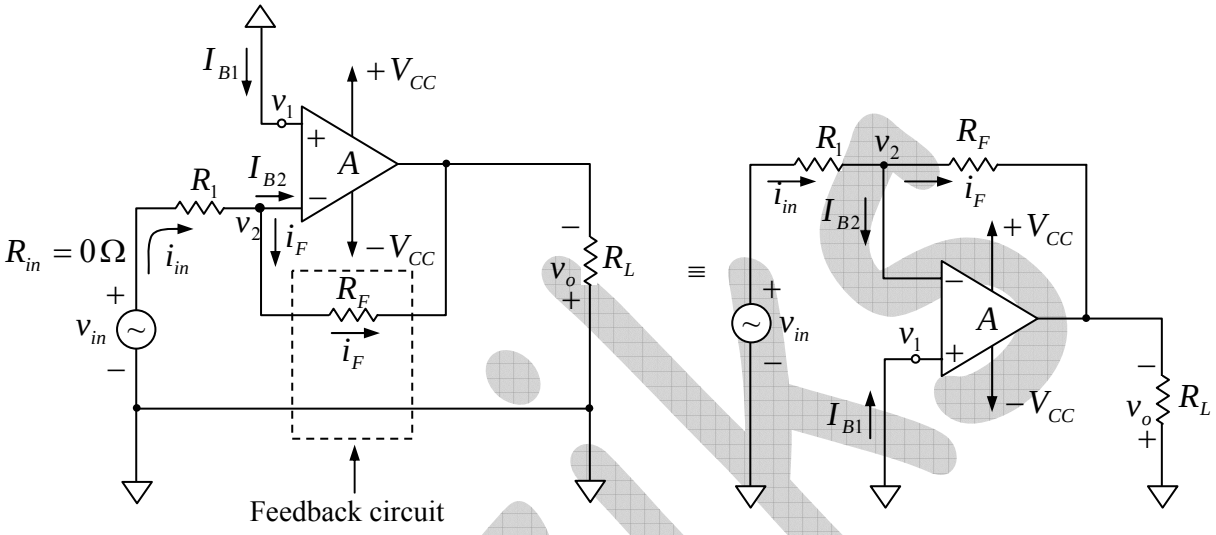


Figure 4.13: Voltage-shunt feedback amplifier.

4.5.1 Closed-Loop Voltage Gain

The closed-loop voltage gain A_F of the voltage-shunt feedback amplifier can be obtained by writing Kirchhoff's current equation at the input node v_2 as follows: $i_{in} = i_F + I_B$

Since R_i is very large, the input bias current I_B is negligibly small.

Therefore $i_{in} \cong i_F \Rightarrow \dfrac{v_{in}-v_2}{R_1} = \dfrac{v_2-v_0}{R_F}$ $\because v_1 - v_2 = \dfrac{v_0}{A}$ and $v_1 = 0\,V$, $v_2 = -\dfrac{v_0}{A}$.

$$\Rightarrow \frac{v_{in}+v_o/A}{R_1} = \frac{-(v_o/A)-v_o}{R_F} \quad \Rightarrow A_F = \frac{v_o}{v_{in}} = -\frac{AR_F}{R_1+R_F+AR_1} \quad \text{(Exact)}$$

$$\Rightarrow \quad \boxed{A_F = \frac{v_o}{v_{in}} = -\frac{R_F}{R_1}} \quad \text{(Ideal)} \qquad \because AR_1 >> R_1 + R_F$$

The negative sign in equation indicates that the input and output signals are out of phase by 180^o (or of opposite polarities). In fact, because of this phase inversion, the configuration is commonly called an inverting amplifier with feedback.

This equation shows that the gain of the inverting amplifier is set by selecting a ratio of feedback resistance R_F to the input resistance R_1. In fact, the ratios R_F / R_1 can be set to any value whatsoever, even to less than 1. Because of this property of the gain equation, the inverting amplifier configuration with feedback lends itself to a majority of applications as against those of the non-inverting amplifier.

$\because \dfrac{A\left(R_F \big/ R_1 + R_F\right)}{1+\dfrac{AR_1}{R_1+R_F}} = -\dfrac{AK}{1+AB}$ where $K = \dfrac{R_F}{R_1+R_F}$ is a voltage attenuation factor,

$B = \dfrac{R_1}{R_1+R_F}$ is gain of the feedback circuit.

4.5.2 Inverting Input Terminal at Virtual Ground

The difference input voltage is ideally zero; that is, the voltage at the inverting terminal (v_2) is approximately equal to that at the non-inverting terminal (v_1). In other words, the inverting terminal voltage v_2 is approximately at ground potential. Therefore, the inverting terminal is said to be at ***virtual ground***. This concept is extremely useful in the analysis of closed-loop inverting amplifier circuits. For example, ideal closed-loop gain can be obtained using the virtual-ground concept as follows:

$\because i_{in} \cong i_F \Rightarrow \dfrac{v_{in}-v_2}{R_1} = \dfrac{v_2-v_o}{R_F} \Rightarrow A_F = \dfrac{v_o}{v_{in}} = -\dfrac{R_F}{R_1}$ $\quad \because v_1 = v_2 = 0\ V$.

4.5.3 Input Resistance with Feedback

$$R_{iF} = R_1 + \frac{R_F}{1+A}\Big\|(R_i) \quad \text{(Exact)}$$

Since R_i and A are very large $\dfrac{R_F}{1+A}\Big\|R_i \cong 0\,\Omega$, hence $R_{iF} = R_1$ (ideal).

4.5.4 Output Resistance with Feedback

$$R_{oF} = \frac{R_o}{1+AB}$$

4.5.5 Bandwidth with Feedback

$$f_F = f_0(1+AB)$$

where f_0 = break frequency of the op-amp $= \dfrac{\text{unity gain bandwidth}}{\text{open - loop voltage gain}} = \dfrac{UGB}{A}$

Thus $f_F = \dfrac{UGB}{A}(1+AB)$ and $f_F = \dfrac{(UGB)(\mathrm{K})}{A_F}$ where $K = \dfrac{R_F}{R_1 + R_F}$

and $A_F = \dfrac{AK}{1+AB} = \left(\dfrac{A}{1+AB}\right)K$.

4.5.6 Total Output Offset Voltage with Feedback

$$(\text{total output offset voltage with feedback}) = \frac{\text{total output offset voltage without feedback}}{1+AB}$$

i.e. $$V_{ooT} = \frac{\pm V_{sat}}{1+AB}.$$

4.5.7 Current-to-Voltage Converter

Let us reconsider the ideal voltage-gain equation of the inverting amplifier,

$$\frac{v_0}{v_{in}} = -\frac{R_F}{R_1} \quad \Rightarrow v_0 = -\left(\frac{v_{in}}{R_1}\right)R_F$$

However, since $v_1 = 0\,V$ and $v_1 = v_2$,

$\dfrac{v_{in}}{R_1} = i_{in}$ and $v_o = -i_{in}R_F$.

This means that if we replace the v_{in} and R_1 combination by a current source i_{in} as shown in figure 4.14, the output voltage v_o becomes proportional to the input current i_{in}. In other words, the circuit converts the input current into a proportional output voltage.

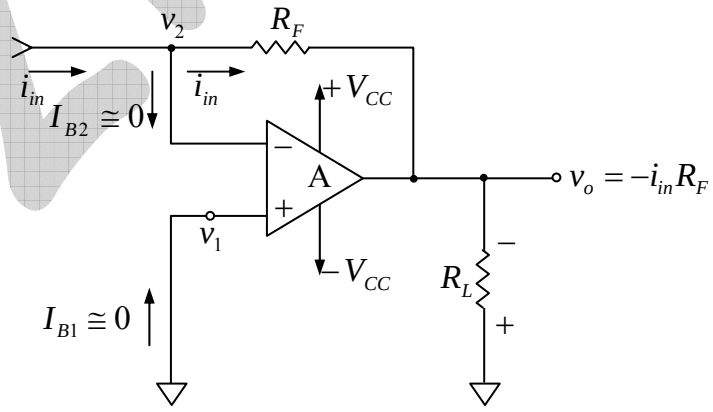


Figure 4.14: Current-to-voltage converter.

One of the most common uses of the current-to-voltage converter is in sensing current from photo detectors and in digital-to-analog converter applications.

Table: Summary of results obtained for non-inverting and inverting amplifiers

Parameter	Noninverting	Inverting amplifier
1. Voltage gain	$A_F = \frac{A(R_1 + R_F)}{R_1 + R_F + AR_1}$ (exact) $= \frac{A}{1+AB}$ $= 1 + \frac{R_F}{R_1}$ (ideal)	$A_F = \frac{-AR_F}{R_1 + R_F + AR_1}$ (exact) $= \frac{-AK}{1+AB}$, where $K = \frac{R_F}{R_1 + R_F}$ $= -\frac{R_F}{R_1}$ (ideal)
2. Gain of the feedback circuit	$B = \frac{R_1}{R_1 + R_F}$	$B = \frac{R_1}{R_1 + R_F}$
3. Input resistance	$R_{iF} = R_i(1+AB)$	$R_{iF} = R_1 + \left(\frac{R_F}{1+A} \Big\| R_i\right)$
4. Output resistance	$R_{oF} = \frac{R_o}{1+AB}$	$R_{oF} = \frac{R_o}{1+AB}$
5. Bandwidth	$f_F = f_0(1+AB)$ $f_F = \frac{UGB}{A_F}$	$f_F = f_0(1+AB)$ $f_F = \frac{(UGB)(K)}{A_F}$
6. Total output offset voltage	$V_{ooT} = \frac{\pm V_{sat}}{1+AB}$	$V_{ooT} = \frac{\pm V_{sat}}{1+AB}$

4.6 Differential Amplifiers

4.6.1 Differential Amplifier with One Op-Amp

Figure below shows the differential amplifier with one op-amp. A close examination of the differential amplifier is a combination of inverting and non-inverting amplifier.

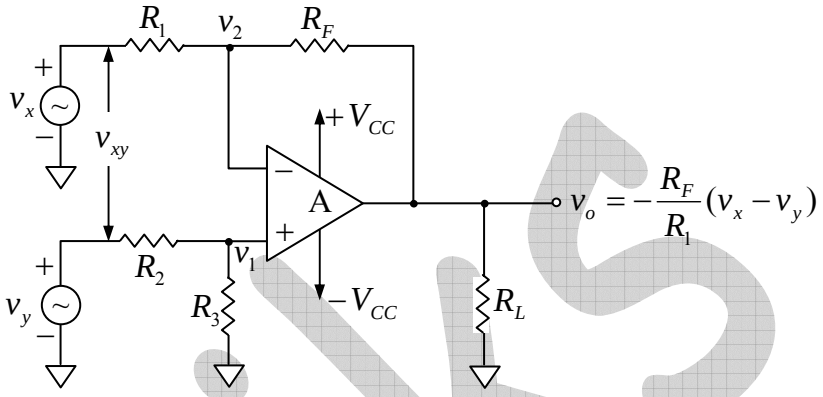


Figure 4.15: Differential amplifier with one op-amp. $R_1 = R_2$ and $R_F = R_3$.

When $v_y = 0\,V$, the configuration becomes an inverting amplifier; hence the output due to v_x only is $v_{ox} = -\dfrac{R_F(v_x)}{R_1}$.

Similarly, when $v_x = 0\,V$, the configuration is a non-inverting amplifier having a voltage-divider network composed of R_2 and R_3 at the non-inverting input.

Therefore, $v_1 = \dfrac{R_3(v_y)}{R_2 + R_3}$ and the output due to v_y then is $v_{oy} = \left(1 + \dfrac{R_F}{R_1}\right)v_1$.

$$\Rightarrow v_{oy} = \frac{R_3}{R_2 + R_3}\left(\frac{R_1 + R_F}{R_1}\right)v_y = \frac{R_F(v_y)}{R_1} \qquad \because R_1 = R_2 \text{ and } R_F = R_3$$

Thus the net output voltage is $v_0 = v_x + v_y \Rightarrow v_o = -\dfrac{R_F}{R_1}(v_x - v_y) = -\dfrac{R_F(v_{xy})}{R_1}$

$$\Rightarrow A_D = \frac{v_o}{v_{xy}} = -\frac{R_F}{R_1}$$

Note that the gain of the differential amplifier is the same as that of the inverting amplifier.

4.6.2 Differential Amplifier with Two Op-Amps

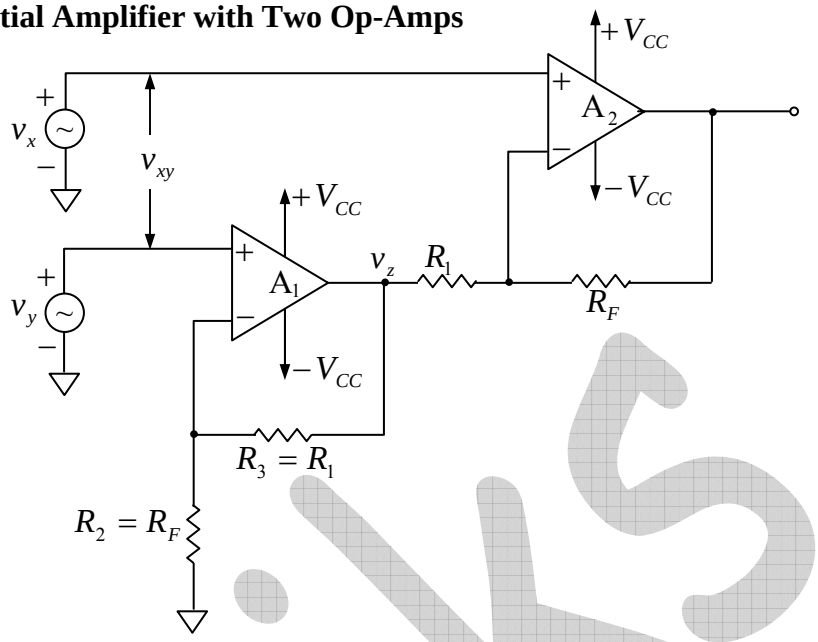


Figure 4.16: Differential amplifier with two op-amps.

The output v_z of the first stage is $v_z = \left(1 + \frac{R_3}{R_2}\right) v_y$.

By applying the superposition theorem to the second stage, we can obtain the output voltage: $v_o = -\frac{R_F (v_z)}{R_1} + \left(1 + \frac{R_F}{R_1}\right) v_x \Rightarrow v_o = -\left(\frac{R_F}{R_1}\right)\left(1 + \frac{R_3}{R_2}\right) v_y + \left(1 + \frac{R_F}{R_1}\right) v_x$

Since $R_1 = R_3$ and $R_F = R_2$, $v_o = -\left(1 + \frac{R_F}{R_1}\right)(v_x - v_y)$.

Therefore, $A_D = \frac{v_o}{v_{xy}} = 1 + \frac{R_F}{R_1}$ where $v_{xy} = (v_x - v_y)$.

4.7 The Practical Op-Amp

Offset minimizing resistance $R_{OM} = \dfrac{R_1 R_F}{R + R_F}$ is used in non-inverting and inverting configuration as shown in figure given below.

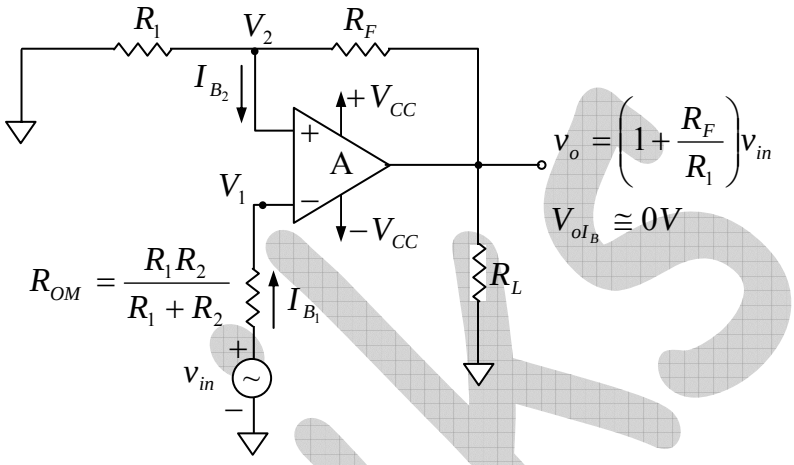


Figure 4.17: Non-inverting amplifier with offset minimizing resistor R_{OM}.

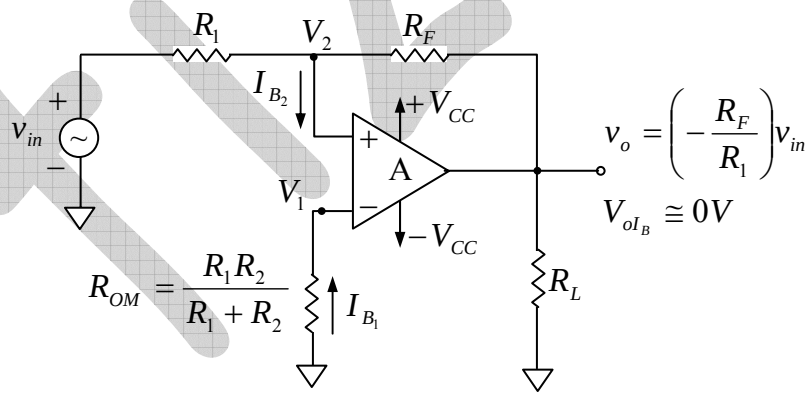


Figure 4.18: Inverting amplifier with offset minimizing resistor R_{OM}.

4.8 Summing, Scaling and Averaging Amplifier

This section shows how the inverting, non-inverting and differential configurations are useful in such applications as summing, scaling and averaging amplifiers.

4.8.1 Inverting Configuration

Figure below shows the inverting configuration with three inputs V_a, V_b and V_c. Depending on the relationship between the feedback resistor R_F and the input resistors R_a, R_b and R_c the circuit can be used as a summing amplifier, scaling amplifier or averaging amplifier.

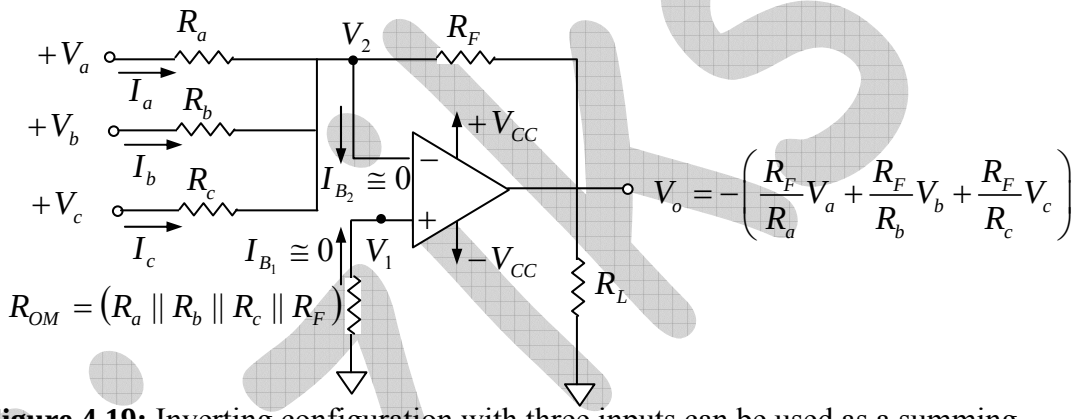


Figure 4.19: Inverting configuration with three inputs can be used as a summing amplifier, scaling amplifier, or averaging amplifier.

The circuit's function can be verified by examining the expression for the output voltage V_o which is obtained from Kirchhoff's current equation written at node V_2.

$$I_a + I_b + I_c = I_B + I_F$$

Since R_i and A of the op-amp are ideally infinity, $I_B = 0\ A$ and $V_1 = V_2 \cong 0\ V$.

Therefore, $$\frac{V_a}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c} = -\frac{V_o}{R_F} \Rightarrow V_o = -\left(\frac{R_F}{R_a}V_a + \frac{R_F}{R_b}V_b + \frac{R_F}{R_c}V_c\right)$$

Summing amplifier

If in the circuit, $R_a = R_b = R_c = R_F$ then $V_o = -(V_a + V_b + V_c)$ hence the circuit is called summing amplifier.

Scaling or weighted amplifier

If each input voltage is amplified by a different factor, in other words, weighted differently at the output, the circuit is then called a scaling or weighted amplifier. This condition can be accomplished if R_a, R_b and R_c are different in value. Thus the output voltage of the scaling amplifier is

$$V_o = -\left(\frac{R_F}{R_o}V_a + \frac{R_F}{R_b}V_b + \frac{R_F}{R_c}V_c\right) \text{ where } \frac{R_F}{R_a} \neq \frac{R_F}{R_b} \neq \frac{R_F}{R_c}.$$

Average circuit

The circuit can be used as an averaging circuit, in which the output voltage is equal to the average of all the input voltages. This is accomplished by using all input resistors of equal value $R_a = R_b = R_c = R$. In addition, the gain by which each input is amplified must be equal to 1 over the number of inputs; that is, $\frac{R_F}{R} = \frac{1}{n}$ where n is the number of inputs.

Thus, if there are three inputs, we want $\frac{R_F}{R} = \frac{1}{3}$, consequently $V_o = -\left(\frac{V_a + V_b + V_c}{3}\right)$.

4.8.2 Non-inverting Configuration

If input voltage sources and resistors are connected to the non-inverting terminal as shown in figure below, the circuit can be used either as a summing or averaging amplifier through selection of appropriate values of resistors, that is, R_1 and R_F.

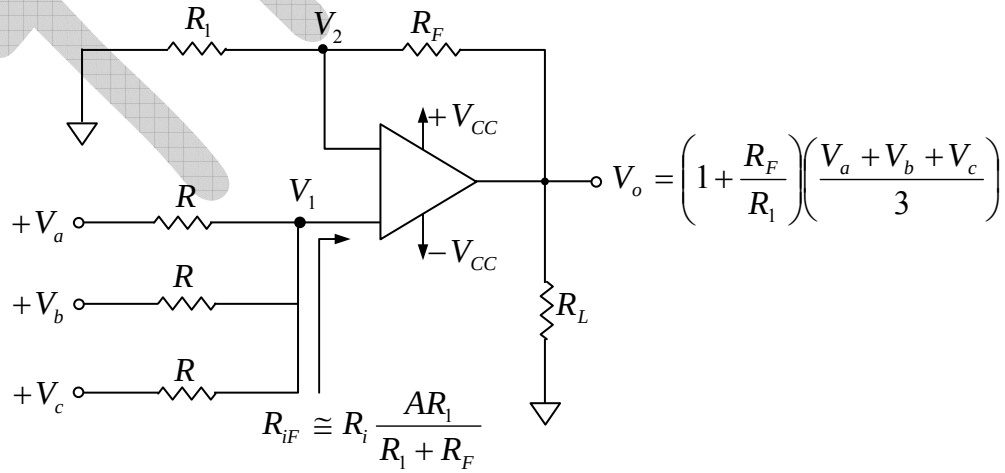


Figure 4.20: Non-inverting configuration.

Again, to verify the functions of the circuit, the expression for the output voltage must be obtained. Recall that the input resistance R_{iF} is very large. Therefore, using the superposition theorem, the voltage V_1 at the non-inverting terminal is

$$V_1 = \frac{R/2}{R+R/2}V_a + \frac{R/2}{R+R/2}V_b + \frac{R/2}{R+R/2}V_c \quad \text{or} \quad V_1 = \frac{V_a}{3} + \frac{V_b}{3} + \frac{V_c}{3} = \frac{V_a+V_b+V_c}{3}$$

$$\Rightarrow V_o = \left(1+\frac{R_F}{R_1}\right)V_1 = \left(1+\frac{R_F}{R_1}\right)\frac{V_a+V_b+V_c}{3}$$

Averaging amplifier

Above equation shows that the output voltage is equal to the average of all input voltages times the gain of the circuit $1+\frac{R_F}{R_1}$, hence the name averaging amplifier. Depending on the application gain is 1 $(R_F = 0)$ the output voltage will be equal to the average of all input voltages.

Summing amplifier If $1+\frac{R_F}{R_1} = 3$ then $R_F = 2R_1 \Rightarrow V_o = V_a + V_b + V_c$

4.8.3 Differential Configuration (Subtractor)

A basic differential amplifier can be used as a subtractor as shown in figure below.

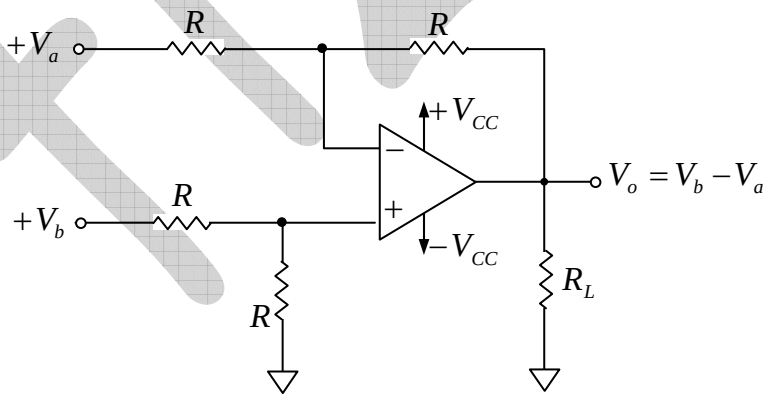


Figure 4.21: Basic differential amplifier used as a subtractor.

From this figure, the output voltage of the differential amplifier with a gain of 1 is

$$V_o = -\frac{R}{R}(V_a - V_b) \Rightarrow V_o = V_b - V_a$$

Summing amplifier

A four-input summing amplifier may be constructed using the basic differential amplifier if two additional input sources are connected, one each to the inverting and non-inverting input terminals through resistor R (as shown in figure).

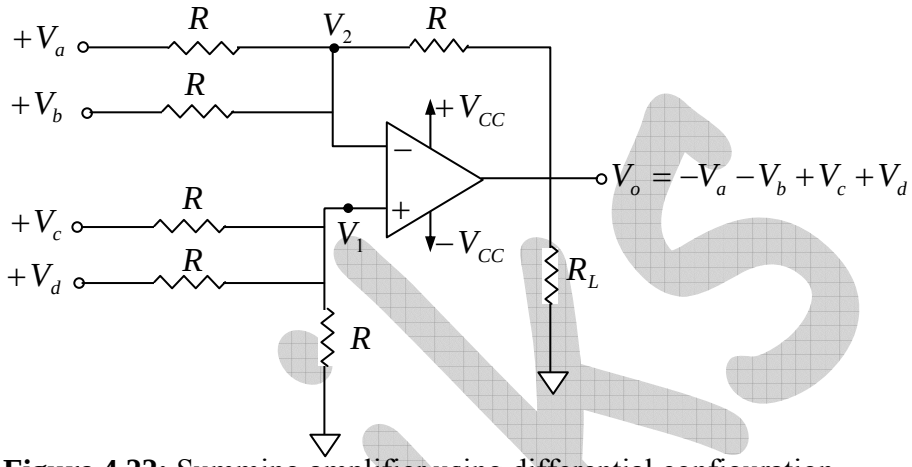


Figure 4.22: Summing amplifier using differential configuration.

The output voltage equation for this circuit can be obtained by using the superposition theorem. For instance, to find the output voltage due to V_a alone, reduce all other input voltages V_b, V_c and V_d to zero as shown in figure below.

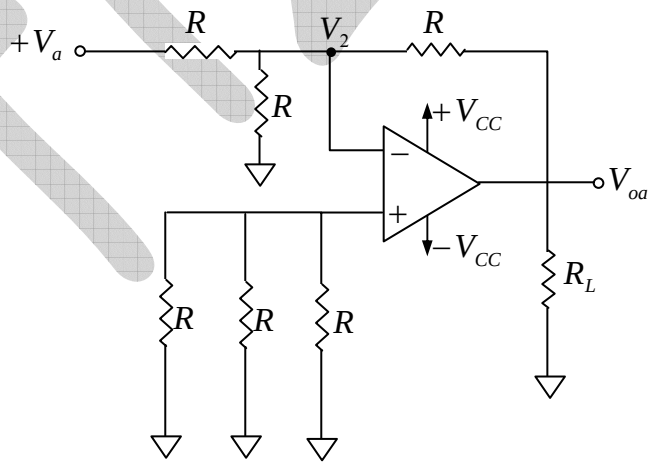


Figure 4.23: Deriving the output voltage equation from the summing amplifier.

In fact, this circuit is an inverting amplifier in which the inverting input is at virtual ground $(V_2 = 0\ V)$.

Therefore, the output voltage is $V_{oa} = -\frac{R}{R}V_a = -V_a$.

Similarly, the output voltage due to V_b alone is $V_{ob} = -V_b$.

Now if input voltages V_a, V_b and V_c are set to zero, the circuit becomes a non-inverting amplifier in which the voltage V_1 at the non-inverting input is

$$V_1 = \frac{R/2}{R + R/2}V_c = \frac{V_c}{3}.$$

This means that the output voltage due to V_c alone is

$$V_{oc} = \left(1 + \frac{R}{R/2}\right)V_1 = (3)\left(\frac{V_c}{3}\right) = V_c.$$

Similarly, the output voltage due to all four input voltages is given by

$$V_o = V_{oa} + V_{ob} + V_{oc} + V_{od} = -V_a - V_b + V_c + V_d$$

Notice that the output voltage is equal to the sum of the input voltages applied to the non-inverting terminal plus the negative sum of the input voltages applied to the inverting terminal.

4.9 The Integrator

A circuit in which the output voltage waveform is the integral of the input voltage waveform is the *integrator* or the *integration amplifier*. Such a circuit is obtained by using a basic inverting amplifier configuration if the feedback resistor R_F is replaced by a capacitor C_F (figure shown below)

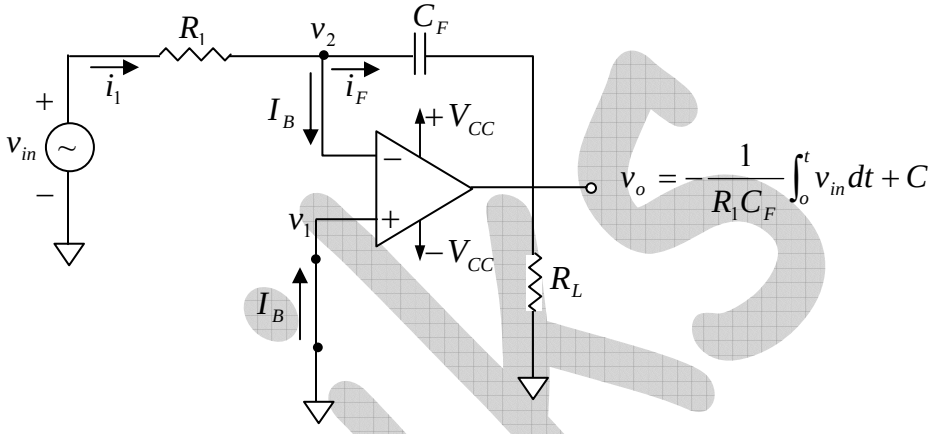


Figure 4.24(a): The integrator circuit.

The expression for the output voltage v_o can be obtained by writing Kirchhoff's current equation at node v_2: $i_1 = I_B + i_F \Rightarrow i_1 \cong i_F$ (Since I_B is negligibly small)

Recall that the relationship between current through and voltage across the capacitor is

$$i_1 \cong i_F \Rightarrow \frac{v_{in} - v_2}{R_1} = C_F\left(\frac{d}{dt}\right)(v_2 - v_o) \qquad \because i_c = C\frac{dv_c}{dt}$$

$$\Rightarrow \frac{v_{in}}{R_1} = C_F\frac{d}{dt}(-v_o) \qquad \because v_1 = v_2 \cong 0$$

The output voltage can be obtained by integrating both sides with respect to time:

$$\int_0^t \frac{v_{in}}{R_1}dt = \int_0^t C_F\frac{d}{dt}(-v_o)dt = C_F(-v_o) + v_o\big|_{t=0} \Rightarrow \boxed{v_o = -\frac{1}{R_1 C_F}\int_0^t v_{in}dt + C}$$

where C is the integration constant and is proportional to the value of the output voltage v_o at time $t = 0$ seconds.

The waveforms are drawn with the assumption that $R_1C_F = 1$ second and $V_{ooT} = 0\ V$, that is, $C = 0$.

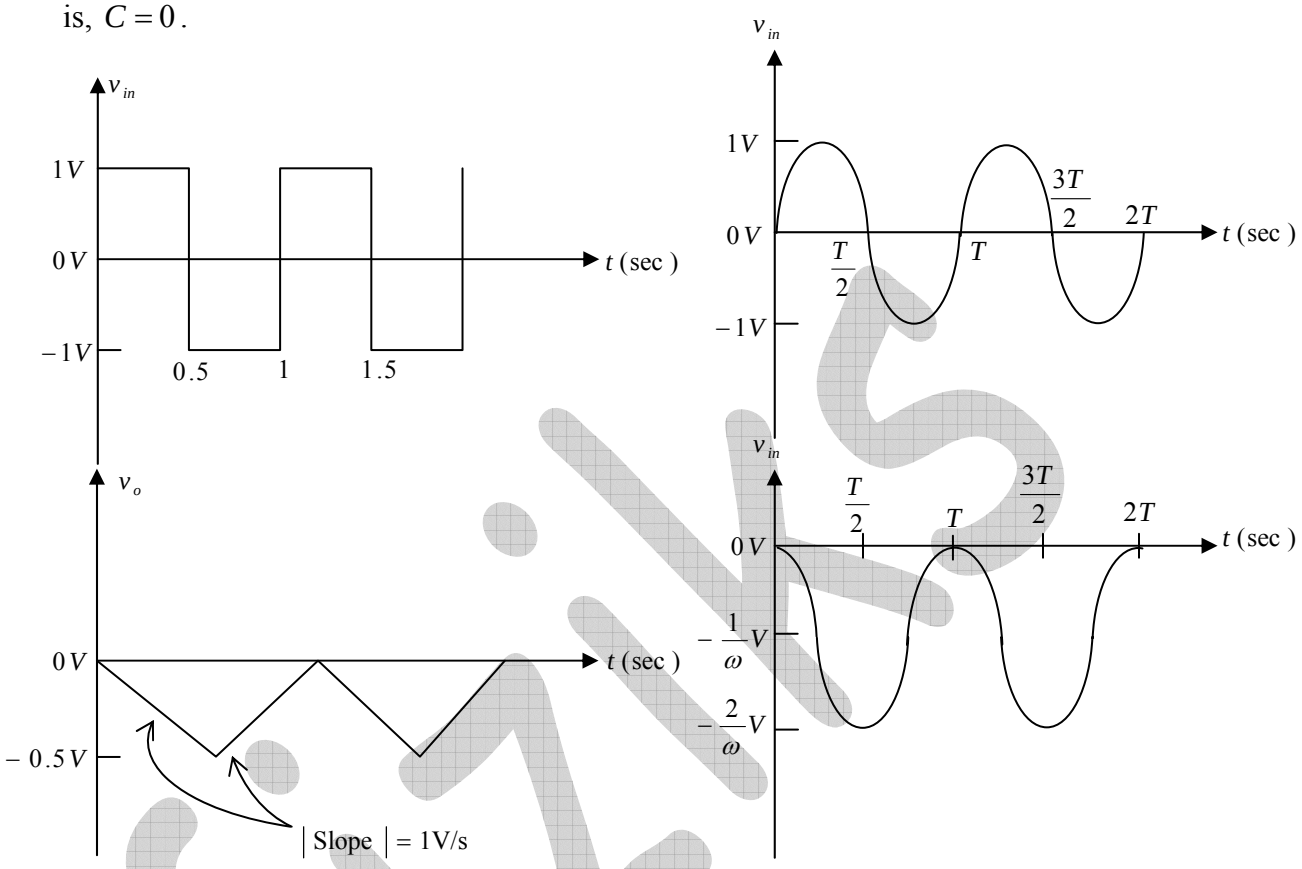


Figure 4.24(b): Input and ideal output waveforms using a sine wave and square wave, respectively. $R_1C_F = 1$ second and $V_{ooT} = 0\ V$ assumed.

When $v_{in} = 0$ the integrator works as an open-loop amplifier. This is because the capacitor C_F acts as an open circuit $\left(X_{C_F} = \infty\right)$ to the input offset voltage V_{io}. In other words, the input offset voltage V_{io} and the part of the input current charging capacitor C_F produce the error voltage at the output of the integrator. Therefore, in the practical integrator shown in figure below, to reduce the error voltage at the output, a resistor R_F limits the low-frequency gain and hence minimizes the variations in the output voltage.

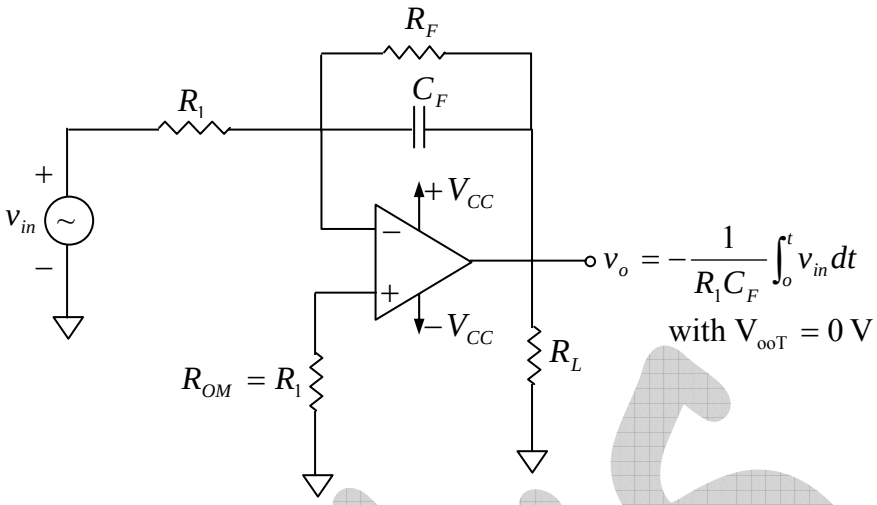


Figure 4.25: Practical integrator.

The frequency response of the basic integrator is shown in figure below.

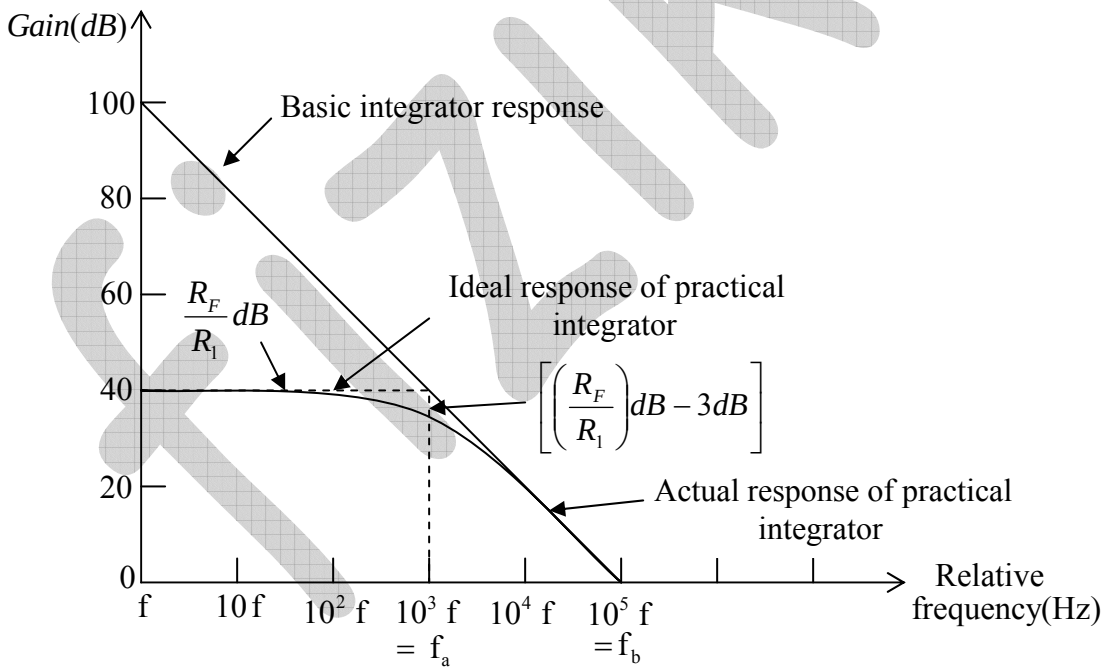


Figure 4.26: Frequency response of basic and practical integrators.

$$f_a = \frac{1}{2\pi R_F C_F} \quad \text{and} \quad f_b = \frac{1}{2\pi R_1 C_F}.$$

In this figure f_b is the frequency at which the gain is $0\ dB$ and is given by

$$f_b = \frac{1}{2\pi R_1 C_F}.$$

The frequency response of the practical integrator is shown in the figure by a dashed line. In this figure, f is some relative operating frequency and for frequencies f to f_a the gain R_F / R_1 is constant. However after f_a gain decreases at a rate $20\ dB/decade$. In other words, between f_a and f_b the circuit acts as an integrator. The gain-limiting frequency f_a is given by $f_a = \frac{1}{2\pi R_F C_F}$.

Generally, the value of f_a and in turn $R_1 C_F$ and $R_F C_F$ values should be selected such that $f_a < f_b$. For example if $f_a = f_b/10$ then $R_F = 10R_1$. In fact, the input signal will be integrated properly if the time period T of the signal is larger than or equal to $R_F C_F$. That is, $T \geq R_F C_F$ where $R_F C_F = \frac{1}{2\pi f_a}$.

4.10 The Differentiator

Figure below shows the differentiator or differentiation amplifier. As its name implies, the circuit performs the mathematical operation of differentiation: that is the output waveform is the derivative of the input waveform. The differentiator may be constructed from a basic inverting amplifier if an input resistor R_1 is replaced by capacitor C_1.

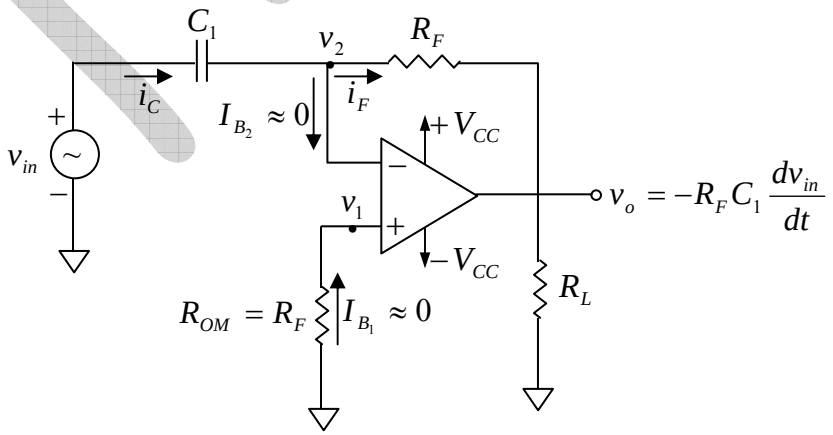


Figure 4.27: Basic differentiator Circuit.

The expression for the output voltage can be obtained from Kirchhoff's current equation written at node v_2 as follows: $i_C = I_B + i_F \Rightarrow i_C \cong i_F \qquad \because I_B \cong 0$

$$\Rightarrow C_1 \frac{d}{dt}(v_{in} - v_2) = \frac{v_2 - v_o}{R_F} \Rightarrow \boxed{v_o = -R_F C_1 \frac{dv_{in}}{dt}}$$

The gain of the circuit $\left(R_F / X_{C_1}\right)$ increases with increase in frequency at a rate of $20\ dB/decade$. This makes the circuit unstable. Also, the input impedance X_{C_1} decreases with increase in frequency, which makes the circuit very susceptible to high frequency noise. When amplified, this noise can completely override the differentiated output signal. This frequency response of the basic differentiator is shown in figure below.

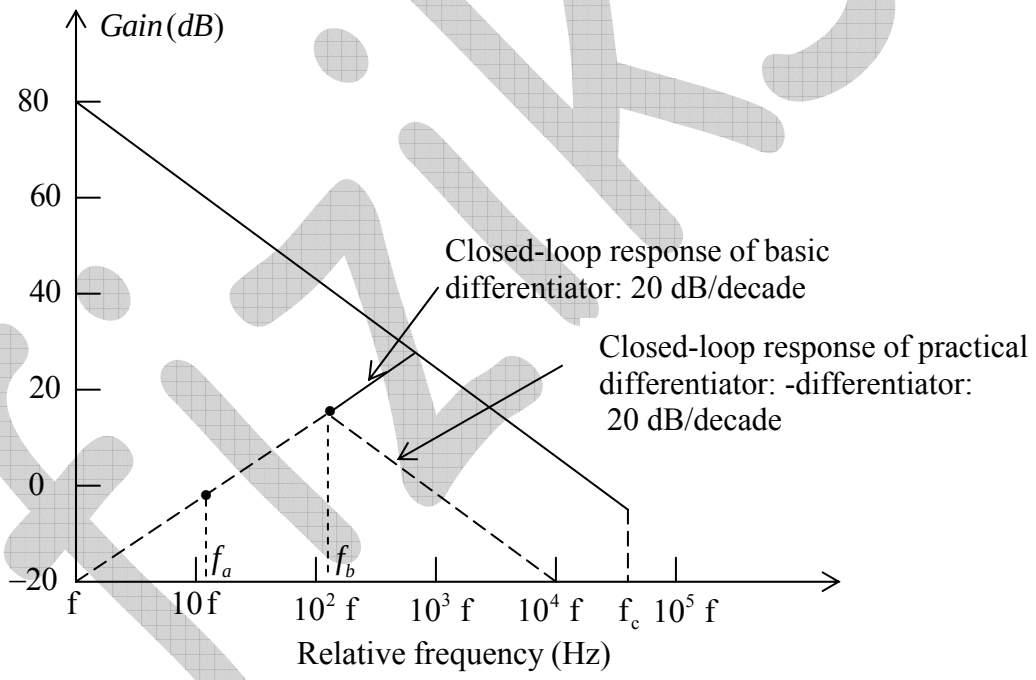


Figure 4.28: Frequency response.

In this figure, f_a is the frequency at which the gain is $0\ dB$ and is given by $f_a = \frac{1}{2\pi R_F C_1}$. Both the stability and the high-frequency noise problems can be corrected by the addition of two components: R_1 and C_F as shown in figure below. This circuit is a practical differentiator.

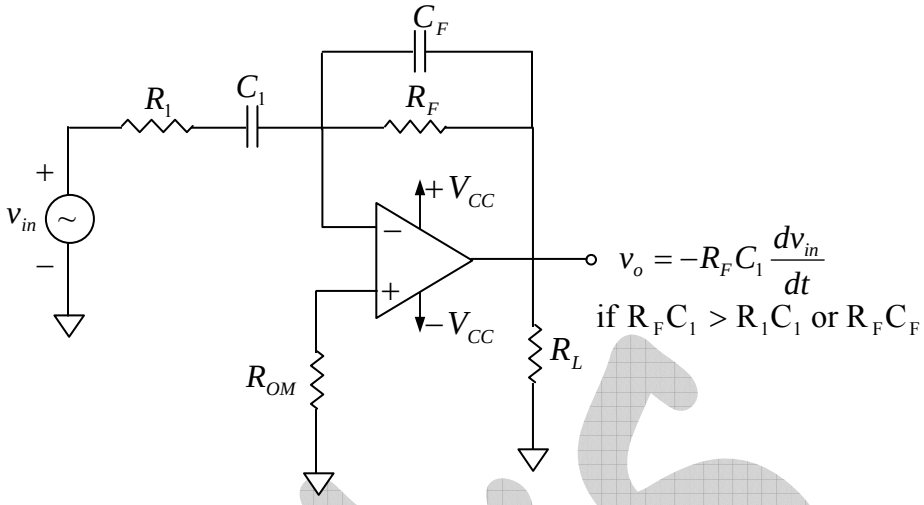


Figure 4.29(a): Practical differentiator.

The frequency response is shown in figure by a dashed line. From frequency f to f_b the gain increases at $20\ dB/decade$. However after f_b the gain decreases at $20\ dB/decade$. This $40\ dB/decade$ change in gain is caused by the R_1C_1 and R_FC_F combinations. The gain-limiting frequency f_b is given by

$$f_b = \frac{1}{2\pi R_1 C_1} \quad \text{where } R_1C_1 = R_FC_F .$$

Thus, R_1C_1 and R_FC_F help to reduce significantly the effect of high-frequency input, amplifier noise, and offsets. Above all, it makes the circuit more stable by preventing the increase in gain with frequency. Generally, the value of f_b and in turn R_1C_1 and R_FC_F values should be selected such that

$$f_a < f_b$$

where $$f_a = \frac{1}{2\pi R_1 C_1},\ f_b = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi R_F C_F}.$$

The input signal will be differentiated properly if the time period T of the input signal is larger than or equal to R_FC_1. That is,

$$T \geq R_F C_1 .$$

Figure below show the sine wave and square wave inputs and resulting differentiated outputs, respectively, for the practical differentiator.

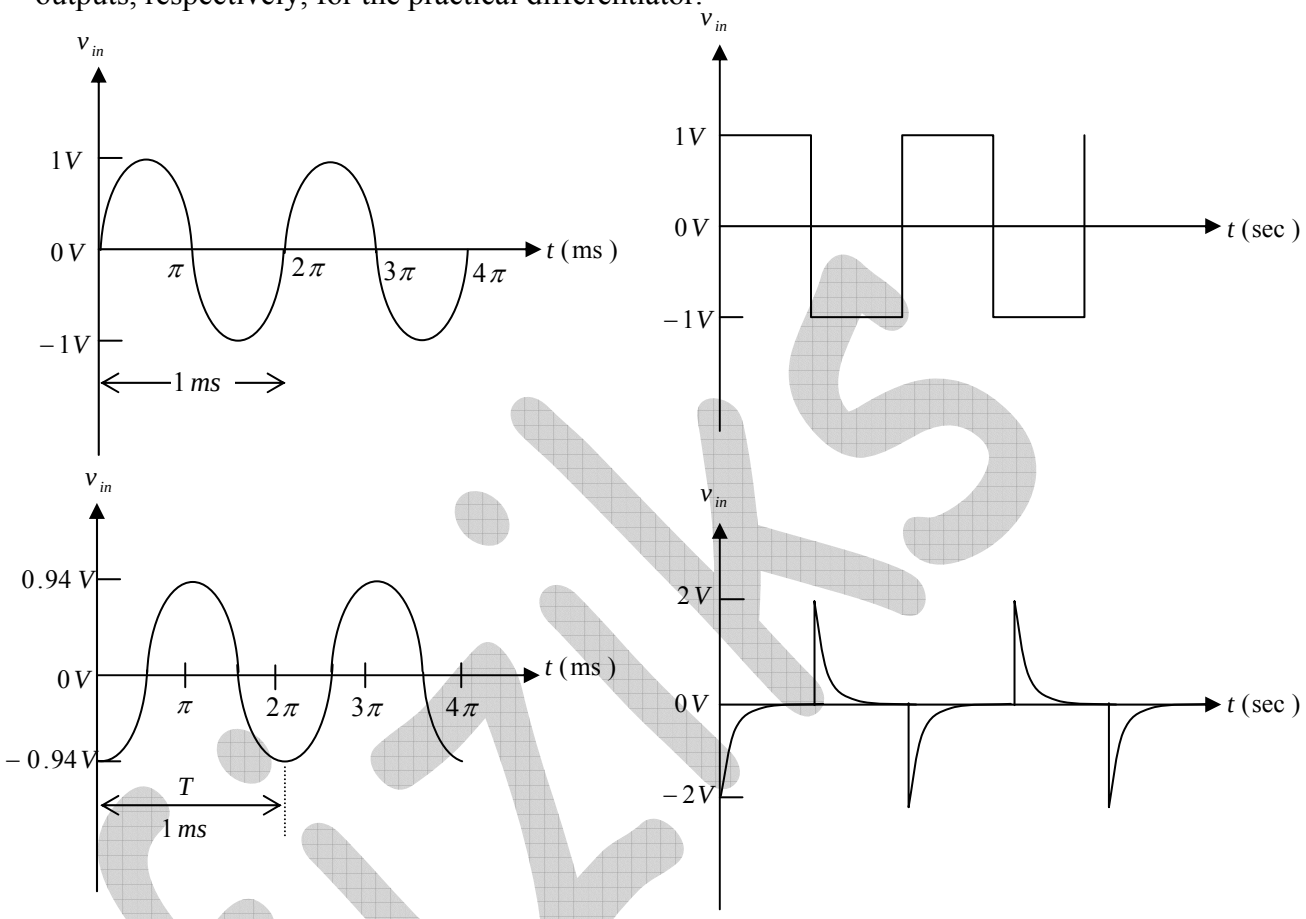


Figure 4.29(b): Input and ideal output waveforms using a sine wave and square wave, respectively.

A workable differentiator can be designed by implementing the following steps:

1. Select f_a equal to the highest frequency of the input signal to be differentiated. Then, assuming a value of $C_1 < 1\ \mu F$, calculate the value of R_F.

2. Choose $f_b = 20\, f_a$ and calculate the values of R_1 and C_F so that $R_1 C_1 = R_F C_F$.

The differentiator is most commonly used in wave shaping circuits to detect high-frequency components in an input signal and also as a rate-of-change detector in FM modulators.

4.11 Active Filters

An electric filter is often a frequency-selective circuit that passes a specified band of frequencies and blocks or attenuates signals of frequencies outside this band. Filters may be classified in a number of ways:

1. Analog or digital 2. Passive or active

3. Audio (AF) or radio frequency (RF)

Analog filters are designed to process analog signals, while digital filters process analog signals using digital techniques. Depending on the type of elements used in their construction, filters may be classified as passive or active. Elements used in *passive filters* are resistors, capacitors, and inductors. *Active filters*, on the other hand employ transistors or op-amps in addition to the resistors and capacitors. The type of element used dictates the operating frequency range of the filter. For example, *RC* filters are commonly used for audio or low frequency operation, whereas *LC* or crystal filters are employed at *RF* or high frequencies.

An active filter offers the following advantages over a passive filter:

1. *Gain and frequency adjustment flexibility*. Since the op-amp is capable of providing a gain, the input signal is not attenuated as it is in a passive filter. In addition, the active filter is easier to tune or adjust.

2. *No loading problem*. Because of the high input resistance and low output resistance of the op-amp, the active filter does not cause loading of the source or load.

3. *Cost*. Typically, active filters are more economical than passive filters. This is because of the variety of cheaper op-amps and the absence of inductors.

The most commonly used filters are:

1. **Low-Pass filter** **2.High-Pass filter** **3.Band-Pass filter**
4. **Band-Reject filters** **5. All-Pass filter**

All of these filters uses an op-amp as the active element and resistors and capacitors as the passive element. Figure below shows the frequency response characteristics of the five types of filters. The ideal response is shown by dashed curves, while the solid lines indicate the practical filter response.

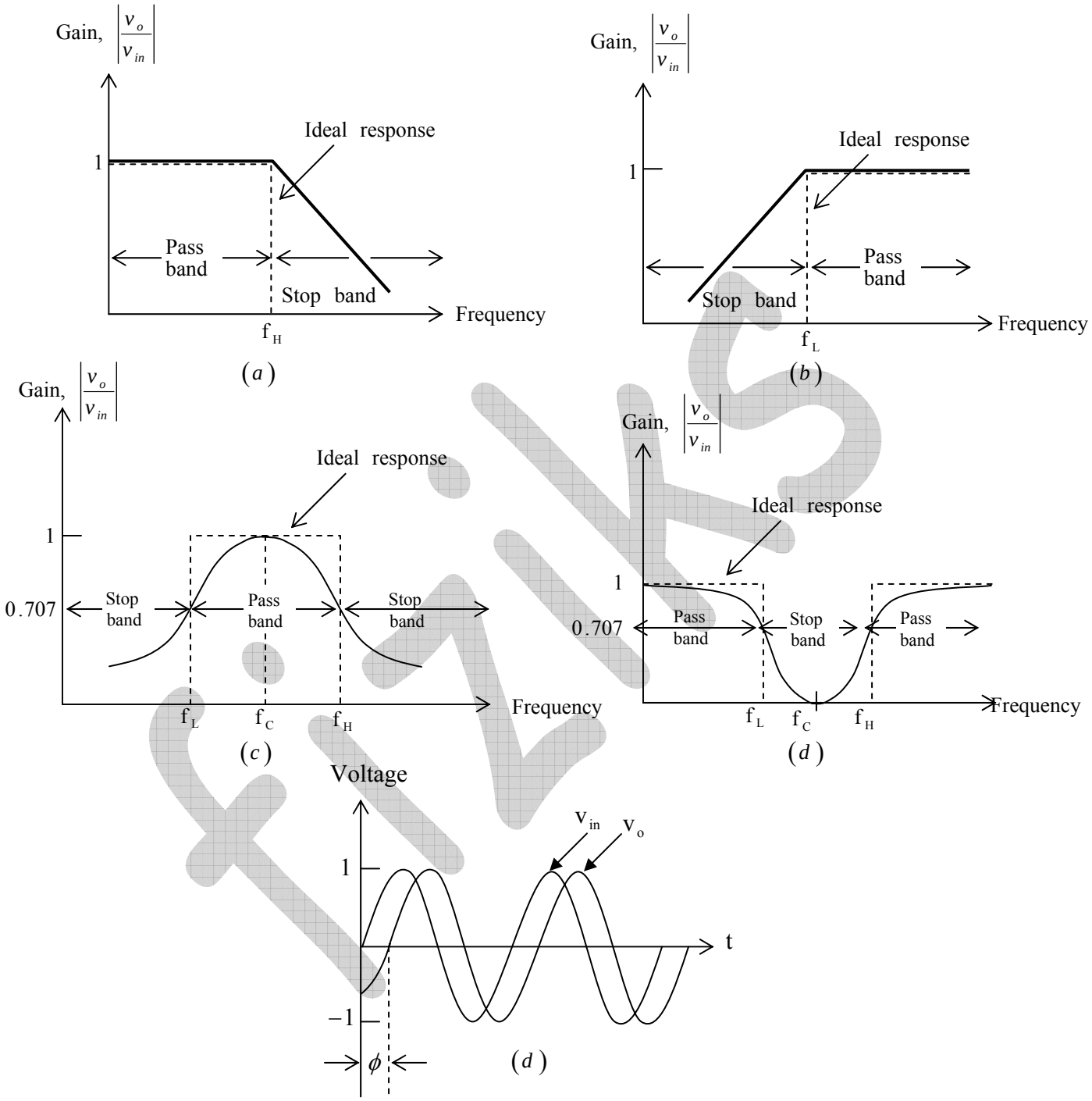


Figure 4.30: Frequency response of the major active filters. (a) Low pass. (b) High pass. (c) Band pass. (d) Band rejects. (e) Phase shift between input and output volages of an all-pass filter.

4.11.1 First-Order Low-Pass Filter

Figure below shows a first-order low-pass Butterworth filter that uses an *RC* network for filtering. Note that the op-amp is used in the non-inverting configuration; hence it does not load down the *RC* network. Resistors R_1 and R_F determine the gain of the filter.

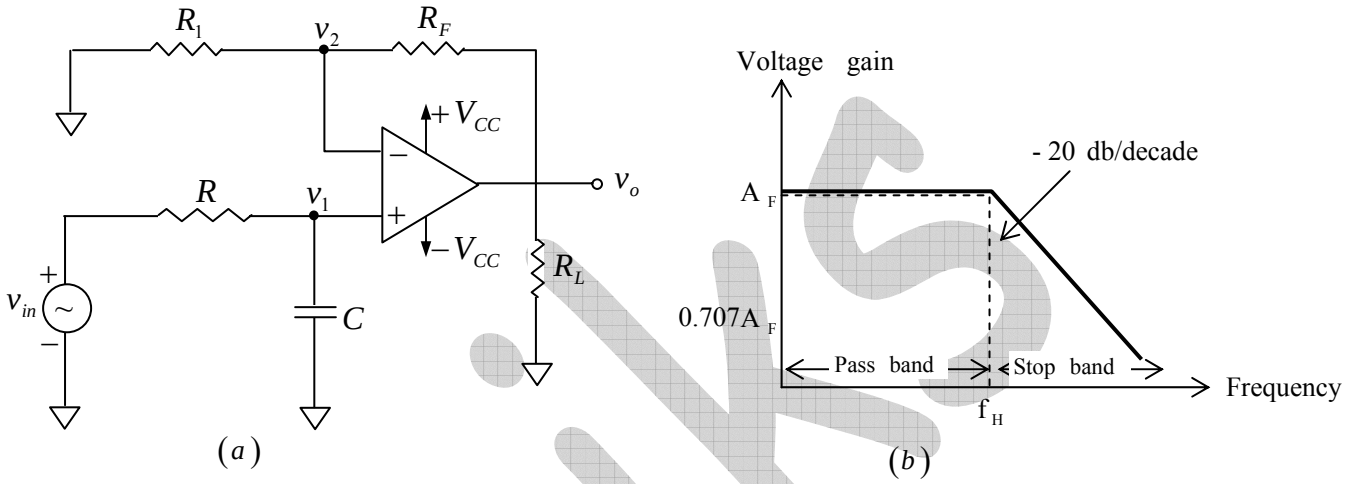


Figure 4.31: First-order low-pass Butterworth filter. (a) Circuit. (b) Frequency response.

According to the voltage-divider rule, the voltage at the non-inverting terminal (across capacitor *C*) is $v_1 = \dfrac{-jX_C}{R - jX_C} v_{in}$ where $j = \sqrt{-1}$ and $-jX_C = \dfrac{1}{j2\pi\, fC}$.

On simplifying, we get $v_1 = \dfrac{v_{in}}{1 + j2\pi fRC}$ and the output voltage $v_o = \left(1 + \dfrac{R_F}{R_1}\right) v_1$.

That is, $v_o = \left(1 + \dfrac{R_F}{R_1}\right) \dfrac{v_{in}}{1 + j2\pi fRC} \Rightarrow \dfrac{v_o}{v_{in}} = \dfrac{A_F}{1 + j(f / f_H)}$

Where $\dfrac{v_o}{v_{in}}$ = gain of the filter as a function of frequency, $A_F = 1 + \dfrac{R_F}{R_1}$ = pass-band gain of the filter, f = frequency of the input signal and $f_H = \dfrac{1}{2\pi RC}$ = high cutoff frequency of the filter.

The gain magnitude and phase angle equations of the low-pass filter can be obtained as follows:

$$\boxed{\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1+(f/f_H)^2}}} \quad \text{and} \quad \phi = -\tan^{-1}\left(\frac{f}{f_H}\right) \text{ in degrees.}$$

The operation of the low-pass filter can be verified from the gain magnitude equation:

1. At very low frequencies, that is, $f < f_H$, $\left|\frac{v_o}{v_{in}}\right| \cong A_F$

2. At $f = f_H$, $\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{2}} = 0.707 A_F$

3. At $f > f_H$, $\left|\frac{v_o}{v_{in}}\right| < A_F$.

4.11.2 Second-Order Low-Pass Filter

A stop-band response having a $40\ dB/decade$ roll-off is obtained with the second-order low-pass filter. A first-order low-pass filter can be converted into a second-order type simply by using an additional *RC* network, as shown in figure below.

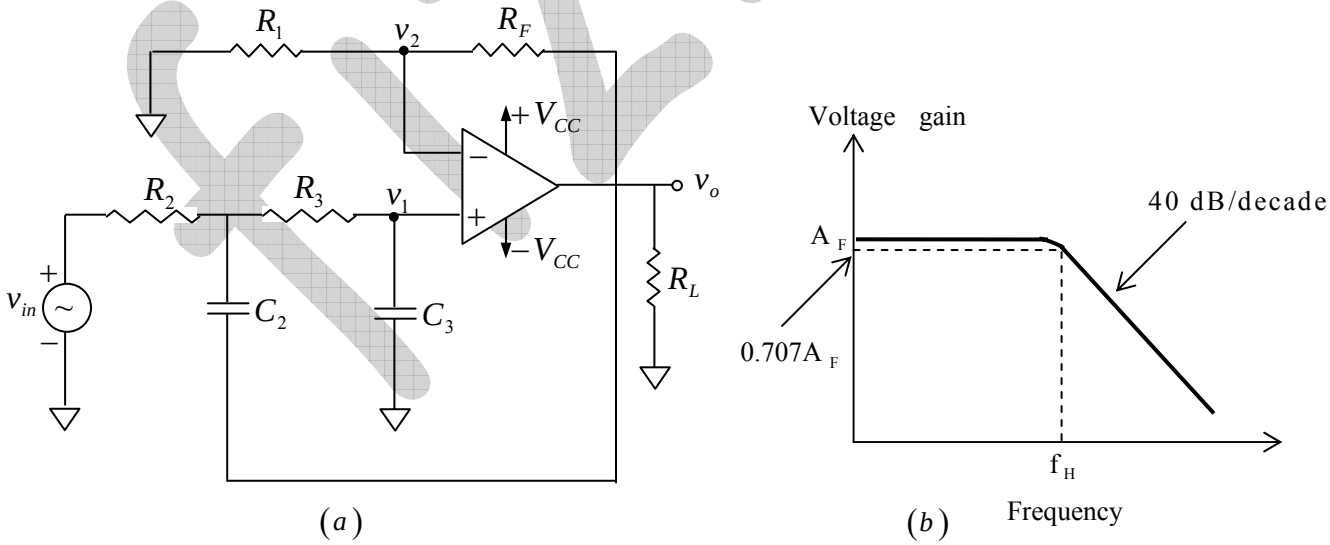


Figure 4.32: Second-order low-pass Butterworth filter.

(a) Circuit. (b) Frequency response.

Second-order filters are important because higher-order filters can be designed using them. The gain of the second-order filter is set by R_1 and R_F while the high cutoff frequency f_H is determined by R_2, C_2, R_3 and C_3 as follows: $f_H = \dfrac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$.

Furthermore, for a second-order low-pass Butterworth response, the voltage gain magnitude equation is:

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1+(f/f_H)^4}}$$

where $A_F = 1+\dfrac{R_F}{R_1}$ = pass-band gain of the filter, f = frequency of the input signal (Hz)

4.11.3 First-Order High-Pass Filter

A first-order high-pass filter is formed from a first-order low-pass type by interchanging components R and C. Figure below shows a first-order high-pass Butterworth filter with a low cutoff frequency of f_L. This is the frequency at which the magnitude of the gain is 0.707 times its pass-band value. Obviously, all frequencies higher than f_L are pass-band frequencies, with the highest frequency determined by the closed-loop bandwidth of the op-amp.

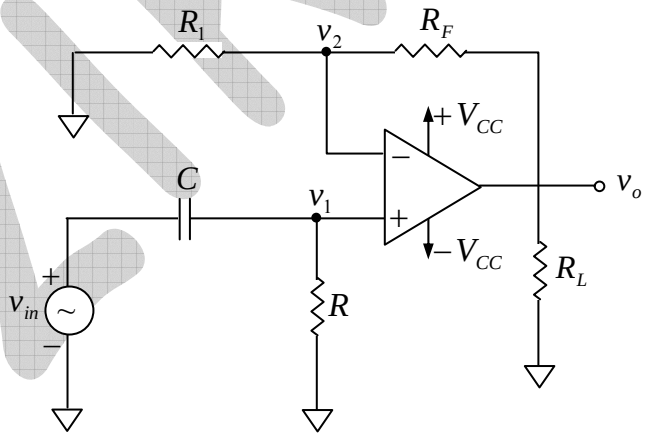


Figure 4.33: First-order high-pass filter.

$$v_o = \left(1+\frac{R_F}{R_1}\right)\frac{j2\pi fRC}{1+j2\pi fRC}v_{in} \Rightarrow \frac{v_o}{v_{in}} = A_F\left[\frac{j(f/f_L)}{1+j(f/f_L)}\right] \text{ where } A_F = 1+\frac{R_F}{R_1}.$$

Hence the magnitude of the voltage gain is $\left|\dfrac{v_o}{v_{in}}\right| = \dfrac{A_F(f/f_L)}{\sqrt{1+(f/f_L)^2}}$ where $f_L = \dfrac{1}{2\pi RC}$

4.11.4 Second-Order High-Pass Filter

As in the case of the first-order filter, a second-order high-pass filter can be formed from a second-order low-pass filter simply by interchanging the frequency determining resistors and capacitors. Figure below shows the second-order high-pass filter.

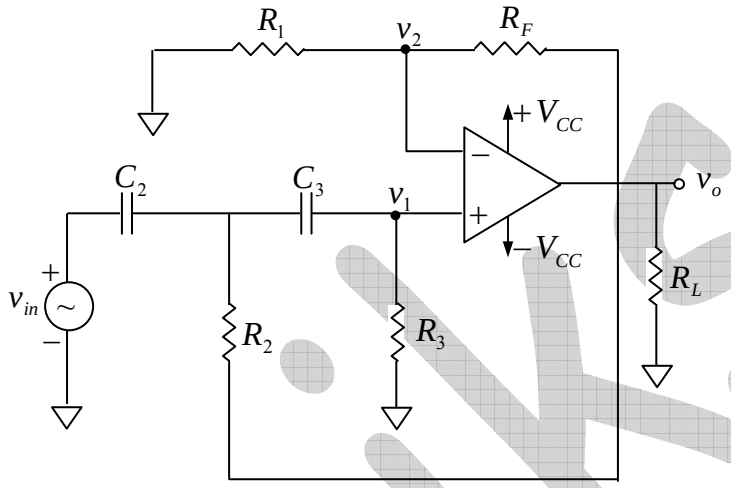


Figure 4.34: Second order high-pass Butterworth filter.

The voltage gain magnitude equation of the second-order high-pass filter is as follows:

$$\left|\frac{v_0}{v_m}\right| = \frac{A_F}{\sqrt{1+\left(f_L/f\right)^4}}.$$

where A_F = pass-band gain for the second-order Butterworth response,

f = frequency of the input signal (Hz), f_L = low cutoff frequency (Hz).

Since second-order low-pass and high-pass filters are the same circuits except that the positions of resistors and capacitors are interchanged, the design and frequency scaling procedures for the high-pass filter are the same as those for the low-pass filter.

$$f_L = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$$

4.11.5 Band-Pass Filter

A wide band-pass filter can be formed by simply cascading high-pass and low-pass sections and is generally the choice for simplicity of design and performance. The order of the band-pass filter depends on the order of the high-pass and low-pass filter sections.

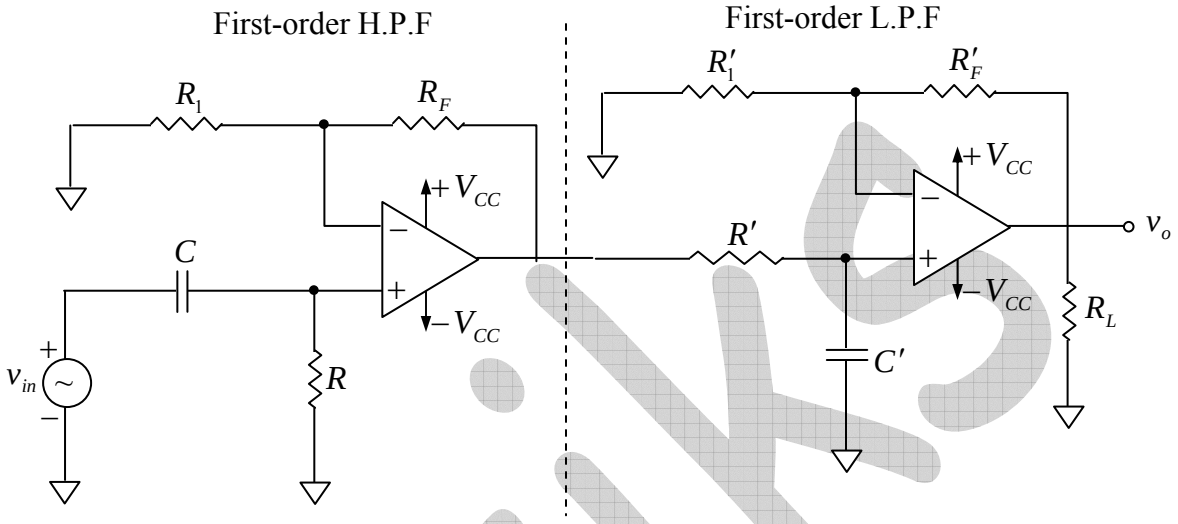


Figure 4.35: First-order wide band pass filter circuit.

4.11.6 All-Pass Filter

As the name suggests, an all-pass filter passes all frequency components of the input signal without attenuation, while providing predictable phase shifts for different frequencies of the input signal.

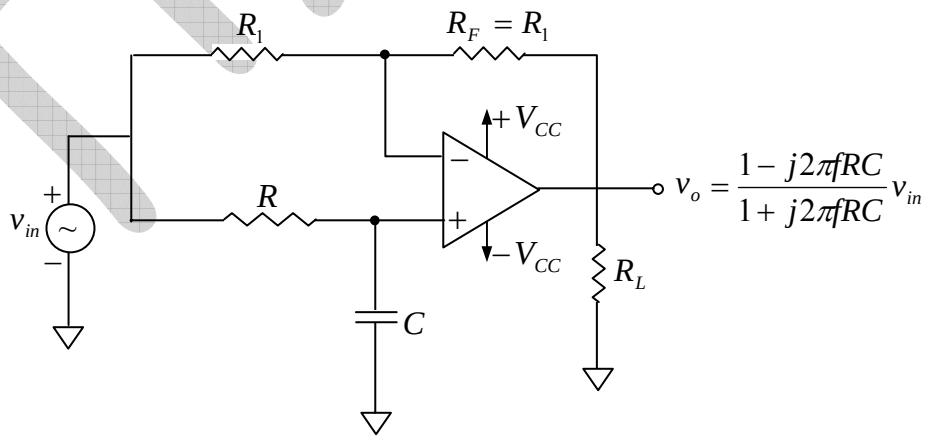


Figure 4.36 (a): All-pass filter circuit.

The output voltage v_o of the filter can be obtained by using the superposition theorem.

$$v_0 = -v_{in} + \frac{-jX_C}{R - jX_C} v_{in} (2) \quad \text{where } X_C = \frac{1}{j\omega C}.$$

$$\Rightarrow v_0 = v_{in}\left(-1 + \frac{2}{j2\pi fRC + 1}\right) \quad \Rightarrow \frac{v_0}{v_{in}} = \frac{1 - 2j2\pi fRC}{1 + j2\pi fRC}$$

Above equation indicates that the amplitude of $\frac{v_0}{v_{in}}$ is unity; that is, $|v_0| = |v_{in}|$ throughout the useful frequency range, and the phase shift between v_0 and v_{in} is a function of input frequency f the phase angle ϕ is given by

$$\phi = -2\tan^{-1}\left(\frac{2\pi fRC}{1}\right)$$

where ϕ is in degrees, f in hertz, R in ohms, and C in farads. This equation is used to find the phase angle ϕ if f, R, and C are known. Figure below shows a phased shift of 90° between the input v_{in} and output v_0. That is v_0 lags v_{in} by 90°. For fixed values of R and C, the phase angle ϕ changes from 0 to -180° as the frequency f is varied from 0 to ∞. In all-pass filter circuit, if the positions of R and C are interchanged, the phase shift between input and output becomes positive. That is, output v_0 leads input v_{in}.

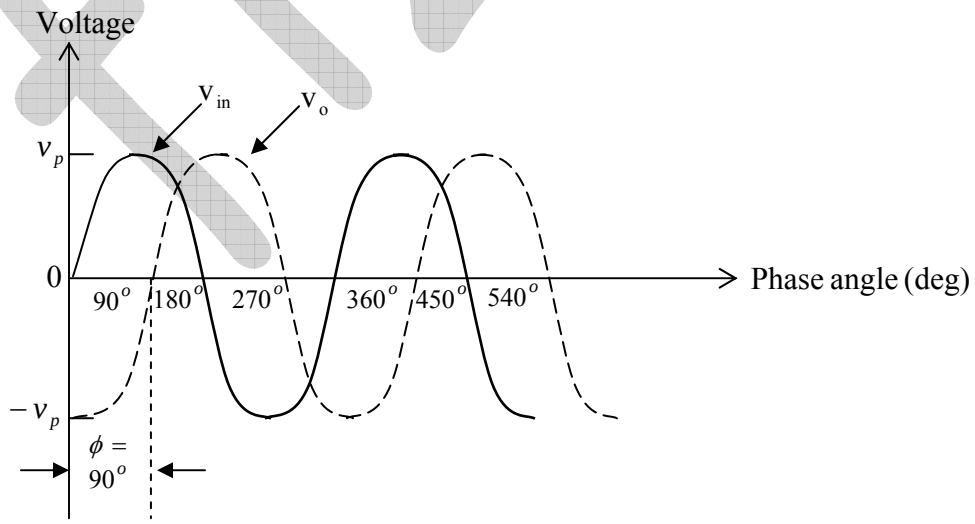


Figure 4.36(b): phase shift between input and output voltages.

4.12 Oscillators

Basically, the function of an oscillator is to generate alternating current or voltage waveforms. More precisely, an oscillator is a circuit that generates a repetitive waveform of fixed amplitude and frequency without any external input signal. Oscillators are used in radio, television, computers, and communications. Although there are different types of oscillators, they all work on the same basic principal.

4.12.1 Oscillator Principles

An oscillator is a type of feedback amplifier in which part of the output is fed back to the input via a feedback circuit. If the signal fed back is of proper magnitude and phase, the circuit produces alternating currents or voltages. To visualize the requirements of an oscillator, consider the block diagram of figure shown below. Here the input voltage is zero ($v_{in} = 0$). Also, the feedback is positive because most oscillators use positive feedback. Finally, the closed-loop gain of the amplifier is denoted by A_v rather than A_F.

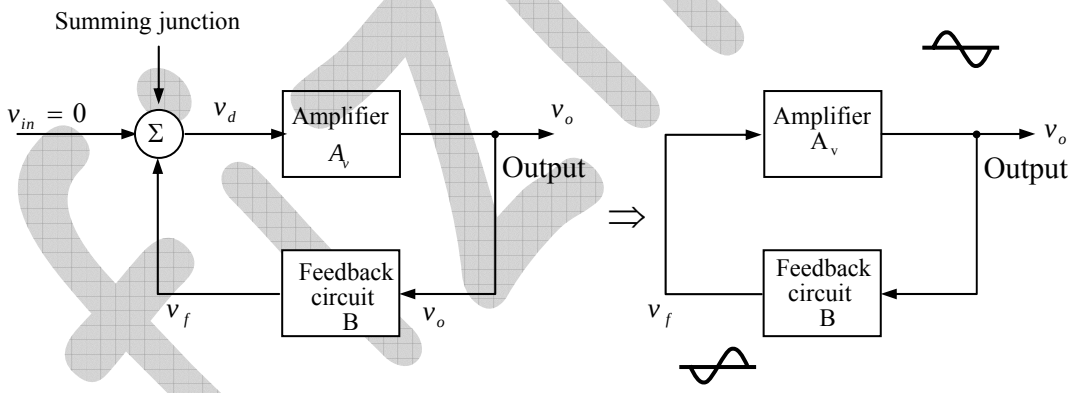


Figure 4.37: Oscillators block diagram.

In the block diagram of above figure, $v_d = v_f + v_{in}$, $v_o = A_v v_d$, $v_f = Bv_o$

Using these relationships, the following equation is obtained: $\frac{v_0}{v_{in}} = \frac{A_v}{1 - A_v B}$

However, $v_{in} = 0$ and $v_o \neq 0$ implies that $A_v B = 1$.

Expressed in polar form, $A_v B = 1$ or total phase shift $A_v B = 0^0$ or 360^0 .

Two requirements for oscillations:

(1) The magnitude of the loop gain $A_v B$ must be at least 1, and

(2) The total phase shift of the loop gain $A_v B$ must be equal to 0^o or 360^o.

For instance, as indicated in figure, if the amplifier causes a phase shift of 180^o, the feedback circuit must provide an additional phase shift of 180^o so that the total phase shift around the loop is 360^o. The waveforms shown are sinusoidal and are used to illustrate the circuit's action. The type of waveform generated by an oscillator depends on the components in the circuit and hence may be sinusoidal, square, or triangular. In addition, the frequency of oscillation is determined by the components in the feedback circuit.

Oscillator Types

Because of their widespread use, many different types of oscillators are available. These oscillator types are summarized in Table given below.

Table: OSCILATOR TYPES

Type of components used	Frequency of oscillator	Types of waveform generated
RC oscillator LC oscillator Crystal oscillator	Audio frequency (AF) Radio frequency (RF)	Sinusoidal Square wave Triangular wave Sawtooth wave, etc

Frequency Stability

The ability of the oscillator circuit to oscillate at one exact frequency is called frequency stability. Although a number of factors may cause changes in oscillator frequency, the primary factors are temperature changes and the change in dc power supply. Temperature and power supply changes cause variations in the op-amp's gain, in junction capacitances and resistances of the transistors in an op-amp, and in external circuit components. In most cases these variations can be kept small by careful design, by using regulated power supplies, and by temperature control.

Multiple Choice Questions (MCQ)

Q1. Consider a voltage follower circuit with $v_o = V_m \sin \omega t$ volts. Then slew rate of operational amplifier is:

(a) $\frac{V_m \omega}{10^6}$ $V/\mu s$ (b) $V_m \omega$ $V/\mu s$

(c) $\frac{V_m \omega}{10^6}$ V/s (d) $\frac{V_m \omega}{10^{12}}$ V/s

Q2. An op-amp has $CMRR = 100dB$, differential gain $A_D = 10$ and common mode voltage applied to it is $2mV$. Then the value of common mode output voltage is:

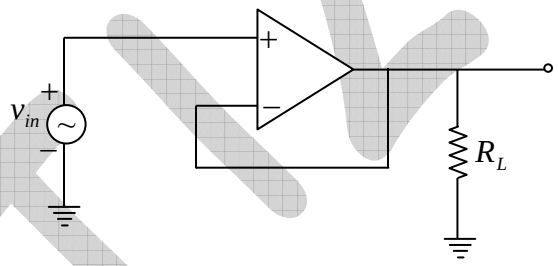
(a) $0.2\,\mu V$ (b) $0.4\,\mu V$ (c) $0.6\,\mu V$ (d) $0.8\,\mu V$

Q3. An amplifier has a voltage gain of 500 and input impedance $20\,k\Omega$, without any feedback. Now a negative feedback with $\beta = 0.1$ is applied. Its gain and input impedance with feedback will respectively be

(a) 9.8 and $392\,\Omega$ (b) 9.8 and $1020\,k\Omega$

(c) 50 and $1020\,k\Omega$ (d) 50 and $2\,k\Omega$

Q4. The circuit shown in figure is used as an



(a) Oscillator (b) High voltage gain amplifier

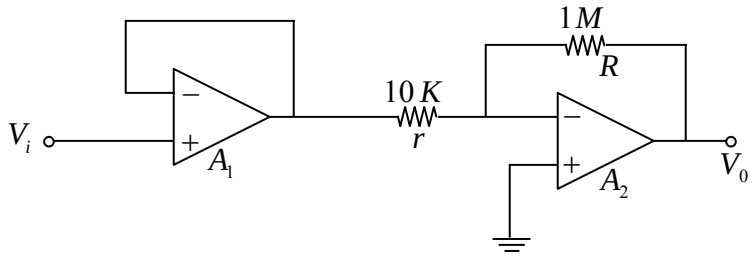
(c) Buffer amplifier (d) Square wave generator

Q5. An op-amp connected in voltage follower mode has following open loop parameters: $A = 200000$, $R_i = 2M\Omega$, $R_o = 75\Omega$. Then the input and output impedance of the voltage follower circuit is:

(a) $200M\Omega$, 75Ω (b) $200G\Omega$, 75Ω

(c) $400G\Omega$, $375\mu\Omega$ (d) $400G\Omega$, $375m\Omega$

Q6. Consider the amplifier circuit comprising of the two op-amps A_1 and A_2 as shown in the figure.



If the input ac signal source has an impedance of $50\,k\Omega$, which of the following statement is true?

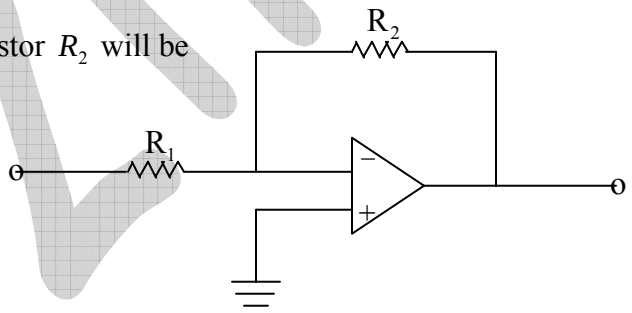
(a) A_1 is required in the circuit because the source impedance is much less than r

(b) A_1 is required in the circuit because the source impedance is much less than R

(c) A_1 can be eliminated from the circuit without affecting the overall gain

(d) A_1 is required in the circuit if the output has to follow the phase of the input signal

Q7. In an inverting operational amplifier, the voltage gain is $20\,dB$. The resistance R_1 is $20\,k\Omega$. The value of feedback resistor R_2 will be



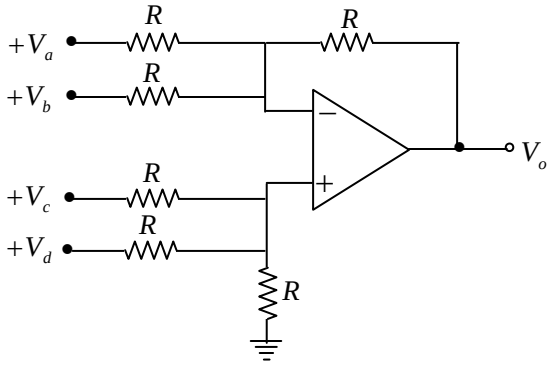
(a) $2\,M\Omega$

(b) $200\,k\Omega$

(c) $20\,k\Omega$

(d) $2\,k\Omega$

Q8. For the circuit shown in figure, the output voltage V_0 is:



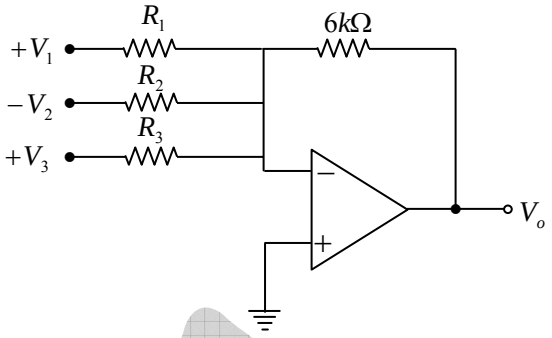
(a) $V_a + V_b - V_c - V_d$

(b) $V_a - V_b + V_c - V_d$

(c) $-V_a - V_b + V_c + V_d$

(d) $-V_a - V_b - V_c - V_d$

Q9. For the circuit shown in figure the output is $V_o = -V_1 + 2V_2 - 3V_3$. The values of resistances R_1, R_2 and R_3 is



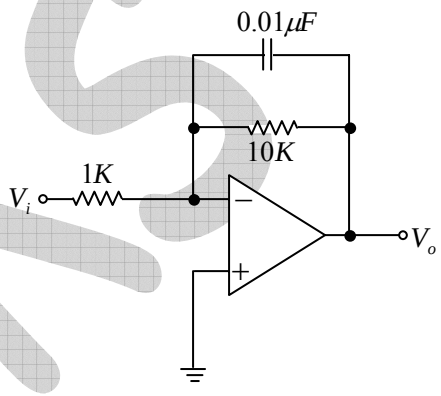
(a) $R_1 = 6k\Omega, R_2 = 2k\Omega, R_3 = 3k\Omega$

(b) $R_1 = 2k\Omega, R_2 = 6k\Omega, R_3 = 3k\Omega$

(c) $R_1 = 6k\Omega, R_2 = 3k\Omega, R_3 = 2k\Omega$

(d) $R_1 = 6k\Omega, R_2 = 3k\Omega, R_3 = 3k\Omega$

Q10. In the op-amp circuit shown in the figure, V_i is a sinusoidal input signal of frequency 10 Hz and V_o is the output signal. The magnitude of the gain is close to the values



(a) 4 (b) 9

(c) 15 (d) 20

Q11. In order to obtain a solution of the third order differential equation, involving voltages $v(t)$, an OP-AMP circuit would require at least

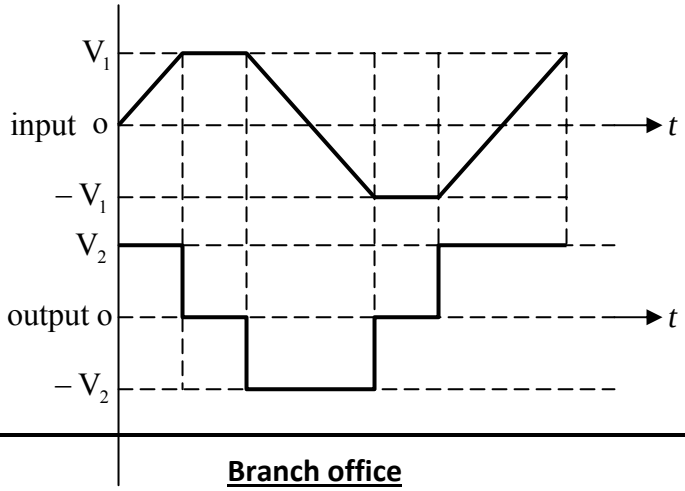
(a) two integrator and one adder

(b) three integrator only

(c) three differentiator and one adder

(d) three integrator and one adder.

Q12. The circuit for which the input and output waveforms are shown below is



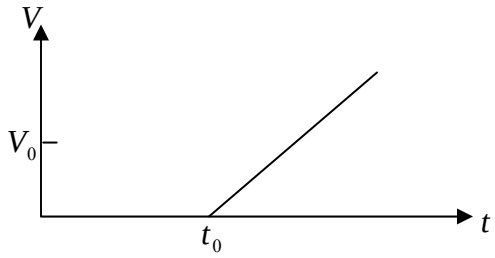
(a) Clipping circuit

(b) Integrator

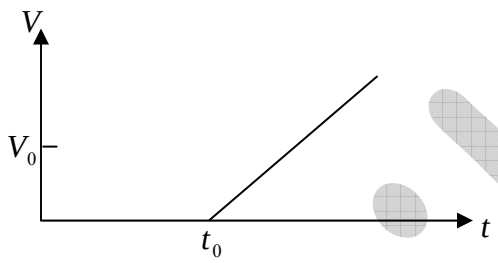
(c) Differentiator

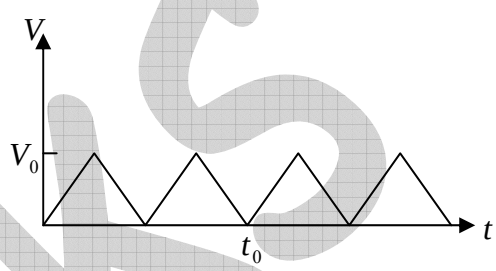
(d) Schmitt trigger

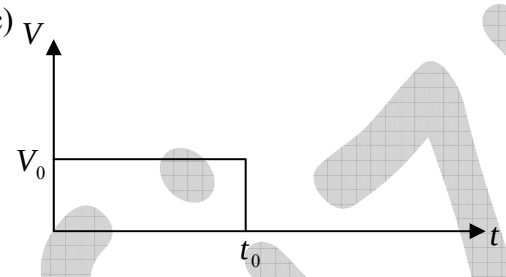
Q13. The input given to an ideal OP-AMP differentiator circuit is

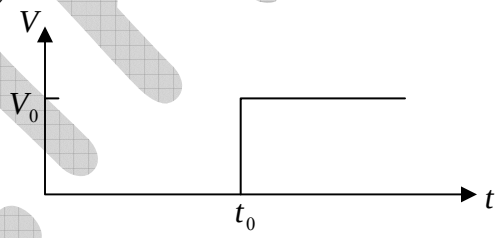


The correct output of the differentiator circuit is

(a)


(b)


(c)


(d)


Q14. A low pass filter is formed by a resistance R and a capacitance C. At the cut-off angular frequency $\omega_C = \frac{1}{RC}$ the voltage gain and the phase of the output voltage relative to the input voltage respectively are

(a) $\frac{1}{\sqrt{2}}$ and 45^o

(b) $\frac{1}{\sqrt{2}}$ and -45^o

(c) $\frac{1}{2}$ and -90^o

(d) $\frac{1}{2}$ and 90^o

Q15. The input to a lock-in amplifier has the form $V_i(t) = V_i \sin(\omega t + \theta_i)$ where V_i, ω, θ_i are the amplitude, frequency and phase of the input signal respectively. This signal is multiplied by a reference signal of the same frequency ω, amplitude V_r and phase θ_r. If the multiplied signal is fed to a high pass filter of cut-off frequency ω, the final output signal is

(a) $\frac{1}{2}V_i V_r \cos(\theta_i - \theta_r)$ (b) $\frac{V_i V_r}{2}\cos(2\omega t + \theta_i + \theta_r)$

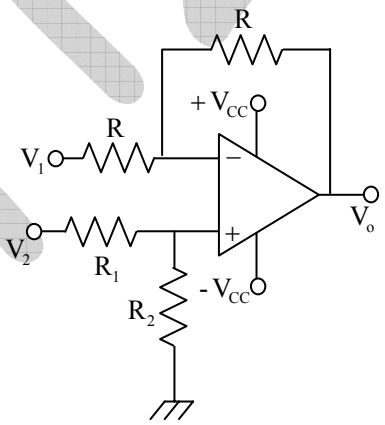
(c) $V_i V_r \sin(\theta_i - \theta_r)$ (d) $V_i V_r \cos\left(\frac{1}{2}\omega t + \theta_i + \theta_r\right)$

Numerical Answer Type Questions (NAT)

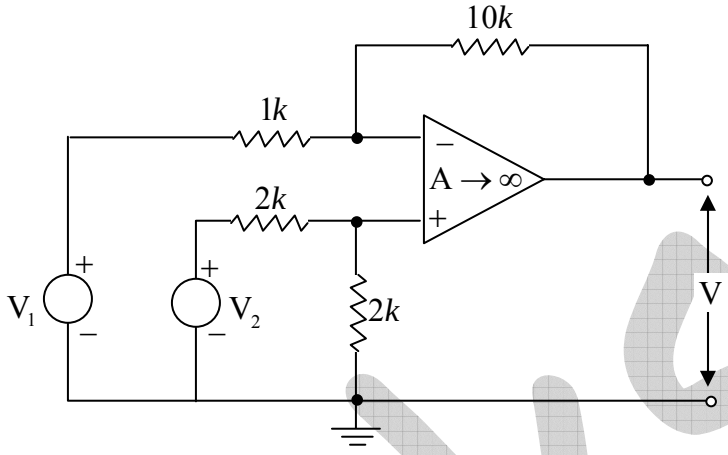
Q16. An op-amp has differential gain $A_D = 10$. Common mode voltage applied to it is $2mV$ and corresponding output voltage is $0.2\mu V$. Then the value of $CMRR(dB)$ is……..

Q17. An amplifier has a voltage gain of 500 and input impedance $20\ k\Omega$, without any feedback. Now a negative feedback is applied and its gain and input impedance are 9.8 and 1020 $k\Omega$ respectively. Then the feedback ratio is ……

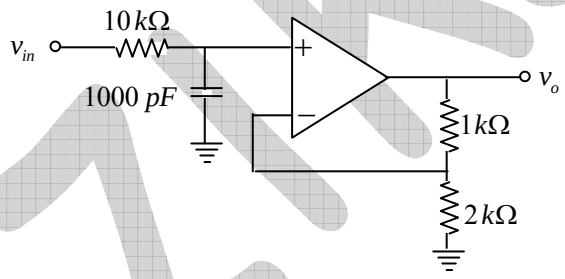
Q18. In the following circuit, for the output voltage to be $V_0 = (-V_1 + V_2/3)$ the ratio R_1/R_2 is…….



Q19. For the circuit given below $V_1 = 0.2\,V$ and $V_2 = 0.8\,V$. Assume that the operational amplifier is ideal then the output voltage V is…….



Q20. For the given circuit the frequency above which the gain will decrease by 20 dB per decade is……. kHz

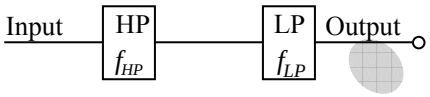


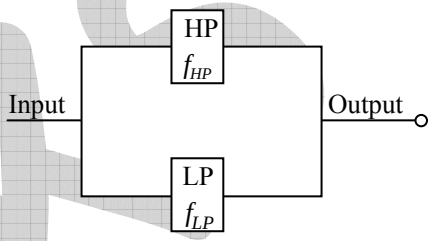
Multiple Select type Questions (MSQ)

Q21. An amplifier has a voltage gain of 500 and input impedance $20\ k\Omega$, without any feedback. Now a negative feedback with $\beta = 0.1$ is applied. Its gain and input impedance with feedback will respectively be

(a) 10 (b) 20 (c) $500\ k\Omega$ (d) $1000\ k\Omega$

Q22. Consider a Low Pass (LP) and a High Pass (HP) filter with cut-off frequencies f_{LP} and f_{HP}, respectively, connected in series or in parallel configurations as shown in the Figures A and B below.

(A) 

(B) 

Which of the following statements are correct?

(a) For $f_{HP} < f_{LP}$, A acts as a Band Pass filter

(b) For $f_{HP} > f_{LP}$, A stops the signal from passing through

(c) For $f_{HP} < f_{LP}$, B stops the signal from passing through

(d) For $f_{HP} > f_{LP}$, B acts as a Band Reject filter

Q23. Which of the following statements are true regarding first order low pass Op-amp filter (where A_F is passband gain, f_H is cutoff frequency and f is operating frequency)

(a) The magnitude of gain $\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1+(f/f_H)^2}}$

(b) The magnitude of gain $\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1+(f_H/f)^2}}$

(c) The phase angle (ϕ) is $-\tan^{-1}\left(\frac{f}{f_H}\right)$

(d) The phase angle (ϕ) is $-\tan^{-1}\left(\frac{f_H}{f}\right)$

Q24. Which of the following statements are true regarding first order high pass Op-amp filter (where A_F is passband gain, f_L is cutoff frequency and f is operating frequency)

(a) The magnitude of gain $\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1+(f/f_L)^2}}$

(b) The magnitude of gain $\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1+(f_L/f)^2}}$

(c) The phase angle (ϕ) is $+\tan^{-1}\left(\frac{f}{f_L}\right)$

(d) The phase angle (ϕ) is $+\tan^{-1}\left(\frac{f_L}{f}\right)$

Q25. Which of the following are true regarding oscillators?

(a) Loop gain $(A_v.B)$ of the circuit must be atleast 1

(b) Total phase shift must be 0^0 or 360^0

(c) It uses positive feedback

(d) It uses negative feedback

Solution
(MCQ)

Ans.1: (a)

Ans.2: (a)

$$CMRR(in\,dB) = 20\log_{10}\left(\frac{A_D}{A_{CM}}\right) \Rightarrow 100 = 20\log_{10}\left(\frac{10}{A_{CM}}\right) \Rightarrow \frac{10}{A_{CM}} = 10^5 \Rightarrow A_{CM} = 10^{-4}$$

$$\because A_{CM} = \frac{V_{oCM}}{V_{CM}} = 10^{-4} \Rightarrow V_{oCM} = 10^{-4} \times 2 \times 10^{-3} V = 0.2\ \mu V$$

Ans.3: (b)

$$A_F = \frac{A}{1+AB} = \frac{500}{1+500\times 0.1} = \frac{500}{51} = 9.8 \text{ and}$$

$$R_{iF} = R_i(1+AB) = 20k \times (1+500\times 0.1) = 1020\ k\Omega$$

Ans.4: (c)

Ans.5 :(c)

$$1+AB \approx A \Rightarrow R_{iF} = R_i(1+AB) = R_i A = 2\times 10^6 \times 2\times 10^5 = 400G\Omega$$

$$R_{oF} = \frac{R_0}{A} = \frac{75}{2\times 10^5} = 375\mu\Omega$$

Ans.6: (c)

A_1 can be eliminated from the circuit without affecting the overall gain

Ans.7: (b)

$$Gain(in\,dB) = 20\log_{10}\left(\frac{V_0}{V_{in}}\right) \Rightarrow 20 = 20\log_{10}\left(\frac{V_0}{V_{in}}\right) \Rightarrow \frac{V_0}{V_{in}} = -\frac{R_2}{R_1} = 10 \Rightarrow R_2 = 200k\Omega$$

Ans.8: (c)

$$V_o = V_{0a} + V_{0b} + V_{0c} + V_{0d} = -\frac{R}{R}V_a - \frac{R}{R}V_b + \left(1+\frac{R}{R/2}\right)\frac{R/2}{R+R/2}V_c + \left(1+\frac{R}{R/2}\right)\frac{R/2}{R+R/2}V_d$$

$$\Rightarrow V_o = -V_a - V_b + V_c + V_d$$

Ans.9: (c)

$$V_o = V_{01} + V_{02} + V_{03} = -\left(\frac{6k}{R_1}V_1 - \frac{6k}{R_2}V_2 + \frac{6k}{R_3}V_3\right) = \frac{6k}{R_1}\times -V_1 + \frac{6k}{R_2}\times V_2 + \frac{6k}{R_3}\times -V_3$$

Ans.10: (b)

$$\frac{v_0}{v_{in}} = -\frac{X_C \parallel R_F}{R_1} = -\frac{X_C R_F}{R_1\left(X_C + R_F\right)} = -\frac{R_F}{R_1\left(1 + R_F / X_C\right)} = -\frac{R_F}{R_1\left(1 + j\omega C R_F\right)}$$

$$\left|\frac{v_0}{v_{in}}\right| = \frac{R_F / R_1}{\sqrt{1+\left(2\pi f C R_F\right)^2}} = \frac{10/1}{\sqrt{1+4\pi^2 f^2 C^2 R_F^2}} = \frac{10}{\sqrt{1+4\times 10\times 100\times 10^{-16}\times 10^8}} \Rightarrow \left|\frac{v_0}{v_{in}}\right| \approx 10$$

Ans.11: (d)

Ans.12: (c)

Ans.13: (d)

Ans.14: (b)

$$\frac{v_0}{v_{in}} = \frac{X_C}{R + X_C} = \frac{1}{\frac{R}{X_C} + 1} = \frac{1}{1 + j\omega CR}$$

At $\omega = \omega_C = \frac{1}{RC} \Rightarrow \frac{v_0}{v_{in}} == \frac{1}{1+j} = \frac{1}{\sqrt{2}e^{j45^0}} = \frac{1}{\sqrt{2}}e^{-j45^0}$

Ans.15: (b)

$$V = V_r \sin\left(\omega t + \theta_r\right) \times V_i \sin\left(\omega t + \theta_i\right) = \frac{V_i V_r}{2}\left[\cos\left(\theta_i - \theta_r\right) - \cos\left(2\omega t + \theta_i + \theta_r\right)\right]$$

Output of high pass filter $= \frac{V_i V_r}{2}\cos\left(2\omega t + \theta_i + \theta_r\right)$

(NAT)

Ans.16: 100

$$A_{CM} = \frac{0.2\times10^{-6}}{2\times10^{-3}} = 10^{-4},\ A_D = 10 \Rightarrow CMRR(in\ dB) = 20\log_{10}\left(\frac{10}{10^{-4}}\right) = 100\ dB$$

Ans.17: 0.1

$$A_F = \frac{A}{1+AB} \Rightarrow 9.8 = \frac{500}{1+500\times B} \Rightarrow B = 0.1 \text{ and}$$

$$R_{iF} = R_i(1+AB) \Rightarrow 1020\ k\Omega = 20k\times(1+500\times B) \Rightarrow B = 0.1.$$

Solution.18: 5

When $V_2 = 0,\quad v_{01} = -V_1$ when $V_1 = 0,\quad v_{02} = \left(1+\frac{R}{R}\right)\left(\frac{R_2}{R_1+R_2}\right)V_2$

Since $V_0 = -V_1 + \frac{V_2}{3} \Rightarrow 2\cdot\frac{R_2}{R_1+R_2} = \frac{1}{3} \Rightarrow \frac{R_1}{R_2} = 5$

Ans.19: (c)

$$V = V_{01} + V_{02} = -\frac{10}{1}V_1 + \left(1+\frac{10}{1}\right)\times\frac{2}{2+2}V_2 = -10V_1 + \frac{11}{2}V_2 = -2 + 4.4 = +2.4$$

Ans.20: 15.9

$$f_H = \frac{1}{2\pi RC} = 15.9\,kHz \approx 16\,kHz$$

(MSQ)

Ans.21: (a), (d)

$$A_F = \frac{A}{1+AB} = \frac{500}{1+500\times0.1} = \frac{500}{51} = 9.8 \text{ and}$$

$$R_{iF} = R_i(1+AB) = 20k\times(1+500\times0.1) = 1020\ k\Omega$$

Ans.22: (a), (b) and (d)

Ans.23: (a), (c)

Ans.24: (b), (d)

Ans.25: (a), (b) and (c)

5. Digital Electronics

5.1 Number System

5.1.1 Decimal Numbers

Decimal number system uses ten digits, that is, a system with **base of ten**. Each of the ten decimal digits, 0 through 9, represents the certain quantity. The position of each of the digits in the decimal number indicates the magnitudes of the quantity represented and can be assigned a "weight".

The value of a decimal number is the sum of the digits times their respective column weights.

Example: $23 = 2\times10 + 3\times1 = 20 + 3 = 23$

The digit 2 has a weight of 10, as indicated by its position, and the digit 3 has a weight of 1, as indicated by its position.

Example: $568 = 5\times100 + 6\times10 + 8\times1 = 500 + 60 + 8 = 568$

The digit 5 has weight of 100, the digit 6 has a weight of 10, and the digit 8 has a weight of 1.

5.1.2 Binary Numbers

The binary system with its two digits is a **base-two** system. The two binary digits (bits) are 1 and 0. The position of the 1 or 0 in a binary number indicates its "weight". The weight of each successively higher position (to the left) in a binary number is an increasing power of two.

Highest decimal number with n bits $= 2^n - 1$

For instance, with two bits we can count from 0 through $3\left(2^2 - 1 = 3\right)$.

Binary-to-Decimal Conversion

The value of a given binary number in terms of its decimal equivalent can be determined by adding the products of each bit and its weight. The right-most bit is the least significant bit (LSB) in a binary number and has a weight of $2^0 = 1$. The weights increase by a power of two for each bit from right to left.

Example: Convert $(1101101)_2$ to decimal number.

Solution:

Binary weight:	2^6 2^5 2^4 2^3 2^2 2^1 2^0
Weight value:	64 32 16 8 4 2 1
Binary number:	1 1 0 1 1 0 1

$$1\times 64+1\times 32+0\times 16+1\times 8+1\times 4+0\times 2+1\times 1=64+32+0+8+4+0+1=109_{10}$$

The binary numbers we have seen so far have been whole numbers. Fractional number can also be represented in binary by placing bits to the right of the **binary point**, just as fractional decimal digits are placed to the right of the decimal point.

The column weights of a binary number are

$$2^n...2^3 2^2 2^1 2^0 . 2^{-1} 2^{-2}...2^{-n}$$

$$binary\ point \leftarrow$$

This indicates that all the bits to the left of the binary point have weights that are positive powers of two and to the right of the binary point have weights that are negative powers of two.

Example: Convert $(0.1011)_2$ to decimal number.

Binary weight:	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Weight value:	0.5	0.25	0.125	0.0625
Binary number:	1	0	1	1

$$1\times 0.5+0\times 0.25+1\times 0.125+1\times 0.0625=0.5+0+0.125+0.0625=0.6875_{10}\ .$$

Example: Convert $(11101.011)_2$ to decimal number.

Solution:

$$(16+8+4+1).(0.25+0.125)=29.375_{10}$$

Binary addition and subtraction

There are four basic rules for adding binary digits: $0+0=0,\ 0+1=1$
$1+0=1,\ 1+1=10_2$

There are four basic rules for subtracting binary digits: $0-0=0,\ 1-1=0$
$1-0=1,\ 10_2-1=1.$

1's and 2's compliments

The 1's compliment of a binary number is found by simply changing all 1's to 0's and all 0's to 1's. The 2's compliment of a binary number is found by adding 1 to the 1's compliment.

Example:

Binary Number	1's compliment	2's compliments
10101	01010	01011
10111	01000	01001
111100	000011	000100

Decimal-to-Binary Conversion

In a repeated division-by-2 process we begin by dividing decimal number by 2 and then dividing each resulting quotient by 2 until there is a 0 quotient. The remainders generated by each division form the binary number. The first remainder to be produced is the least significant bit (LSB) in the binary number.

Example: Convert decimal number 12 to binary number.

Step 1	**Step 2**	**Step 3**	**Step 4**
$2\overline{)12}$	$2\overline{)6}$	$2\overline{)3}$	$2\overline{)1}$
6	3	1	0
12	6	2	0
0	0	1	1
LSB			**MSB**

$\rightarrow (1100)_2$

Converting Decimal Fraction to Binary

In a repeated multiplication-by-2 process we begin by multiplying decimal number by 2 and then multiplying each resulting fractional part by 2 until the fractional product is zero. The carry generated by each multiplication form the binary number. The first carry produced is the most significant bit (MSB) in the binary number.

Example: Convert decimal number 0.3125 to binary number.

Step 1 $0.3125\times 2 = 0.625 \rightarrow \text{carry} = 0\ (MSB)$

Step 2 $0.625\times 2 = 1.25 \rightarrow \text{carry} = 1$

Step 3 $0.25\times 2 = 0.50 \rightarrow \text{carry} = 0$

Step 4 $0.50\times 2 = 1.00 \rightarrow \text{carry} = 1\ (LSB)$

$$\rightarrow (0.0101)_2$$

5.1.3 Octal Numbers

The octal number system is composed of eight digits, which are

$$0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7\ .$$

Counting in octal is the same as counting in decimal, except any number with an 8 or a 9 is omitted. It is a system with **base eight**.

Octal-to-Decimal Conversion

Since the octal number system has a base of eight, each successive digit position is an increasing power of eight, beginning in the right-most column with 8^0. The evaluation of an octal number in terms of its decimal equivalent is accomplished by multiplying each digit by its weight and summing the products.

Example: Convert $(2374)_8$ to decimal number.

Solution:

Octal weight:	8^3	8^2	8^1	8^0
Weight value:	512	64	8	1
Octal number:	2	3	7	4

$$2\times 512 + 3\times 64 + 7\times 8 + 4\times 1 = 1276_{10}$$

The octal numbers we have seen so far have been whole numbers. Fractional octal numbers are represented by digits to the right of the octal point.

The column weights of an octal number are

$$8^n \ldots 8^3 8^2 8^1 8^0 . 8^{-1} 8^{-2} \ldots 8^{-n}$$

This indicates that all the digits to the left of the octal point have weights that are positive powers of eight and to the right of the octal point have weights that are negative powers of eight.

Example: Convert $(0.325)_8$ to decimal number.

Solution:

Octal weight:	8^{-1}	8^{-2}	8^{-3}
Weight value:	0.125	0.015625	0.001953
Octal number:	3	2	5

$$3\times 0.125 + 2\times 0.015625 + 5\times 0.001953 = 0.416015_{10}$$

Decimal-to-Octal Conversion

In a repeated division-by-8 process we begin by dividing decimal number by 8 and then dividing each resulting quotient by 8 until there is a 0 quotient. The remainders generated by each division form the octal number. The first remainder to be produced is the least significant digit (LSD) in the octal number.

Example: Convert decimal number 359 to octal number.

Solution:

Step 1	**Step 2**	**Step 3**
$8\overline{)359}$	$8\overline{)44}$	$8\overline{)5}$
44	5	0
32	40	0
39	4	5
32		
7		
LSD		**MSD**

$$\rightarrow (547)_8$$

Octal-to-Binary Conversion

The primary application of octal numbers in is in the representation of binary numbers. Since it takes only one octal digit to represent three bits, octal numbers are much easier to "read" than binary numbers.

Because all three-bit binary numbers are required to represent the eight octal digits, it is very easy to convert from octal to binary and from binary to octal. The octal number system is often used in digital systems, especially for input/output applications.

Each octal digit is represented by three bits as indicated:

Octal Digit	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

To convert an octal number to a binary number, simply replace each octal digit by the appropriate three bits. This is illustrated in the following example.

Example: Convert each of the following octal numbers to binary

(a) 13_8 (b) 25_8 (c) 47_8 (d) 170_8 (e) 752_8 (f) 5276_8

(g) 37.12_8

Solutions:

(a) 13_8 ↙↘ 001011_2

(b) 25_8 ↙↘ 010101_2

(c) 47_8 ↙↘ 100111_2

(d) 170_8 ↙↓↘ 001111000_2

(e) 752_8 ↙↓↘ 111101010_2

(f) 5276_8 ↙↙↘↘ 1010101111110_2

(g) 37.12_8 ↙↙↘↘ 011111.001010_2

Binary-to-octal Conversion

Conversion of a binary number to an octal number is also a straightforward process. Beginning at the binary point, simply break the binary number into groups of three bits and convert each group into the appropriate octal digit.

Example: Represent each of the following binary numbers by its octal equivalent:

(a) 110101_2 (b) 101111001_2 (c) 1011100110_2 (d) 1001101.1011_2

Solutions:

(a) $\underbrace{110}_{6}\underbrace{101}_{5_8}{}_2$

(b) $\underbrace{101}_{5}\underbrace{111}_{7}\underbrace{001}_{1_8}{}_2$

(c) $\underbrace{001}_{1}\underbrace{011}_{3}\underbrace{100}_{4}\underbrace{110}_{6_8}{}_2$

(d) $\underbrace{001}_{1}\underbrace{001}_{1}\underbrace{101}_{5}\underbrace{101}_{5}\underbrace{100}_{4_8}{}_2$

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5.1.4 Hexadecimal Numbers

The hexadecimal system has a base of sixteen; that is, it is composed of 16 digits and characters. Many digital systems process binary data in groups that are multiples of four bits, making the hexadecimal number very convenient because each hexadecimal digit represents a four-bit binary number (as listed in Table 2-2).

Ten digits and six alphabetic characters make up this number system. A subscript 16 indicates a hexadecimal number.

Decimal	Binary	Binary
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Binary-to-Hexadecimal Conversion

Converting a binary number to hexadecimal is a straightforward procedure. Simply break the binary number into four-bit groups starting at the binary point, and replace each group with the equivalent hexadecimal symbol.

Example: (a) 1100101001010111_2 (b) $1111110001011010 01_2$

(c) 1110011000.111_2

Solution:

(a) $\underbrace{1100}_{C}\underbrace{1010}_{A}\underbrace{0101}_{5}\underbrace{0111}_{7}{}_2 = 7_{16}$

(b) $\underbrace{0011}_{3}\underbrace{1111}_{F}\underbrace{0001}_{1}\underbrace{0110}_{6}\underbrace{1001}_{9}{}_2 = 9_{16}$

(c) $\underbrace{0011}_{3}\underbrace{1001}_{9}\underbrace{1000}_{8}.\underbrace{1110}_{E} = E_{16}$

Hexadecimal-to-Binary Conversion

To convert from a hexadecimal number to a binary number, reverse the process and replace each hexadecimal symbol with the appropriate four bits.

Example: Determine the binary numbers for the following hexadecimal numbers:

(a) $10A4_{16}$ (b) $CF83_{16}$ (c) 9742_{16} (d) $D2E.8_{16}$

Solutions:

(a) $10A4_{16} = 1000010100100_2$

(b) $CF83_{16} = 1100\,1111\,1000\,0011$

(c) $9742_{16} = 1001\,0111\,0100\,0010_2$

(d) $D2E.8_{16} = 1101\,0100\,1110\,1000_2$

Hexadecimal-to-Decimal Conversion

One way to evaluate a hexadecimal number in terms of its decimal equivalent is first convert from binary decimal. The following example illustrates this procedure.

Example: Convert the following hexadecimal numbers to decimal.

(a) $1C_{16}$ (b) $A85_{16}$

Solutions:

(a) $1C_{16}$

$0001\ 1100_2$

$0011100_2 = 24 + 23 + 22$

$= 16 + 8 + 4$

$= 28_{10}$.

(b) $A85_{16}$

$1010\ 1000\ 0101_2$

1010100001012

$1010100001012 = 2^{11} + 2^9 + 2^7 + 2^2 + 2^0$

$= 2048 + 512 + 128 + 4 + 1$

$= 2693_{10}$

Another way to convert a hexadecimal number to its decimal equivalent is by multiplying each hexadecimal digit by its weight and then taking the sum of these products. The weights of a hexadecimal number are increasing powers of 16 (from right to left). For a four-digit hexadecimal number the weights are

16^3	16^2	16^1	16^0
4096	256	16	1

The following example shows this conversion method.

Example: Convert (a) $E5_{16}$ and (b) $B2F8_{16}$ to decimal.

Solutions:

(a) $E5_{16} = E \times 16 + 5 \times 1 = 14 \times 16 + 5 \times 1 = 224 + 5 = 229_{10}$

(b) $B2F8_{16} = B \times 4096 + 2 \times 256 + F \times 16 + 8 \times 1$

$= 11 \times 4096 + 2 \times 256 + 15 \times 16 + 8 \times 1$

$= 45056 + 512 + 240 + 8 = 45816_{10}$

Decimal to Hexadecimal number formed by the reminders of each division. This is similar to the repeated division-by-2 for decimal-to-binary conversion and repeated division-by-8 for decimal-to-octal conversion.

Example: Convert 650_{10} to hexadecimal by repeated division by 16_{10}.

Solution:

	Hexadecimal Remainder	
$16\overline{)650}$ = 40, 64, 10 $\rightarrow$	A	(LSD)
$16\overline{)40}$ = 2, 32, 8 $\rightarrow$	8	
$16\overline{)2}$ = 0, 0, 2 $\rightarrow$	2	(MSD)

Therefore, $650_{10} = 28A_{16}$.

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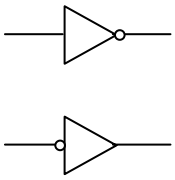
5.2 Logic Gates

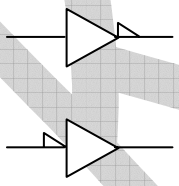
5.2.1 The Inverter

The inverter (**NOT** circuit) performs a basic logic function called *inversion* or *complementation*. The purpose of the inverter is to change one logic level to opposite level. In terms of bits, it changes a 1 to a 0 and a 0 to a 1.

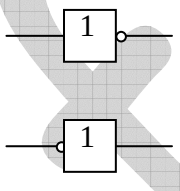
Standard logic symbols for the inverter are shown in figure 5.1. Figure 5.1 (a) and (b) show the distinctive shape symbols and parts (c) and (d) show the *rectangular outline* symbols. *In this text, **distinctive** shape symbols used*; however, the rectangular outline symbols are commonly found in industry publications, and you should become familiar with them as well.



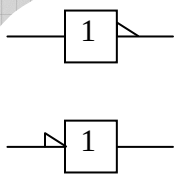
(a) Distinctive shape symbols with negation indicators.



(b) Distinctive shape symbols with polarity indicators.



(c) Rectangular outline symbols with negation indicators.



(d) Rectangular outline symbols with polarity indicators.

Figure 5.1: Standard logic symbols for the inverter.

The Negation and Polarity Indicators

The negation indicator is a "bubble" (◦) appearing on the input or output logic element, as shown in figure 5.1 (a) and (c). When appearing on the input, bubble means that an external 0 produces an internal 1. When appearing on output, the bubble means that an internal 1 produces an external 0. Typically inputs are on the left of a logic symbol and outputs are on the right.

The polarity indicator is a "triangle" (◺) appearing on the input or output of a logic element, as shown in figure 5.1 (b) and (d). When appearing on input, it means that an external LOW level produces an internal HIGH. When appearing on the output, it means that an internal HIGH produces an external LOW level. Either indicator (bubble or triangular) can be used on both distinctive shape symbols and rectangular outlines as indicated. The placement of the negation or polarity indicator does not imply a change in the way an inerter operates.

Because positive logic (HIGH = 1, LOW = 0) is used in this text, both indicators are equivalent and can be interchanged. However, we will use primarily the negation indicator (bubble) throughout this text to represent an inversion.

The placement of the bubble on the input or output of a logic element is determined by the *active state* of the input signal (pulse or level). The active state is the state (1 or 0) when the signal is considered to be present on the input. When the active state of the input is a 0, the bubble is placed on the input. When the active output state is a 0, the bubble is placed on the output.

Inverter Operation

When a HIGH level is applied to an inverter input, a LOW level will appear on its output. When a LOW level is applied to its input, a HIGH will appear on its output. This operation is summarized in Table given below, which shows the output for each possible input in terms of levels and bits. These tables are called *truth tables*.

TABLE: Inverter Truth Tables

Input	Output
HIGH	LOW
LOW	HIGH

Input	Output
0	1
1	0

Pulsed Operation

Figure 5.2 shows the output of an inverter for a pulse input where t_1 and t_2 indicate the corresponding points on the input and output pulse waveforms. Note that when the input is LOW, the output is HIGH, and when the input is HIGH, the output is LOW, thereby producing an inverted output pulse.

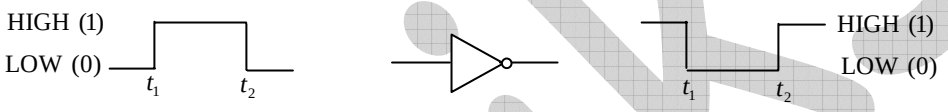
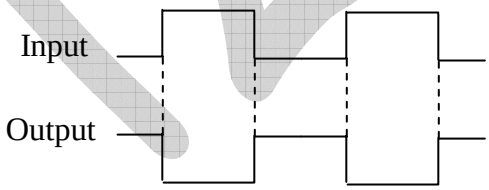


Figure 5.2: Inverter with pulse input.

Example: A pulse waveform is applied to an inverter as shown in figure 5.2. Determine the output waveform corresponding to the input.

Solution: The output waveform is exactly opposite to the input (inverted) at each point.



5.2.2 The AND Gate

The AND gate performs logical multiplication, more commonly known as the AND function. The AND gate is composed of two or more inputs and a single output, as indicated by the standard logic symbols shown in figure 5.3. Inputs are on the left and the output is on the right in each symbol. Gates with two and four inputs are shown; however, an AND gate can have any number of inputs greater than one.

(a) Distinctive shape (b) Rectangular outline

Figure 5.3: Standard logic symbols for the AND gate showing two and four inputs.

Logical Operation of the AND Gate

The operation of the AND gate is such that the output is HIGH only when all of the inputs are HIGH. When any of the inputs are LOW, the output is LOW.

The truth table can be expanded for any number of inputs.

Table: Truth table for a two-input AND gate.

Inputs		Outputs
A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

Pulsed Operation

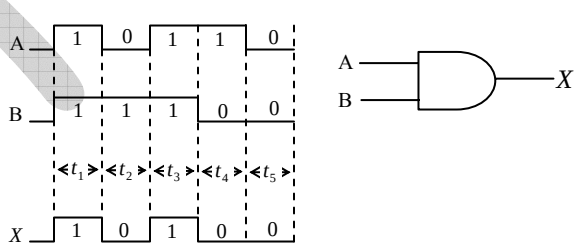


Figure 5.4: Example of pulsed AND gate operation.

5.2.3 The OR Gate

The OR gate performs logical addition, more commonly known as the OR function. An OR gate has two or more inputs and one output, as indicated by standard logic symbols shown in figure 5.5 where OR gates with two and four inputs are illustrated.

Figure 5.5: Standard logic symbols for the OR gate showing two and four inputs.

Logical Operation of the OR Gate

The operation of the OR gate is such that a HIGH on the output is produced when any of the inputs are HIGH. The output is LOW only when all of the inputs are LOW.

Table: Truth table for a two-input OR gate.

Inputs		Outputs
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Pulsed Operation

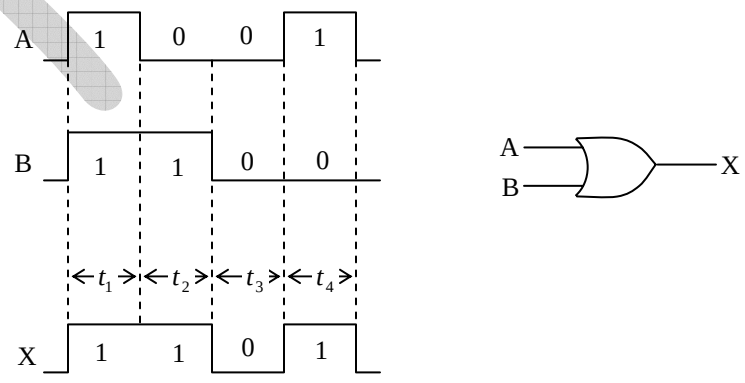
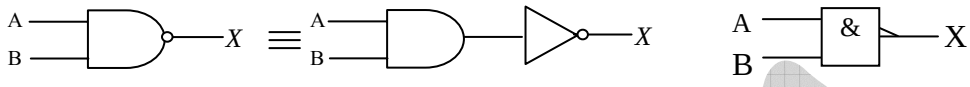


Figure 5.6: Example of pulsed OR gate operation.

5.2.4 The NAND Gate

The term NAND is a contraction of NOT-AND and implies an AND function with a complemented (inverted) output. A standard logic symbol for a two-input NAND gate and its equivalency to an AND gate followed by an inverter are shown in figure 5.7(a). A rectangular outline representation is shown in part (b).



(a) Distinctive shape, two-input NAND gate with negation indicators

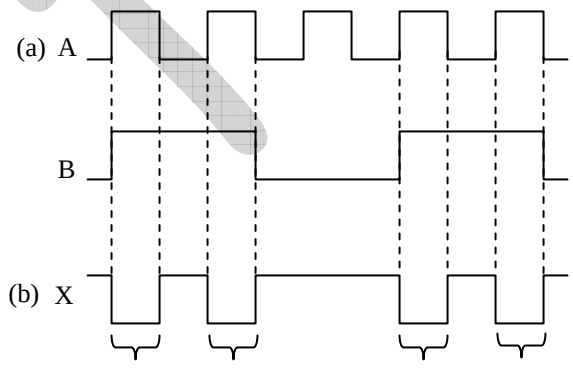
(b) Rectangular outline, two-input NAND gate with polarity indicator

Figure 5.7: Standard NAND gate logic symbols.

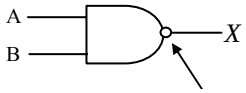
The NAND gate is very popular logic function because it is a *"universal" function*; that is, it can be used to construct an AND gate, an OR gate, an inverter, or any combination of these functions. Truth table for a two-input NAND gate.

Inputs		Outputs
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Pulsed Operation



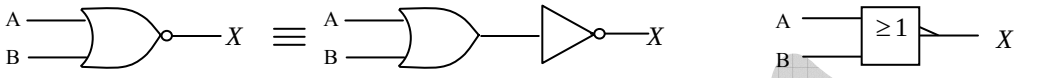
A and B are both HIGH.
Therefore X is LOW.



Bubble indicates an active-LOW output

5.2.5 The NOR Gate

The term NOR is a contraction of NOT-OR and implies an OR function with inverted output. A standard logic symbol for a two input NOR gate and equivalent OR gate followed by an inverter are shown in figure 5.8(a). A rectangular outline symbol is shown in part (b).



(a) Distinctive shape, two-input NOR gate and its NOT/OR equivalent with negation indicators

(b) Rectangular outline, two-input NOR gate with polarity indicator

Figure 5.8: Standard NOR gate logic symbols.

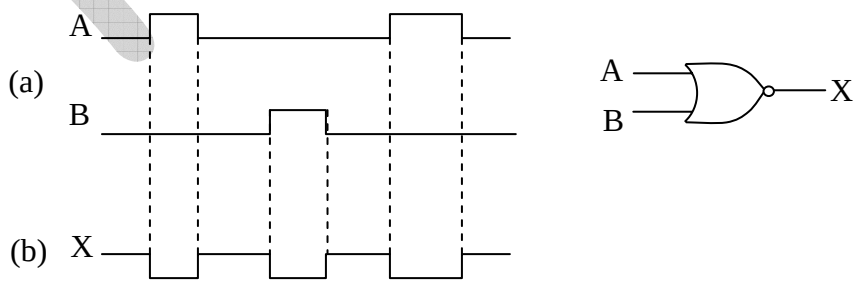
The NOR gate, like the NAND, is a very useful logic gate because of *universal property*.

Truth table for a two-input gate

Inputs		Outputs
A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

Figure 5.8: Standard NOR gate logic symbols.

Pulsed Operation:



5.2.6 OR Gate (Positive Logic and Negative Logic)

OR Gate (Positive logic)

The truth table can be verified for the circuit shown in figure 5.9. In positive logic $V(1)$ means HIGH voltage and $V(0)$ means LOW voltage. When both inputs have LOW voltage output will be $V(0)$ i.e. LOW voltage. When any of the input is HIGH, the output will be $V(1)$ i.e. HIGH as explained below.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

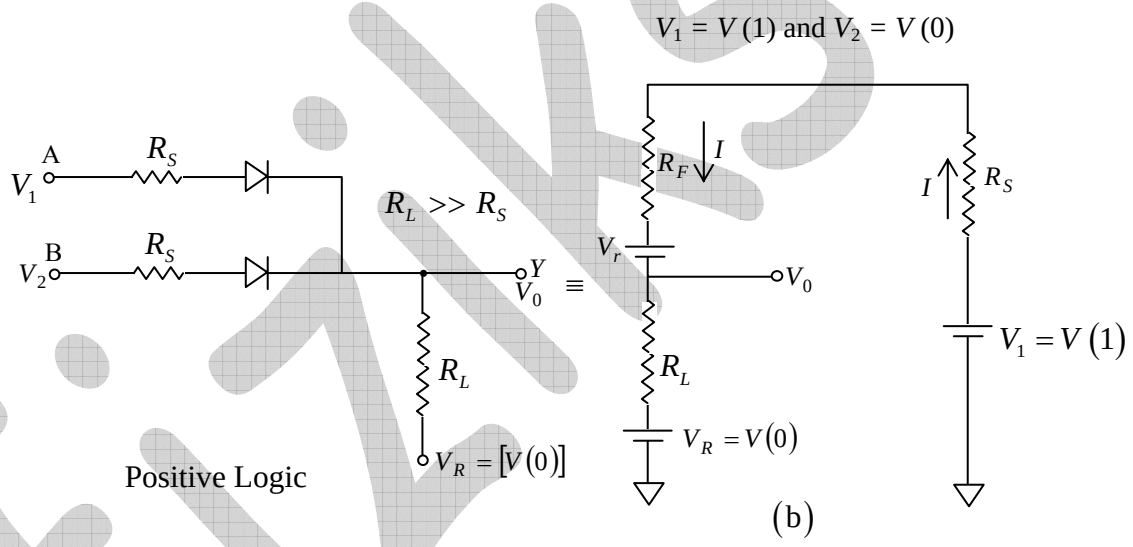


Figure 5.9: (a) Positive logic OR Gate (b) Equivalent circuit for analysis.

Apply K.V.L.

$$-V(1)+V_\gamma+V(0)+I(R_L+R_F+R_S)=0 \Rightarrow I=\frac{V(1)-V(0)-V_r}{R_L+R_F+R_S}\approx\frac{V(1)-V(0)}{R_L}$$

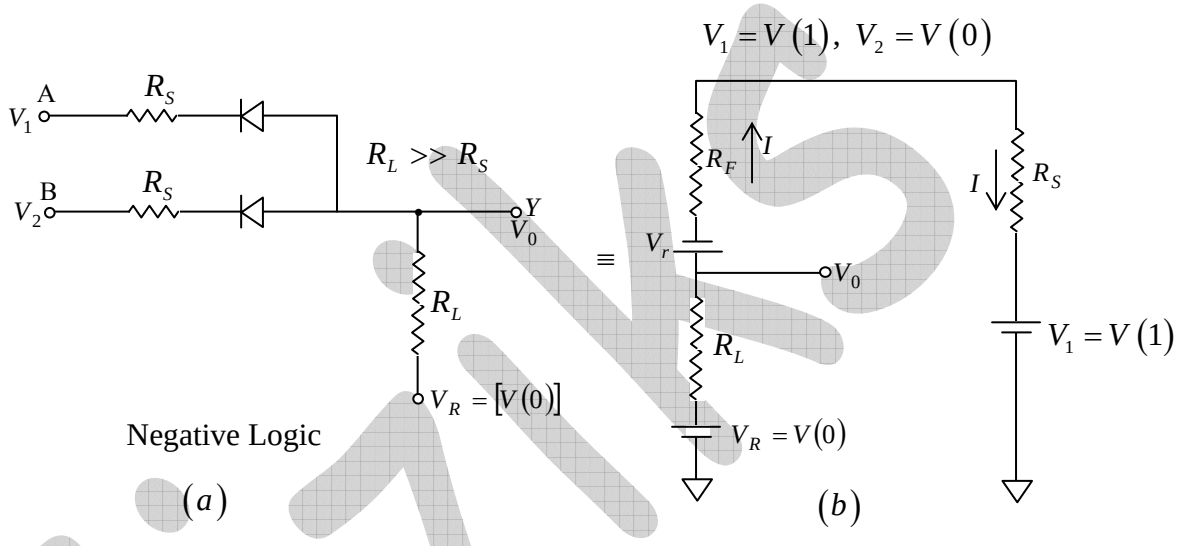
$\because R_L >> R_F + R_S$ and V_γ is very small.

$$V_0 = IR_L + V(0) = [V(1)-V(0)]+V(0) \Rightarrow V_0 = V(1).$$

Similarly when any of the input is 1 output is $V(1)$. When both inputs are 0 output is $V(0)$.

OR Gate (Negative logic)

The truth table can be verified for the circuit shown in figure 5.10. In negative logic $V(1)$ means LOW voltage and $V(0)$ means HIGH voltage. When both inputs have HIGH voltage $[V(0)]$ output will be $V(0)$. When any of the input is LOW $[V(1)]$, the output will be $V(1)$ i.e. LOW as explained below.



Apply K.V.L.

$$-V(0)+V_r+V(1)+I(R_L+R_F+R_S)=0 \Rightarrow I=\frac{V(0)-V(1)-V_r}{R_L+R_F+R_S}\approx\frac{V(0)-V(1)}{R_L}$$

$\because R_L >> R_F + R_S$ and V_γ is very small.

$$V_0 = V(0) - IR_L = V(0) - [V(0) - V(1)] \Rightarrow V_0 = V(1)$$

5.1.7 AND Gate (Positive Logic and Negative Logic)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

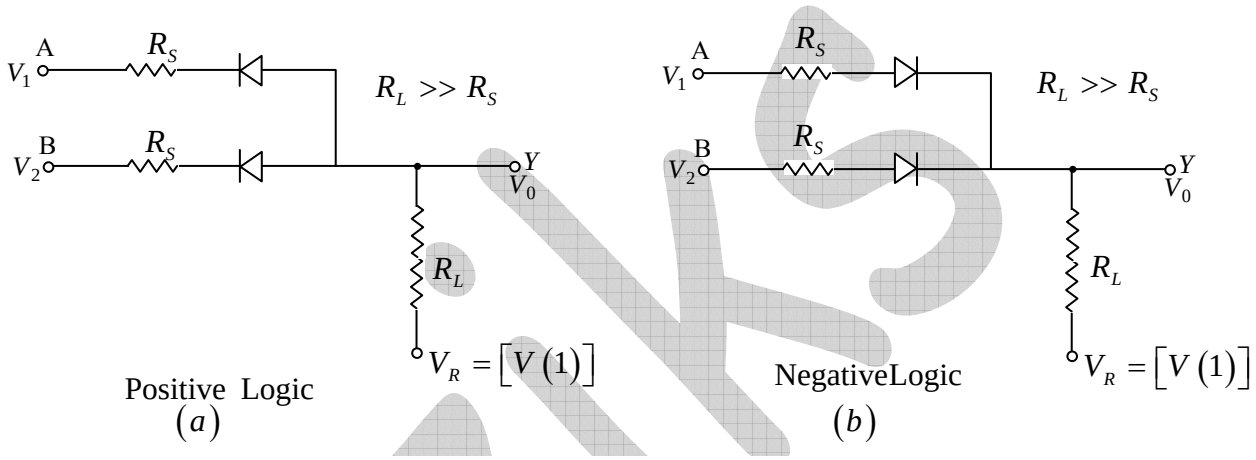


Figure 5.11: (a) Positive logic AND Gate (b) Negative logic AND Gate.

We can easily verify truth table of AND gate by drawing equivalent circuit in each case.

5.3 Logic Expressions

5.3.1 NOT

The operation of an inverter (NOT circuit) can be expressed with symbols as follows: If the input variable is called A and the output variable is called X then $X = \overline{A}$. This expression states that the output is the complement of the input, so that if $A = 0$, then $X = 1$ and if $A = 1$, then $X = 0$.

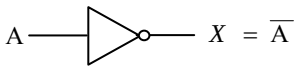


Figure 5.12: The inverter complements an input variable.

5.3.2 AND

The operation of a two-point AND gate can be expressed in equation form as follows: If one input variable is A, the other input variable is B, and the output variable is X, then the Boolean expression for this basic gate function is $X = AB$. Figure 5.13 shows the gate with the input and output variables indicated.

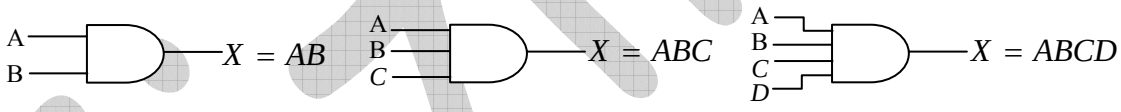
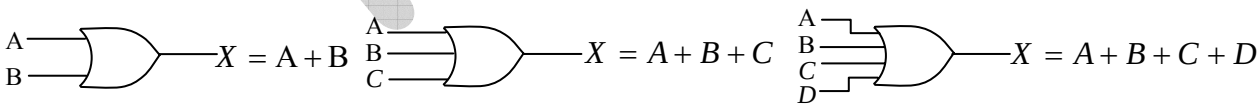


Figure 5.13: Boolean expression for AND functions.

5.3.3 OR

The operation of a two-input OR gate can be expressed in equation form as follows: If one input is A, the other input is B, and the output is X, then the Boolean expression is $X = A + B$. Figure 5.14 shows the gate logic symbol, with input and output variables labeled.



(a) (b) (c)

Figure 5.14: Boolean expressions for OR functions.

5.3.4 NAND

The Boolean expression for a two-input NAND gate is $X = \overline{AB}$. This expression says that the two input variables, A and B, are first ANDed and then complemented, as indicated by the bar over the AND expression.

5.3.5 NOR

Finally, the expression for a two input NOR gate can be written as $X = \overline{A + B}$. This equation says that the two input variables are first ORed and then complemented, as indicated by the bar over the OR expression.

5.4 Rules for Boolean algebra

Basic rules that are useful in manipulating and simplifying Boolean algebra expressions are given below:

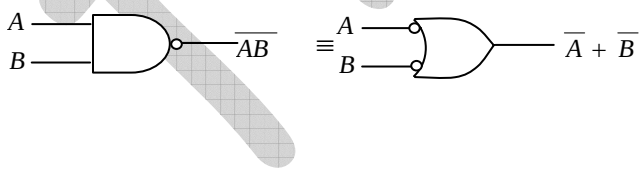
1. $A + 0 = A$	4. $A.1 = A$	7. $A.A = A$	10. $A + AB = A$
2. $A + 1 = 1$	5. $A + A = A$	8. $A.\overline{A} = 0$	11. $A + \overline{A}B = A + B$
3. $A.0 = 0$	6. $A + \overline{A} = 1$	9. $\overline{\overline{A}} = A$	12. $(A + B).(A + C) = A + BC$

5.4.1 Demorgan's Theorems

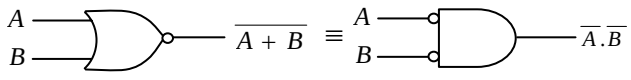
$$\overline{A.B} = \overline{A} + \overline{B}$$

$$\overline{A + B} = \overline{A}.\overline{B}$$

These theorems are illustrated by the gate equivalencies and truth tables in figure 5.15.



A	B	$\overline{AB}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



A	B	$\overline{A + B}$	$\overline{A}.\overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Figure 5.15: Gate equivalencies and corresponding truth takes illustrating DeMorgan's theorems.

Applying DeMorgan's Theorems

The following procedure illustrates the application of DeMorgan's theorems using a specific expression: $X = \overline{\overline{A + B\overline{C}} + D\left(\overline{E + \overline{F}}\right)}$

Step1. Break the bar over the entire expression and change the sign $(+ \text{ to } .)$ between the terms, $\overline{A + B\overline{C}}$ and $D\left(\overline{E + \overline{F}}\right)$: $X = \overline{\overline{A + B\overline{C}}}.\left[\overline{D\left(\overline{E + \overline{F}}\right)}\right]$

Step2. Cancel the double bars over the left term: $X = \left(A + B\overline{C}\right).\left[\overline{D\left(\overline{E + \overline{F}}\right)}\right]$

Step3. Break the bar over the term $D\left(\overline{E + \overline{F}}\right)$ and change the sign $(. \text{ to } +)$ between D and $\left(\overline{E + \overline{F}}\right)$: $X = (A + B\overline{C}).\left[\overline{D} + \left(\overline{\overline{E + \overline{F}}}\right)\right]$

Step4. Cancel the double bars over the term $\left(E + \overline{F}\right)$: $X = (A + B\overline{C}).\left[\overline{D} + E + \overline{F}\right]$

Example: Apply DeMorgan's theorems to each expression.

(a) $\overline{\left(\overline{A + B}\right) + \overline{C}}$ (b) $\overline{\left(\overline{\overline{A} + B}\right) + \overline{CD}}$ (c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

Solution:

(a) $\overline{\left(\overline{A + B}\right) + \overline{C}} = \overline{(A + B)} \cdot \overline{\overline{C}} = (A + B)C$

(b) $\overline{\left(\overline{\overline{A} + B}\right) + \overline{CD}} = \overline{\left(\overline{\overline{A} + B}\right)} \cdot \overline{\overline{CD}} = \left(\overline{A} + B\right)CD$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}} = \left[\overline{(A + B)\overline{C}\,\overline{D}}\right]\left(\overline{E + \overline{F}}\right) = \left(\overline{A}\,\overline{B} + C + D\right)\overline{E}F$

5.5 Boolean Expressions for Gate Networks

The form of a given Boolean expression indicates the type of gate network it describes. For example, let us take the expression $A(B+CD)$ and determine what kind of logic circuit it represents. First, there are four variables: A, B, C and D. C is ANDed with D, given CD; then CD is ORed with B, giving $(B+CD)$. Then this is ANDed with A to produce the final function. Figure 5.16 illustrates the gate network represented by this particular Boolean expression, $A(B+CD)$.

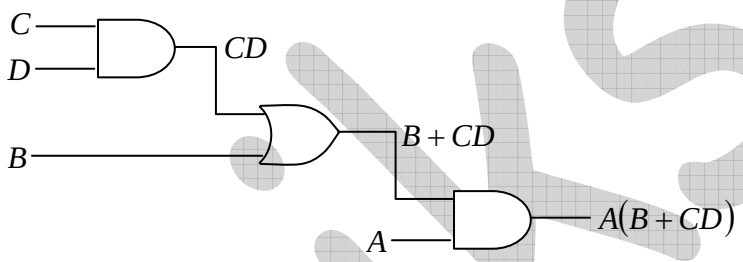


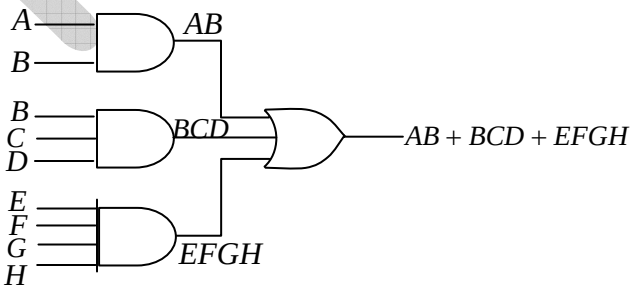
Figure 5.16: Logic gate implementation of the expression $A(B+CD)$.

There are also certain forms of Boolean expressions that are more commonly used than others; the two most important of these are the *sum-of-products* and the *product-of-sum* forms.

5.5.1 Sum-of-Product Form

Example: Implement the expression $AB+BCD+EFGH$ with logic gates.

Solution:



An important characteristic of the sum-of-products form is that the corresponding implementation is always a two-level gate network; that is, the maximum number of gates through which a signal must pass in going from an input to the output is two, excluding inversions.

5.5.2 Product-of-Sums Form

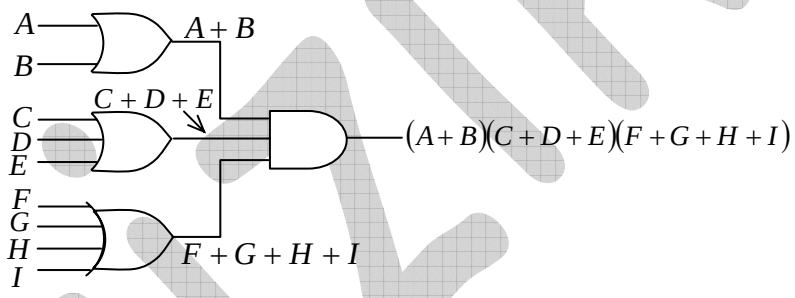
The product-of-sums form can be thought of as the dual of the sum-of-products. It is, in terms of logic functions, the AND of two or more OR functions. For instance, $(A+B)(B+C)$ is a product-of-sums expression. Several other examples are

$$(A+B)(B+C+D)$$
$$(A+B+C)(D+E+F)$$
$$(A+B+C)(D+E+F+G)(A+E+G)$$
$$(A+B+\bar{C})(\bar{D}+\bar{E}+F)(F+G+H)(\bar{A}+\bar{F}+G)$$

Example: Construct the following function with logic gate.

$$(A+B)(C+D+E)(F+G+H+I)$$

Solution:



5.6 Simplification of Boolean Expressions

5.6.1 Boolean Algebra Techniques

Example: Simplify the expression $AB + A(B + C) + B(B + C)$ using Boolean algebra techniques.

Solution: The following is not necessarily the only approach.

Step1. Apply the distributive law to the second and third terms in the expression, as follows:

$$AB + AB + AC + BB + BC$$

Step2. Apply rule $7(B.B = B)$: $AB + AB + AC + B + BC$

Step3. Apply rule $5(AB + AB = AB)$: $AB + AC + B + BC$

Step4. Factor B out of the last two terms: $AB + AC + B(1 + C)$

Step5. Apply rule $2(1 + C = 1)$: $AB + AC + B.1$

Step6. Apply rule $4(B.1 = B)$: $AB + AC + B$

Step7. Factor B out of the first and third terms, as follows: $B(A + 1) + AC$

Step8. Apply rule $2(A + 1 = 1)$: $B.1 + AC$

Step9. Apply rule $4(B.1 = B)$: $B + AC$

5.6.2 The Karnaugh Map

The Karnaugh map provides a systematic method for simplifying a Boolean expression and, if properly used, will produce the simplest sum-of-products expression possible.

The map format: The Karnaugh map is composed of an arrangement of adjacent "cells", each representing one particular combination of variables in product form. Since the total number of combinations of n variables and their complements is 2^n, the Karnaugh map consists of 2^n cells. For example, there are four combinations of the products of two variables $(A$ and $B)$ and their complements $\bar{A}\bar{B}$, $\bar{A}B$, $A\bar{B}$ and AB. Therefore, the Karnaugh map must have four cells, with each cell representing one of the variable combinations, as illustrated in figure 5.17(a). The variable combinations are labeled in the cells only for purposes of illustration. In practice, the map is actually arranged with the variables labeled outside the cells, as shown in figure 5.17(b). The variable above a column of cells applies to each cell in that column.

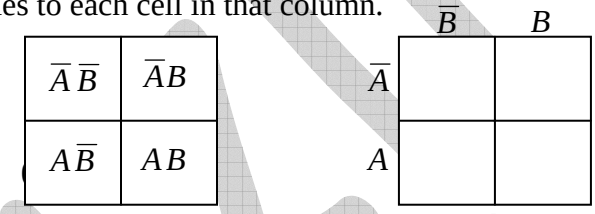


Figure 5.17: Format of two-variable Karnaugh map.

Extensions of the Karnaugh map to three and four variables are shown in figure 5.18. Notice that the cells are arranged such that there is only a single variable change between any adjacent cells (this is the characteristic that determines adjacency).

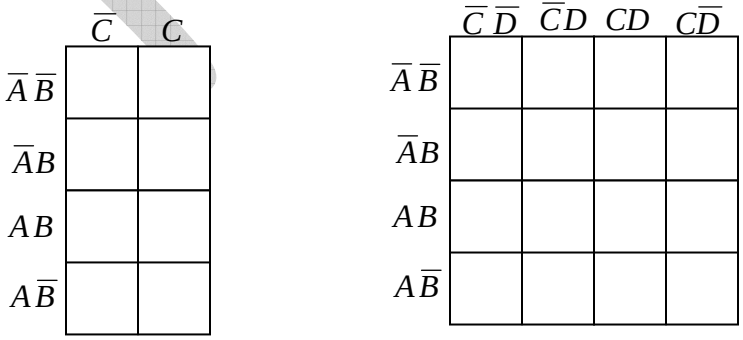


Figure 5.18: Formats for three and four-variable Karnaugh maps.

Plotting a Boolean Expression

Once a Boolean expression is in the sum-of-products form, you can "plot" it on the Karnaugh map by placing 1 in each cell corresponding to a term in the sum-of-products expression. For example, the three-variable expression $\bar{A}BC + AB\bar{C} + ABC$ and the four-variable expression $\bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + A\bar{B}CD$ are plotted on the map as shown in figure 5.19.

	$\bar{C}$	C
$\bar{A}\,\bar{B}$		
$\bar{A}B$		1
AB	1	1
$A\bar{B}$		

	$\bar{C}\,\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\,\bar{B}$				
$\bar{A}B$		1	1	
AB				
$A\bar{B}$		1	1	

Figure 5.19: Examples of plotting a Boolean expression on a Karnaugh map.

Grouping Cells for Simplification

You can group 1s that are in adjacent cells according to the following rules by drawing a loop around those cells:

1. Adjacent cells are cells that differ by only a single variable (for example $ABCD$ and $ABC\bar{D}$ are adjacent).
2. The 1s in adjacent cells must be combined in groups of 1, 2, 4, 8, 16, and so on.
3. Each group of 1s should be maximized to include the largest number of adjacent cells as possible in accordance with rule 2.
4. Every 1 on the map must be included in at least one group. There can be overlapping groups if they include noncommon 1s.

Grouping is illustrated by an example in figure 5.20. Notice how the groups are overlapped to include all the 1s in the largest possible group.

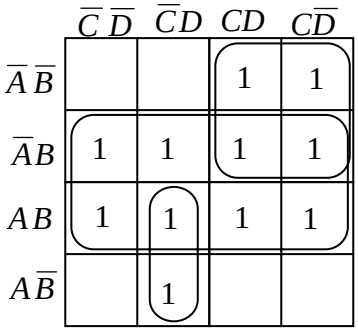


Figure 5.20: Example of grouping1s on a four-variable Karnaugh map.

Simplifying the Expression

When all the 1s representing each term in the original Boolean expression are grouped, the mapped expression is ready for simplification. The following rules apply:

1. Each group of 1s creates a product term composed of all variables that appear in only one form (uncomplemented or complemented) within the group. Variables that appear both uncomplemented and complemented are eliminated.
2. The final simplified expression is formed by summing the product terms of all the groups.

For example, in figure 5.21, the product term for the eight-cell group is B because the cells within that group contain both A and $\overline{A}$, C and $\overline{C}$ and D and $\overline{D}$, so these variables are eliminated. The four-cell group contains B, $\overline{B}$, D and $\overline{D}$, leaving the product term $\overline{A}C$. The two-cell group contains B and $\overline{B}$, leaving $A\overline{C}D$ as the product term. The resulting Boolean expression is the sum of these product terms: $B + \overline{A}C + A\overline{C}D$.

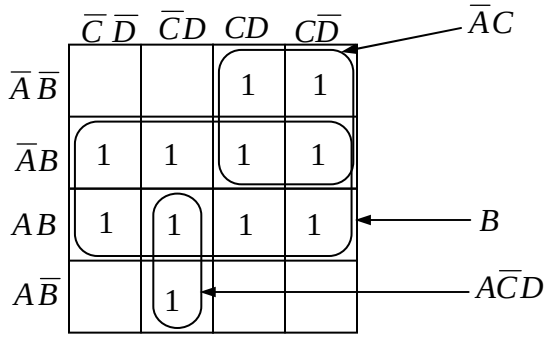
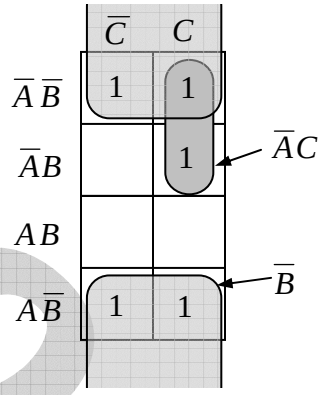


Figure 5.21: This plotted Boolean expression simplifies to $B + \overline{A}C + A\overline{C}D$.

Example: Minimize the expression $X = A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$

Solution: Notice that this expression is already in a sum-of-products form from which the 1s can be plotted very easily, as shown in figure. Four of the 1s appearing in adjacent cells can be grouped. The remaining 1 is absorbed in an overlapping group. The group of four 1s produces a single variable term $\bar{B}$. This is determined by observing that within the group $\bar{B}$ is the only variable that does not change from cell to cell. The group of two 1s produces a two-variable term, $\bar{A}C$. This is determined by observing that within the group, the variables $\bar{A}$ and C do not change from one cell to the next. To get the minimized function, the two terms that are produced are summed (ORed) as $X = \bar{B} + \bar{A}C$.

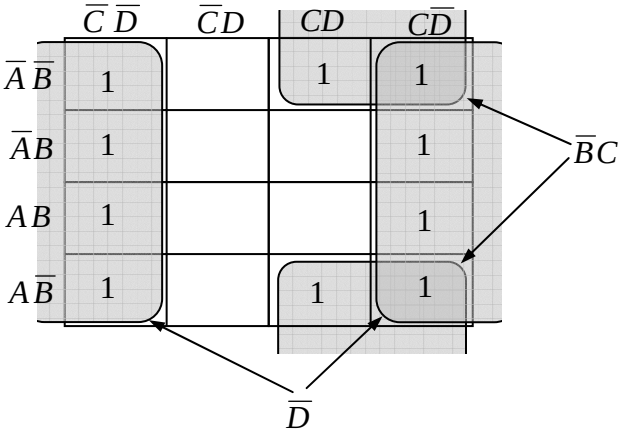


Example: Reduce the following four-variable function to its minimum sum-of-products form:

$$X = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + A\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D}$$

If all variables and their complements were available, this function would take ten-4-input AND gates and one 10-input OR gate to implement.

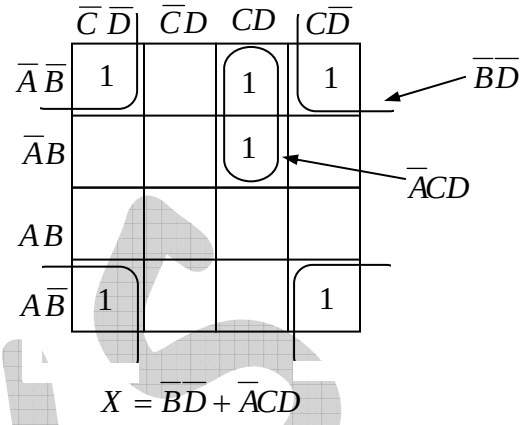
Solution: A group of eight 1s can be factored as shown in figure because the 1s in the outer columns are adjacent. A group of four 1s is formed by the "wrap-around" adjacency of the cells to pick up the remaining two 1s. The minimum form of the original equation is $X = \bar{D} + \bar{B}C$. Thus $X = \bar{D} + \bar{B}C$ takes one 2-input AND gate and one 2-input OR gate. Compare this to the implementation of the original function.



Example: Reduce the following function to its minimum sum-of-products form:

$$X = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}CD + A\bar{B}C\bar{D}$$

Solution: The function is plotted on the four-variable map and factored as indicated in figure. Notice that the four corner cells are adjacent.



Implementing Truth Table Functions

Another use of the Karnaugh map is in implementing logic functions that are specified in truth table format. Since truth tables normally use 1s and 0s (HIGH and LOWs) to represent logic states, the format of the Karnaugh map is modified slightly for this application, as shown in figure 5.22 for two, three and four variables; 1s and 0s are used to label the cells. The variables represented by the 1s and 0s are specified in the upper left corner of the map.

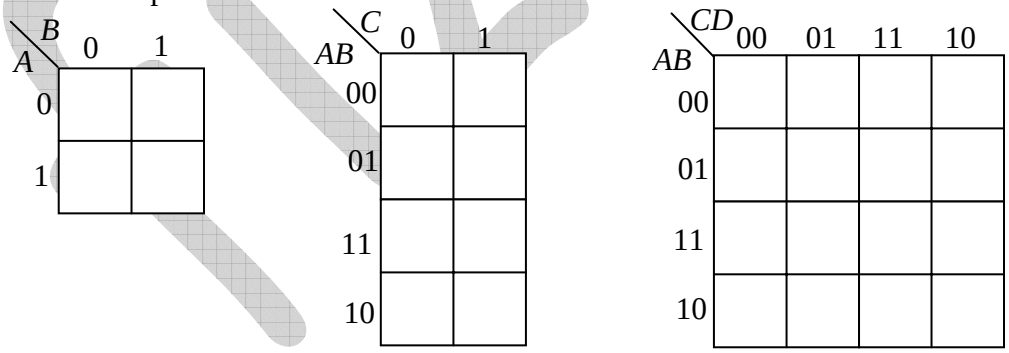


Figure 5.22: Karnaugh maps with a 1/0 labeling format.

Plotting the truth table

When the output variable on the truth table is a 1, a 1 is placed on the Karnaugh map in the cell corresponding to the states of the input variables. For example, if the output variable is a 1 when $A = 1$, $B = 0$, and $C = 1$, then a 1 is placed in the lower right cell of the three-variable map.

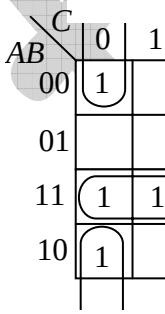
Grouping and simplification

The previously stated rules apply in grouping the 1s. Variables that are both 1 and 0 within a group are eliminated. If a variable is a 1 in all cells of a group, it appears uncomplemented in the product term. If a variable is a 0 in all cells of a group, it appears complemented in the product term.

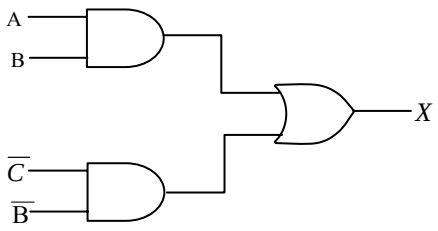
Example: Implement the logic function specified by the truth table using the Karnaugh map method. *X* is the output variable, and *A*, *B* and *C* are the input variables.

Solution: The truth table function is plotted and simplified on the map. The logic gate implementation for the resulting sum-of-products expression is shown in part (b) of the figure.

Input			Output
A	*B*	*C*	*X*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



(a) $X = AB + \overline{B}\,\overline{C}$

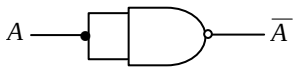


(b) Gate implementation of $X = AB + \overline{B}\,\overline{C}$

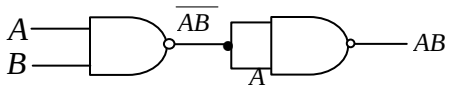
5.6 Universal Gates

5.6.1 The Universal Property of the NAND Gate

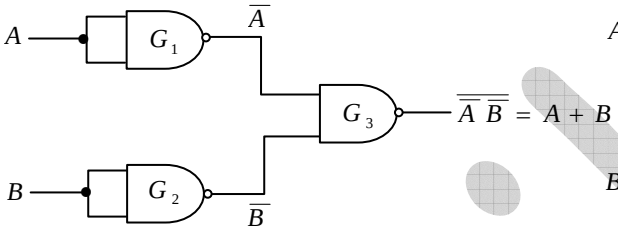
The NAND gate can be used to generate the NOT function, the AND function, the OR function, and the NOR function.



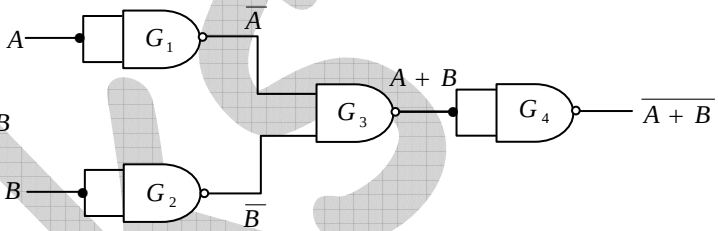
(a) A NAND gate used as an inverter



(b) Two NAND gates used as an AND gate



(c) Three NAND gate used as an OR gate

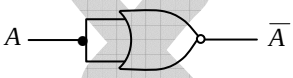


(d) Four NAND gate used as a NOR gate

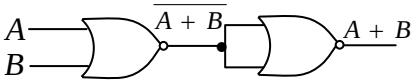
Figure 5.23: Universal application of NAND gates.

5.6.2 The Universal Property of the NOR Gate

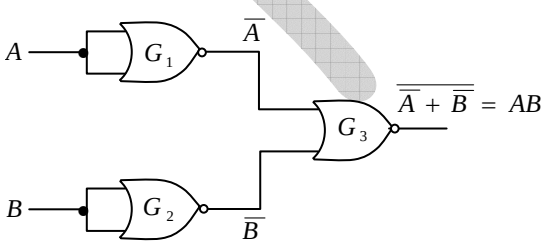
As with the NAND gate, the NOR gate can be used to generate the NOT, AND, OR, and NAND functions.



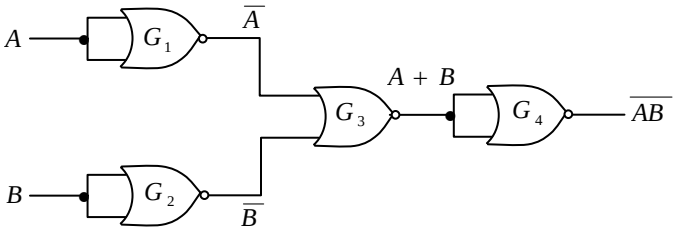
(a) A NOR gate used as an inverter



(b) Two NOR gates used as an OR gate



(c) Three NOR gates used as an AND gate

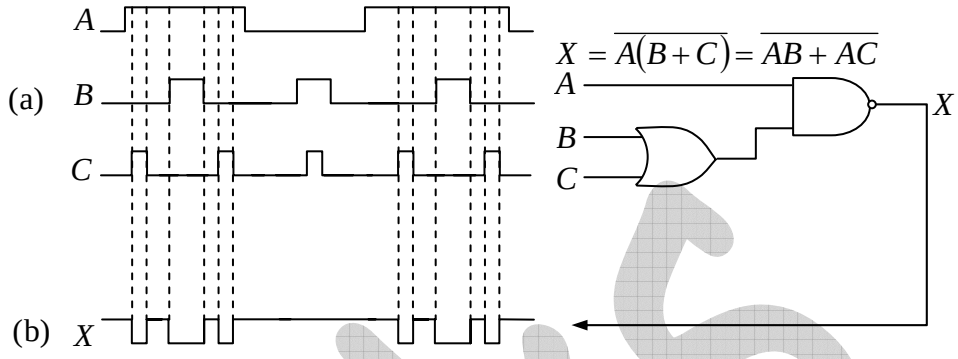


(d) Four NOR gates used as a NAND gate

Figure 5.24: Universal application of NOR gates.

Pulsed Operation

Example: Determine the output waveform for the circuit in figure (a), with the inputs as shown.



Solution: X is shown in the proper time relationship to the inputs in figure (b).

Example: Determine the output waveform for the circuit in figure (a) if the input waveforms are as indicated.

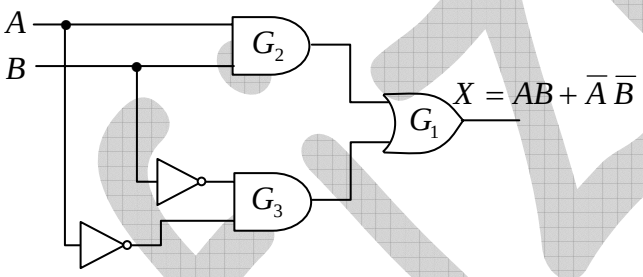


Figure (a)

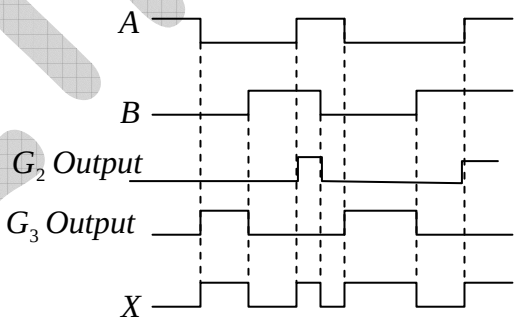


Figure (b)

Solution: When both inputs are HIGH or when both inputs are LOW, the output is HIGH. This is called a coincidence circuit or exclusive-NOR. The intermediate outputs of gates G_2 and G_3 are used to develop the final output.

Multiple Choice Questions (MCQ)

Q1. A number system's conversion equation is given below having p integer digits and q fractional digits:

$$D = C_{p-1}B^{p-1} + C_{p-2}B^{p-2} + ... + C_0B^0 + C_{-1}B^{-1} + C_{-2}B^{-2} + ... + C_{-q}B^{-q}$$

Which of the following conversions would result by the equation given above?

(a) Binary to decimal only

(b) Hexadecimal to decimal only

(c) Octal to decimal only

(d) Binary to decimal and hexadecimal to decimal

Q2. Find the $2^{'s}$ complement of binary number $(10010)_2$ is

(a) $(01101)_2$ (b) $(01110)_2$ (c) $(01100)_2$ (d) $(11100)_2$

Q3. The decimal equivalent of the Hexadecimal number $(1C)_{16}$ is

(a) $(28)_{10}$ (b) $(18)_{10}$ (c) $(38)_{10}$ (d) $(48)_{10}$

Q4. Hexadecimal equivalent of binary number $(1100101001010111)_2$ is

(a) $CB57$ (b) $CB67$ (c) $CA57$ (d) $DA57$

Q5. Simplified form of the Boolean expression $\overline{AB} + \overline{A} + AB$ is

(a) $\overline{A}$ (b) $\overline{B}$ (c) 1 (d) $\overline{A} + \overline{B}$

Q6. Simplified form of Boolean expression $AB + A(\overline{B} + C) + \overline{B}(B + C)$ is

(a) $A + \overline{B}C$ (b) $C + \overline{A}B$

(c) $B + \overline{A}C$ (d) $B + AC$

Q7. The simplified form of a logic function $Y = A\left(B + C\left(\overline{AB + AC}\right)\right)$ is

(a) $\overline{A}\overline{B}$ (b) AB (c) $\overline{A}B$ (d) $A\overline{B}$

Q8. Simplified form of Boolean Expression $AB + A(B + C) + B(B + C)$ is

(a) $A + BC$ (b) $B + AC$ (c) $C + AB$ (d) AC

Q9. Simplified form of Boolean Expression $BC + \overline{B} + \overline{B}(B + A)$ is

(a) $A + BC$ (b) $B + AC$ (c) $C + AB$ (d) $\overline{B} + C$

Q10. Simplify the Boolean function

$$Y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + A\overline{B}\overline{C}\overline{D} + \overline{A}BCD + \overline{A}\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}BC\overline{D}$$

(a) $\overline{B}\overline{D}$ (b) $\overline{B}\overline{D} + \overline{A}C$

(c) $BD + \overline{A}CD$ (d) $\overline{B}\overline{D} + \overline{A}CD$

Q11. Simplify the Boolean function

$$Y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + A\overline{B}\overline{C}\overline{D} + \overline{A}CD + A\overline{B}C\overline{D} + \overline{A}BC\overline{D}$$

(a) $\overline{B}\overline{D}$ (b) $\overline{B}\overline{D} + \overline{A}C$

(c) $BD + \overline{A}CD$ (d) $\overline{B}\overline{D} + \overline{A}CD$

Q12.

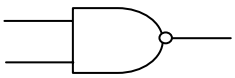


Figure (i)

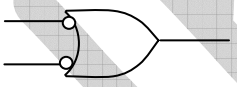
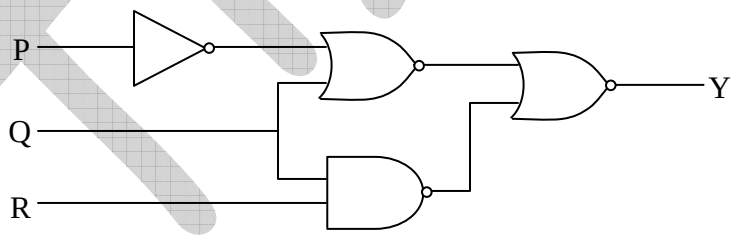


Figure (ii)

Figure (i) and (ii) represent respectively,

(a) NOR, NOR (b) NOR, NAND (c) NAND, NAND (d) OR, NAND

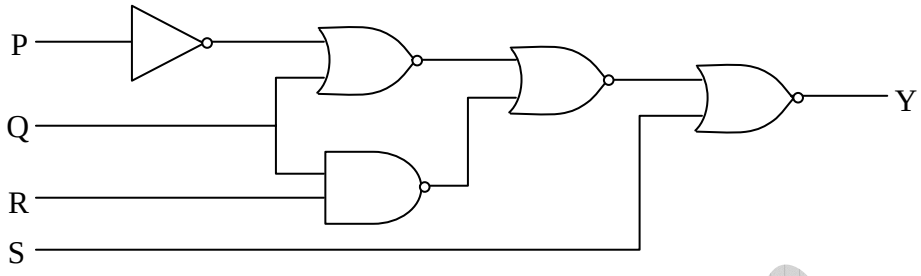
Q13. The logic expression for the output Y of the following circuit is



(a) $\overline{\overline{\overline{P} + Q} + QR}$ (b) $\overline{\overline{P} + Q} + \overline{QR}$

(c) $\overline{\overline{P} + Q + \overline{QR}}$ (d) $\overline{\overline{\overline{P} + Q} + \overline{QR}}$

Q14. The logic expression for the output Y of the following circuit is



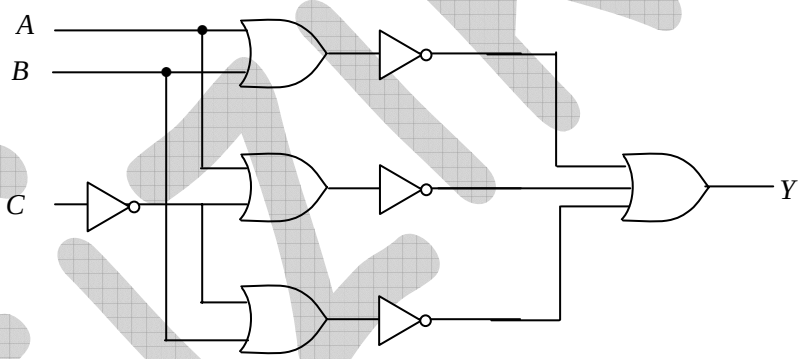
(a) $\overline{\overline{\overline{P}+Q}+\overline{QR}+S}$

(b) $\overline{\overline{\overline{P}+Q}+\overline{QR}+S}$

(c) $\overline{\overline{P}+Q+\overline{QR}+S}$

(d) $\overline{\overline{P}+Q+\overline{QR}+\overline{S}}$

Q15. Which one of the following gives the output *Y* for the logic diagram shown in the figure?



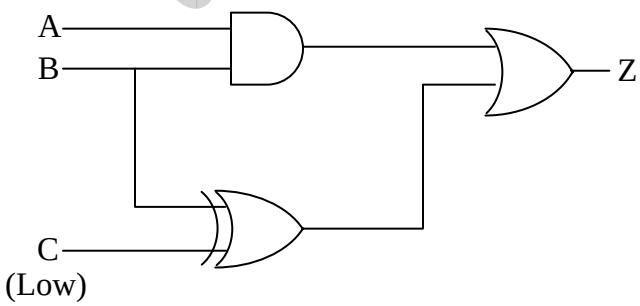
(a) $(A+B+\overline{C})$

(b) $\overline{(A+B)}+\overline{(B+C)}+\overline{(C+A)}$

(c) $\overline{AB}+\overline{A\overline{C}}+\overline{B\overline{C}}$

(d) $\overline{A}\overline{B}+\overline{A}C+\overline{B}C$

Q16. Consider the digital circuit shown below in which the input C is always low (0).



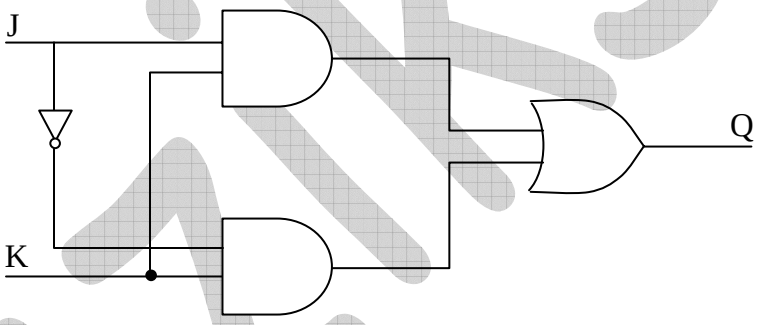
The truth table for the circuit can be written as

A	B	Z
0	0	
0	1	
1	0	
1	1	

The entries in the Z column (vertically) are

(a) 1010 (b) 0101 (c) 1111 (d) 1011

Q17. The truth table for the given circuit is



(a)

J	K	Q
0	0	1
0	1	0
1	0	1
1	1	0

(b)

J	K	Q
0	0	1
0	1	0
1	0	0
1	1	1

(c)

J	K	Q
0	0	0
0	1	1
1	0	0
1	1	1

(d)

J	K	Q
0	0	0
0	1	1
1	0	1
1	1	0

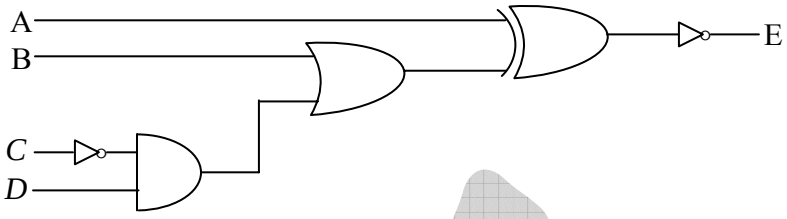
Q18. The truth values of the function E realized by the logic circuit shown below for $(i)\ A=1,\ B=0,\ C=1,\ D=0$ and $(ii)\ A=0,\ B=1,\ C=0,\ D=1$

(a) $(i)\ 1 \quad (ii)\ 0$

(b) $(i)\ 0 \quad (ii)\ 1$

(c) $(i)\ 0 \quad (ii)\ 0$

(d) $(i)\ 1 \quad (ii)\ 1$



Q19. Which one of the following gives the simplified Boolean expression for Y of the 3–variable given truth table?

Input			Output
A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

(a) $\overline{A}\overline{B}+AC$

(b) $A\overline{B}+\overline{A}\overline{C}$

(c) $\overline{A}B+A\overline{C}$

(d) $AB+\overline{A}\overline{C}$

Q20. A half-adder is a digital circuit with two inputs (P,Q) and two outputs (Sum, Carry). Then Sum (S) and carry (C) are given by

(a) $S=\overline{P}\overline{Q}+PQ, C=\overline{PQ}$ (b) $S=\overline{P}Q+P\overline{Q}, C=\overline{PQ}$

(c) $S=\overline{P}Q+P\overline{Q}, C=PQ$ (d) $S=\overline{P}\overline{Q}+PQ, C=PQ$

Numerical Answer Type Questions (NAT)

Q21. Decimal equivalent of the binary number $(1101101)_2$ is……

Q22. Decimal equivalent of the binary number $(11101.011)_2$ is…….

Q23. Decimal equivalent of the binary number $(0.111)_2$ is…….

Q24. Decimal equivalent of the octal number $(2374)_8$ is…….

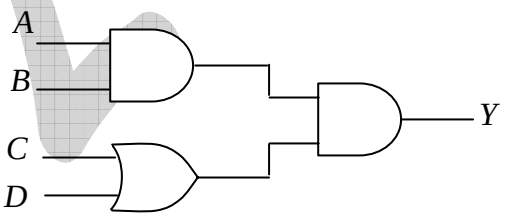
Q25. Octal equivalent of the decimal number $(359)_{10}$ is…….

Multiple Select Questions (MSQ)

Q26. The $(100110)_2$ is numerically equivalent to

(a) $(26)_{16}$ (b) $(36)_{10}$ (c) $(46)_8$ (d) $(38)_{10}$

Q27. If the output (Y) of the logic circuit shown in the figure is 0, the input could be



(a) $A=1,\ B=1,\ C=1,\ D=0$

(b) $A=1,\ B=1,\ C=0,\ D=0$

(c) $A=1,\ B=0,\ C=1,\ D=1$

(d) $A=0,\ B=1,\ C=1,\ D=1$

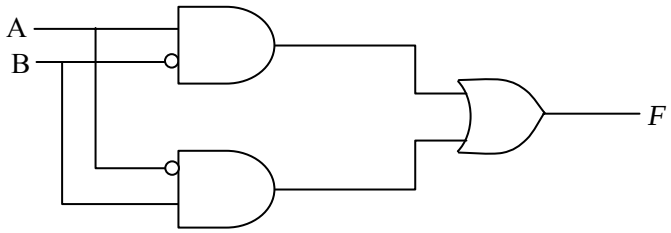
Q28. Which of the following implements same logic output?

(a) $\overline{A}\overline{C}+AB\overline{C}+ABC$ (b) $\overline{A}\overline{B}\overline{C}+\overline{A}B\overline{C}+AB$

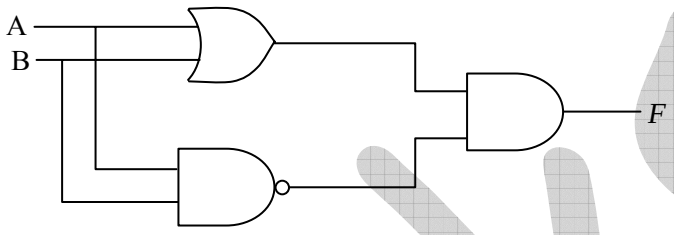
(c) $\overline{A}B+A\overline{C}$ (d) $AB+\overline{A}\overline{C}$

Q29. Which of the following circuit satisfy the Boolean expression $A\overline{B} + \overline{A}B = F$

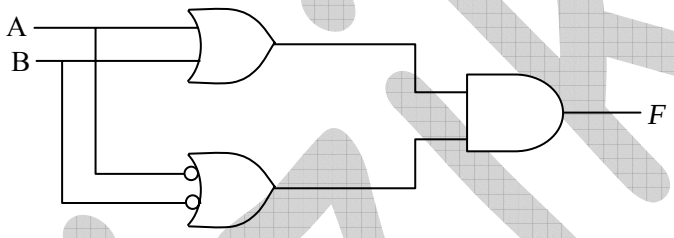
(a)



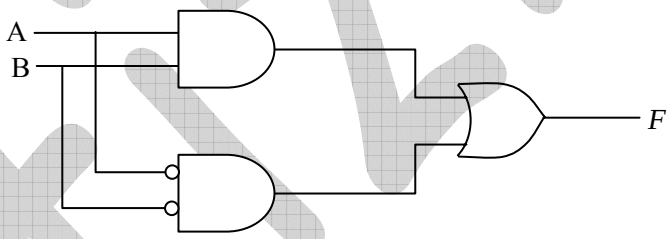
(b)



(c)



(d)



Solution

MCQ

Ans.1: (d)

Ans.2: (b)

Ans.3: (a)

Ans.4: (c)

Ans.5: (c)

$$\overline{AB}+\overline{A}+AB=\overline{AB}+\overline{A}+\overline{\overline{A}}B=\overline{AB}+\overline{A}+B=\overline{A}+\overline{B}+B=\overline{A}+1=1$$

Ans.6: (a)

$$AB+A\left(\overline{B}+C\right)+\overline{B}\left(B+C\right)=AB+A\left(\overline{B}+C\right)+\overline{B}C=A\left(B+\overline{B}+C\right)+\overline{B}C=A+\overline{B}C$$

Ans.7: (b)

$$Y=A\left(B+C\left(\overline{AB+AC}\right)\right)=A\left(B+C\left(\overline{A\left(B+C\right)}\right)\right)=A\left(B+C\left(\overline{A}+\overline{B}\overline{C}\right)\right)=AB$$

Ans.8: (b)

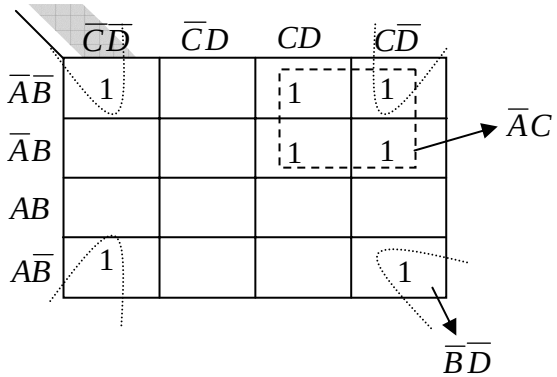
$$AB+A\left(B+C\right)+B\left(B+C\right)=AB+AC+B+BC=AB+AC+B=B+AC$$

Ans.9: (d)

$$BC+\overline{B}+\overline{B}\left(B+A\right)=BC+\overline{B}+A\overline{B}=BC+\overline{B}=\overline{B}+\overline{\overline{B}}C=\overline{B}+C$$

Ans.10: (b)

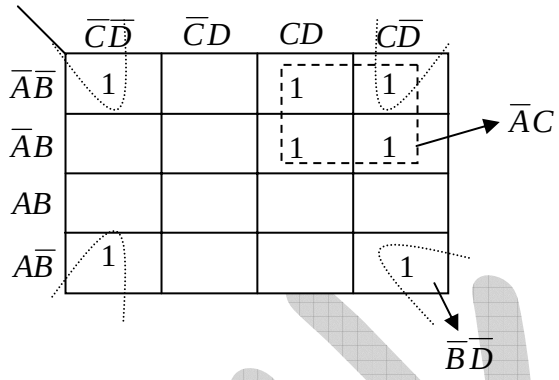
$$Y=\overline{A}\overline{B}\overline{C}\overline{D}+\overline{A}\overline{B}C\overline{D}+A\overline{B}\overline{C}\overline{D}+\overline{A}BCD+\overline{A}\overline{B}CD+A\overline{B}C\overline{D}+\overline{A}BC\overline{D}$$



Ans.11: (b)

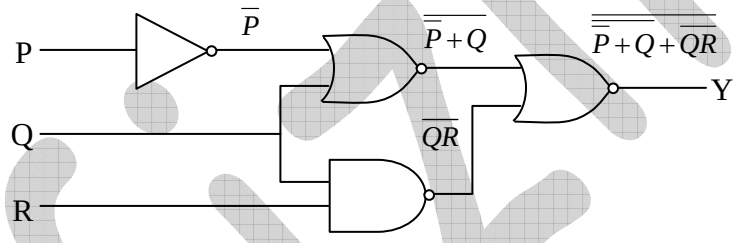
$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}CD\left(B + \bar{B}\right) + A\bar{B}C\bar{D} + \bar{A}BC\bar{D}$$

$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}BCD + \bar{A}\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}BC\bar{D}$$

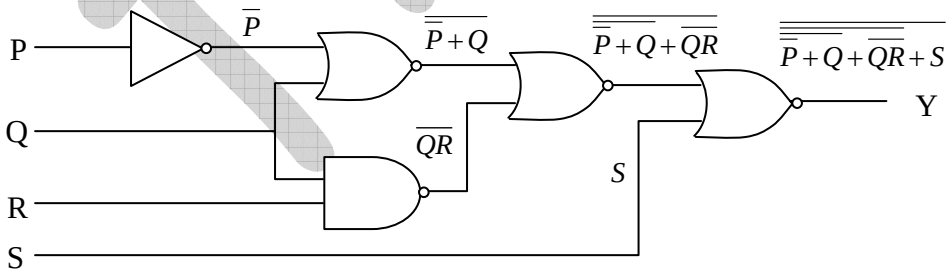


Ans.12: (c)

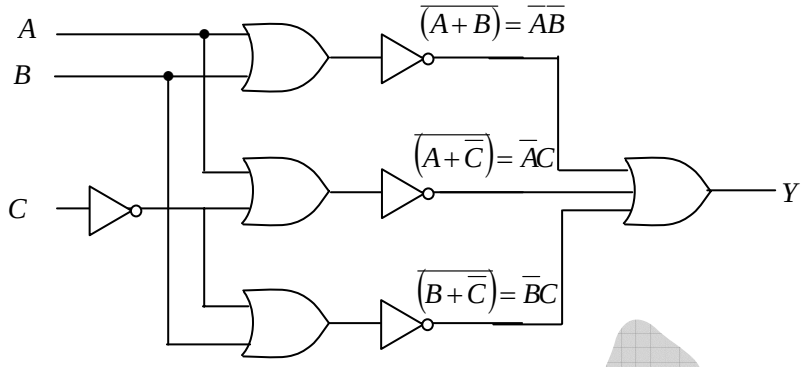
Ans.13: (d)



Ans.14: (a)



Ans.15: (d)



Ans.16: (b)

$Z = A.B + (B \oplus 0)$

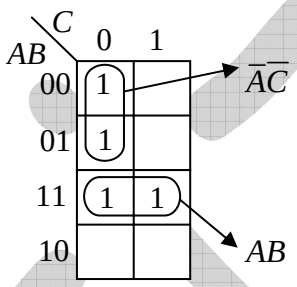
Ans.17: (c)

$Z = J.K + \overline{J}.K = K$

Ans.18: (c)

Ans.19: (d)

Solution:



$X = AB + \overline{A}\overline{C}$

Ans.20: (c)

NAT

Ans.21: 109

Binary weight:	$2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$
Weight value:	64 32 16 8 4 2 1
Binary number:	1 1 0 1 1 0 1

$1\times 64+1\times 32+0\times 16+1\times 8+1\times 4+0\times 2+1\times 1=64+32+0+8+4+0+1=109_{10}$

Ans.22: 29.375

$(16+8+4+1).(0.25+0.125)=29.375_{10}$

Ans.23: 0.875

$0.(0.5+0.25+0.125)=0.875_{10}$

Ans.24: 1276

Octal weight:	8^3	8^2	8^1	8^0
Weight value:	512	64	8	1
Octal number:	2	3	7	4

$2\times 512+3\times 64+7\times 8+4\times 1=1276_{10}$

Ans.25: 547

Step 1	**Step 2**	**Step 3**
$8\overline{)359}$	$8\overline{)44}$	$8\overline{)5}$
44	5	0
32	40	0
39	4	5
32		
7		
LSD		**MSD**

$\rightarrow (547)_8$

MSQ

Ans.26: (a), (c), (d)

Ans.27: (b), (c) and (d) $Y = A.B(C + D)$

Ans.28: (a), (b) and (d)

(a) $Y = \bar{A}\bar{C} + AB\bar{C} + ABC = \bar{A}\bar{C} + AB$

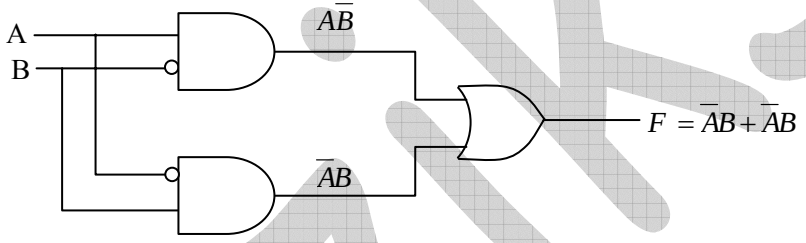
(b) $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + AB = \bar{A}\bar{C} + AB$

(c) $\bar{A}B + A\bar{C}$

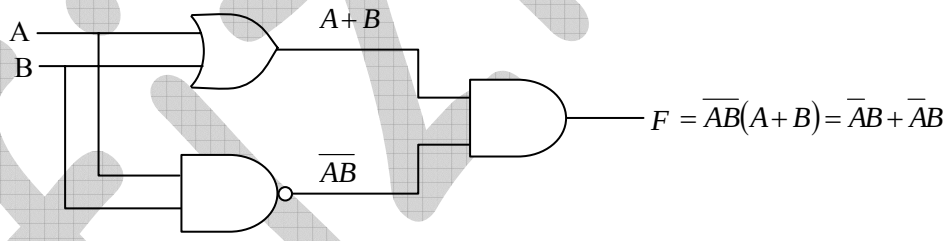
(d) $Y = AB + \overline{A\bar{C}}$

Ans.29: (a), (b) and (c)

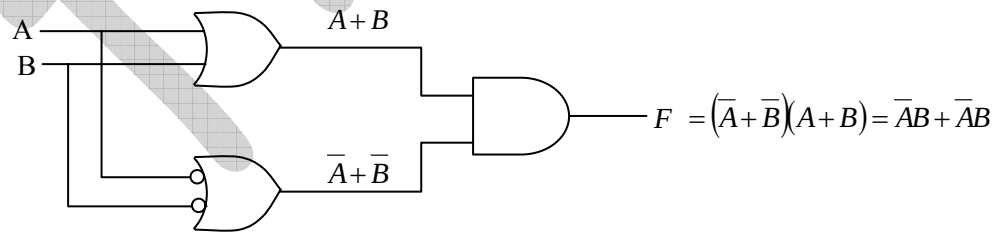
(a)



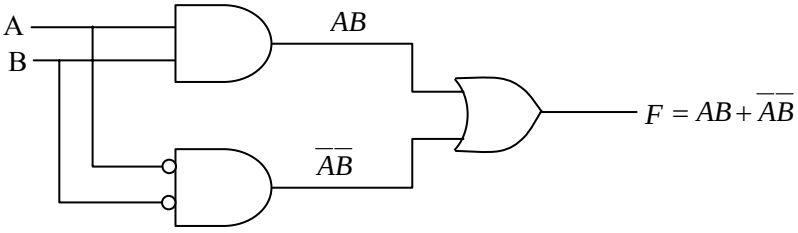
(b)



(c)



(d)



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Solid State physics

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1. Crystal Structure

1.1 Space Lattice

A space lattice is defined as "an infinite array of points in three dimensional space in which each point is identically located with respect to other". It is a periodic arrangement of points arranged in regular manner and having repeat distance in three directions which are termed as "fundamental lattice vectors". If at these lattice positions, we place atom, or group of two, three etc atoms we obtained a crystalline solid.

Let us now consider the case of a two dimensional array of points as shown in figure (i). A two-dimensional array may have (a) square (b) rectangular array.

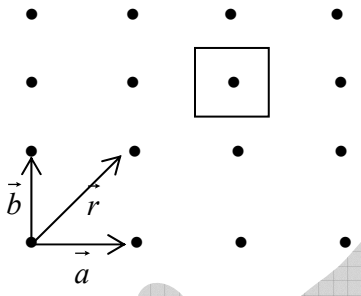


Figure (i) a = b

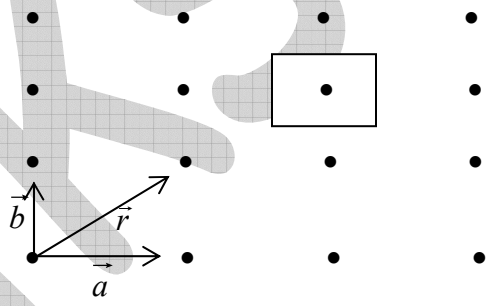


Figure (ii) a ≠ b

In figure (i) and (ii) $\vec{a}$ and $\vec{b}$ are fundamental translational vectors, and $\vec{r}$ is generated vector. If points are located such that $\vec{a} = \vec{b}$, the arrangement will be known as square lattice, if location of the point is such that a ≠ b figure (ii) the arrangement will be called rectangular lattice.

Translation and Primitive Translation Vector

A three dimensional space lattice may have

1. Cubic array when a = b = c

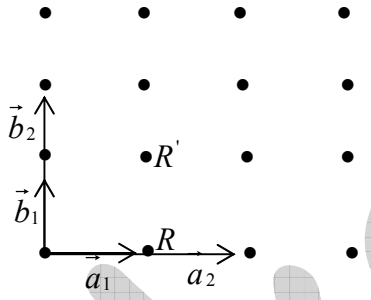
2. Non-cubic array

a lattice exhibit perfect translational symmetry and relative to an arbitrary chosen origin at a lattice pint, any other lattice point has the position vector $r_{123} = n_1\vec{a} + n_2\vec{b} + n_3\vec{c}$

where n_1, n_2, n_3, are arbitrary integers $\vec{a},\vec{b},\vec{c}$ are fundamental units of the translational symmetry.

Considering first the translational vector a_1 and b_1 the point R' can be obtained from R using the translational operator ($T = n_1a + n_2b$) given by

$$T = 0a_1 + 1b_1$$



Which containing integer coefficients, thus R' is related to R by the equation

$R' = R + T = R + 0a_1 + 1b_1,$

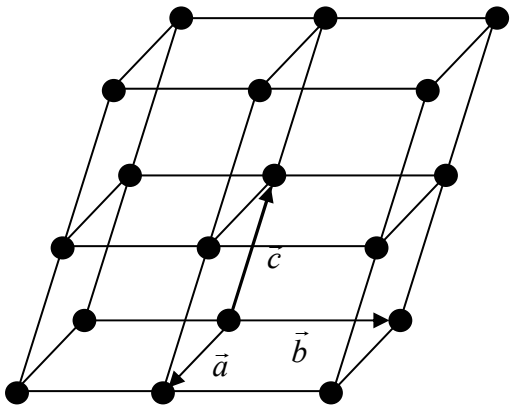
Such translational vector which produced a translational operation containing coefficient integer coefficient called primitive translation vectors, referring to the second set of translation vector a_2 and b_2 the point R' can be obtained from R by using the equation

$$R' = R + \frac{1}{2}a_2 + \frac{1}{2}b_2$$

Which containing non-integral coefficient of a_2 and b_2 such translation vector for which the translation operation contains non-integral coefficient are called non-primitive translation vectors.

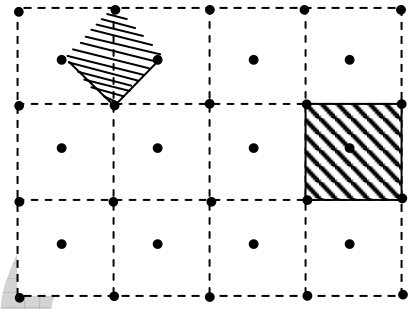
Unit cells

The entire lattice structure of a crystal can be generated by identical blocks known as unit cell. The unit cell may be a group of ions, atoms or molecules. The unit cell is the smallest building block or geometric figure from which the crystal is built up by repeating it in three dimensions as shown below.



Primitive and Non Primitive Unit Cells

A unit cell can be chosen in a number of ways. The following figure shows two ways of choosing a unit cell in two dimensions. In the first way the unit cell contains lattice points only at the corners. This type of unit cells is called as Primitive Cells. In the second way the unit cell contains lattice points in addition to the corners. These are known as Non Primitive Cells.



PRIMITIVE AND NON PRIMITIVE CELL

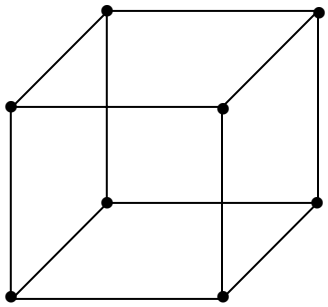
Note: The unit cells differ from the primitive cell in that it is not restricted to being the equivalent of one lattice point. Thus, unit cells may also be primitive cells, but all the primitive cells need not be Unit Cells.

The unit cell is so-called because the entire lattice can be derived by repeating this cell as a unit by means of the translations that serve as the unit cell's edge.

In three dimension there are four type of possible unit cells.

1. Primitive (P-type)

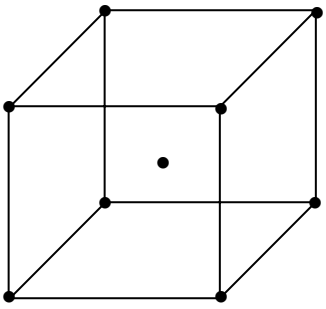
It has lattice point only at corners and each corner is common in eight cells, so each cell represents a single lattice point $8\times\frac{1}{8}=1$ lattice point /unit cell as shown in figure.



2. Body Centered cell (I-type)

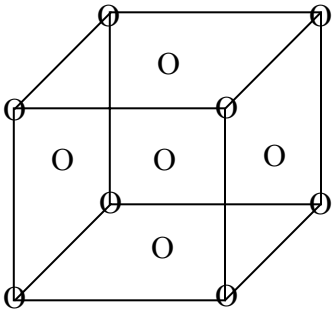
It has lattice points at corners as well as at centre of each cell, so each cell has two lattice points per unit cell.

$8\times\frac{1}{8}+1=1+1=2$ lattice point /unit cell.



3. Face Centered cell (F-type)

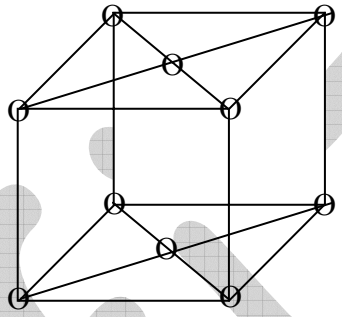
It has a lattice point at corner as well as at centre of each face of the cell as shown in figure.



$8\times\frac{1}{8}+6\times\frac{1}{2}=1+3=4$ lattice point /unit cell.

4. Base Centred (C-type)

It has lattice points at corner as well as centre of top and bottom face of cell, it has two lattice points per unit cell as shown in figure:



$8\times\frac{1}{8}+2\times\frac{1}{2}=1+1=2$

Wigner-Seitz Cell

The primitive cell can also be constructed using the following procedure

(i) Connect a given lattice point to all the nearby lattice points.

(ii) Draw normal at the mid point of lines connecting the lattice points.

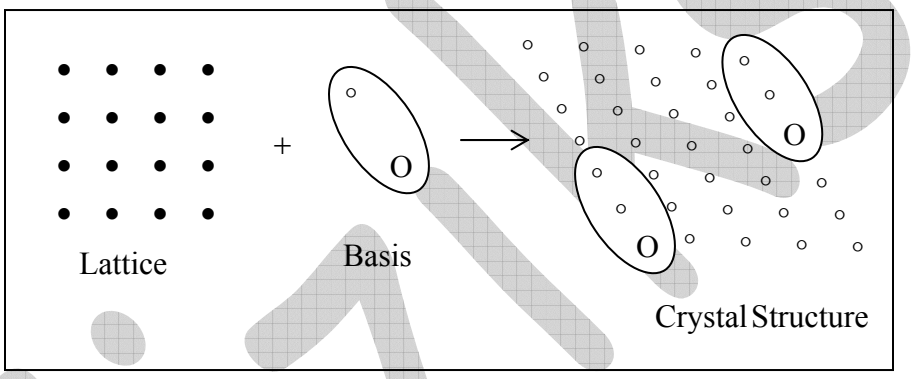
The smallest volume enclosed by the normal is the required primitive cell, such a cell is Wigner-Seitz cell

1.2 Basis

The repeating unit assembly: atom, molecule, ion or radical that is located at each lattice point is called basis or motif. Every basis is identical in composition, arrangement and orientation, when the basis is repeated with correct periodically in all directions, it gives the actually crystal structure, the crystal structural is real, while the lattice is imaginary. Thus,

Lattice + Basis = Crystal structure

Figure shows the basis representing each lattice point. It is observed from the figure that a basis consists of three different atoms.



More precisely we can say that when the space lattice combines with the basis a unit cell of the crystal is generated, the unit cell will be called 'monoatomic' if one atom occupies a lattice point. When two atoms occupy a lattice point, it will make a diatomic unit cell. Similarly, the unit cell will be known as multiatomic when too many occupy a lattice point.

Example: Cu Crystal is a Mono-atomic crystal; NaCl is a diatomic crystal while CaF_2 is a tri-atomic crystal.

Symmetry Operation

A symmetry operation is that which transforms the crystal to itself, i.e. a crystal remains invariant under a symmetry operation. These operations are: (i) Translational (ii) Rotational (iii) Reflection (iv) Inversion.

(i) Translations: The translation symmetry follows from the orderly arrangement of a lattice. It means that a lattice point r, under lattice translation vector operator T, gives another point r' which is exactly identical r' = r + T.

(ii) Rotations: A lattice is said to posses the rotation symmetry if its rotation by an angle θ about an axis (or a point in a two dimensional lattice) transforms the lattice to itself. Also, since the lattice always remains invariant by a rotation of 2 π, the angle 2π must be an integer multiple of θ i.e. nθ = 2π the factor n takes integral values and is known as multiplicity of rotation axis. The possible value of n which are compatible with the requirement of translation symmetry are 1, 2, 3, 4, 6 only. But 5 fold, 7 fold or higher order symmetry are not possible.

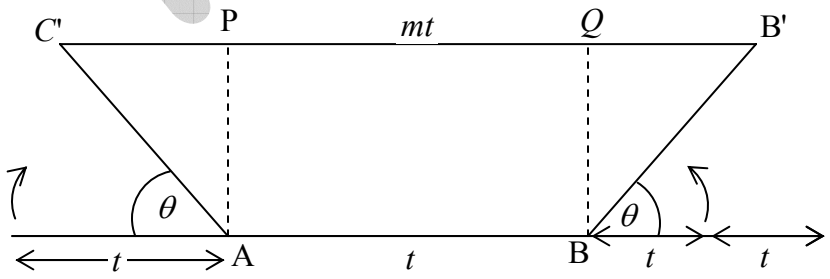
Example: Hexagonal crystal have 6 fold axis of rotation $\theta = 360/60 = 60^o$

Now we can explain why five fold symmetry is not permissible

As we know that the throw of an axis θ = 60 (n fold of the axis) if we consider a regular pentagon, which has an angle of 108^o. As 360^o is not an integer multiple of 108^o pentagons can not be made to meet at a point bearing a constant angle to another.

Hence 5 fold symmetry (i.e. a pentagonal lattice) is not possible.

Let us consider the row lattice as shown in figure and place an n fold axis at each lattice point.



Two opposite rotations by an amount Q about two axis produces two new lattice point C' and B' (and AC' = BB' = t). Therefore, the line joining C' and B' is parallel to the translation t. C'B' = C'P + PQ + QB'

$mt = t\cos\theta + t + t + t\cos\theta$ or $mt = t(1 + 2\cos\theta)$

$\cos\theta = \left(\frac{m-1}{2}\right)$ $\theta = \cos^{-1}\left(\frac{m-1}{2}\right)$ Let m – 1 = N (Integer)

$\theta = \cos^{-1}\left(\frac{N}{2}\right)$ $\therefore \cos\theta = \frac{N}{2}$ $-2 \le N \le 2$ ($\because \cos\theta$ varies from 1 to -1)

throw of an axis $\theta = \frac{2\pi}{n} = \frac{360^o}{n}$ where n is fold of the axis

when N = - 2 $\cos\theta = -1$ $\theta = 180^o$ $n = \frac{360}{180} = 2$

N = - 1 $\cos\theta = -1/2$ $\theta = 120^o$ $n = \frac{360}{120} = 3$

N = 0 $\cos\theta = 0$ $\theta = 90^o$ $n = \frac{360}{90} = 4$

N = + 1 $\cos\theta = 1/2$ $\theta = 60^o$ $n = \frac{360}{60} = 6$

N = + 2 $\cos\theta = +1$ $\theta = 360^o$ $n = \frac{360}{360} = 1$

Therefore, 2, 3, 4, 6 and 1 fold of symmetry axis are possible, 5 fold symmetry and more than 6 fold is not possible.

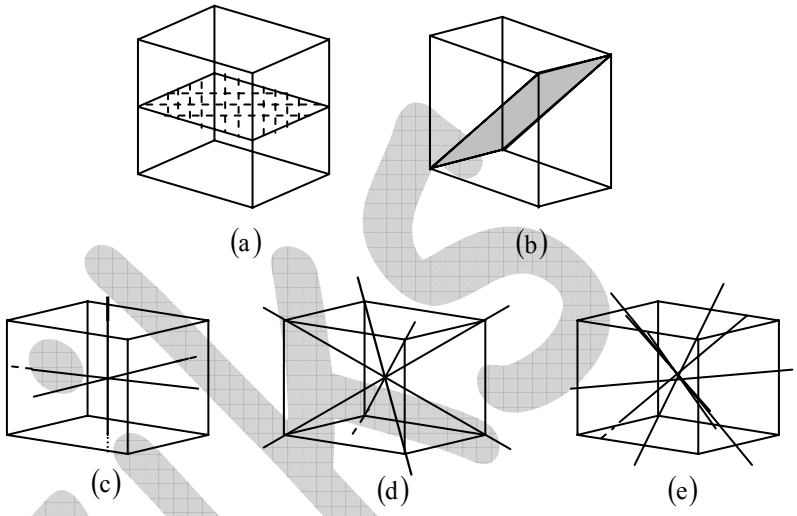
(iii) Reflections: A lattice is said to possess reflection symmetry if there exists a plane (or a line in two dimensions) in the lattice which divide it into two identical halves which are mirror image of each other. Such a plane is represented by m.

(iv) Inversion: Inversion is a point operation which is applicable to three dimensional lattice only. This symmetry element implies that each point located at r relative to the same lattice point. In other words, it means that the lattice possesses a centre of inversion denoted by 1.

Symmetry Element of Cubic Lattice

Total symmetry elements is cubic lattice is 23.

The geometric locus about which a group of repeating operation acts is called symmetry elements. For example, in square lattice axis normal to the plane of paper about which square is rotated by 90^0, 180^0 and 270^0 to preserve the symmetry is a symmetry element. In square lattice, axis in the plane of paper and bisecting the opposite edges of the square which rotate the square by 180^0 to preserve symmetry is called another symmetry element.



Similarly the plane across which the reflection occurs is a symmetry element and called a reflection plane or a mirror etc.

In cubic lattice there are total 13 rotational symmetry elements (3 tetrad axis Fig.c , 4 triad axis Fig d and 6 diad axis Fig e), 9 reflection symmetry elements, a plane of symmetry parallel to the face of the cube and diagonal to the cube is shown in fig a and fig b, and there is only 1 inversion symmetry elements.

1.3 Bravais Space Lattice

There are number of ways in which an actual crystal may be built up and atoms piled together resulting in a great many crystal structures. But each of the structure consists of some fundamental patterns repeated at each point of a space lattice. The scheme of repetitions of a space lattice are very limited in number while the possible crystal structures are almost unlimited. But it was shown by Bravais that there are 14 different arrays or networks of lattice in which points can be arranged in space so that each point has identical surrounding these are known as Bravais Space Lattice.

Bravais Lattice: These are 5 in 2-Dimension and 14 in 3-Dimension

Bravais Lattices in Two-Dimension

The four crystal systems of two-dimensional space are square, rectangular, hexagonal, and oblique. The rectangular crystal system has two Bravais lattice namely rectangular primitive and rectangular centred. In all there are five Bravais lattices:

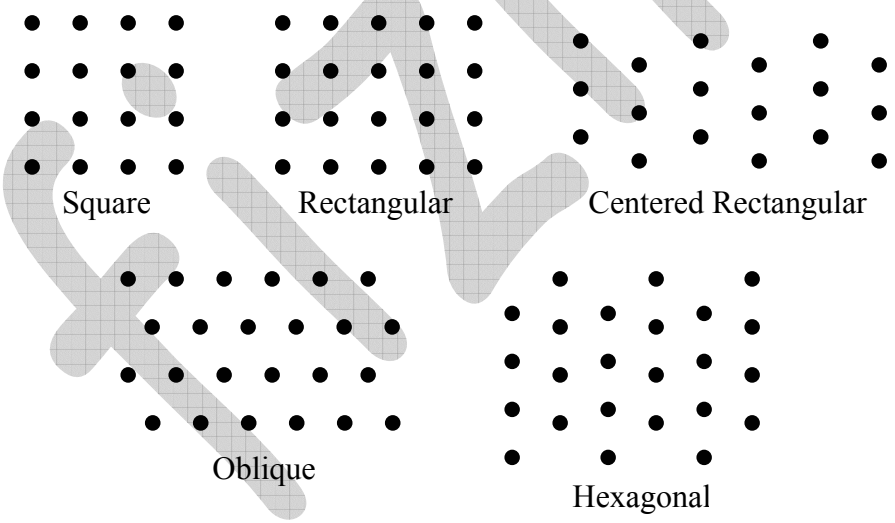
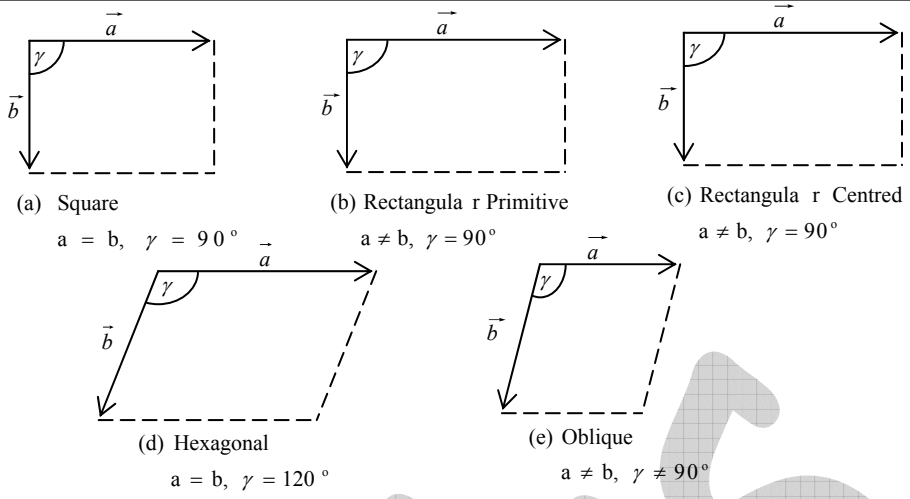


Figure: Five Bravais Lattice in two dimensions



(a) Square
a = b, $\gamma = 90^\circ$

(b) Rectangular Primitive
a ≠ b, $\gamma = 90^\circ$

(c) Rectangular Centred
a ≠ b, $\gamma = 90^\circ$

(d) Hexagonal
a = b, $\gamma = 120^\circ$

(e) Oblique
a ≠ b, $\gamma \neq 90^\circ$

Bravais Lattices in three-dimension: In three dimension Bravais lattice is given as

Crystal System	No.	Unit Cell	Coordinate Description
1 Triclinic	1	Primitive	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma$
2 Monoclinic	2	Primitive	$a \neq b \neq c$ $\alpha = \beta = 90^\circ \neq \gamma$
	3	Body Centered	
3 Orthorhombic	4	Primitive	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90^\circ$
	5	Base Centered	
	6	Body Centered	
	7	Face Centered	
4 Tetragonal	8	Primitive	$a = b \neq c$ $\alpha = \beta = \gamma = 90^\circ$
	9	Body Centered	
5 Trigonal	10	Primitive	$a = b = c$ $\alpha = \beta = \gamma < 120^\circ, \neq 90^\circ$
6 Hexagonal	11	Primitive	$a = b \neq c$ $\alpha = \beta = 90^\circ, \gamma = 120^\circ$
7 Cubic	12	Primitive	$a = b \neq c$ $\alpha = \beta = \gamma = 90^\circ$
	13	Body Centered	
	14	Face Centered	

Lattice Parameters

To completely illustrate the crystal structure, the basic minimum parameters required are:

(1) The inter atomic or intermolecular distance in x, y and z direction (a, b and c).

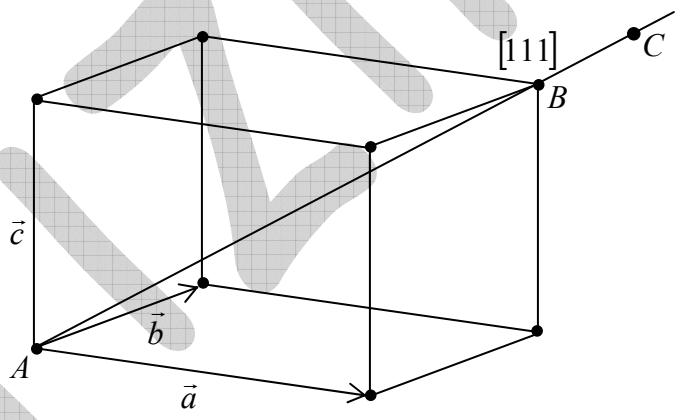
(2) The angles (α , β andγ) between the x and y, y and z and z and x axes.

The angle between the x and y axes is taken as α , the angle between the y and z axes is taken as β, angle between the Z and x axes is taken as γ are called the crystal parameters.

Therefore, the acceptable way to study structure of crystals is in terms of inter-lattice distances and the inter-planar angles, i.e. in terms of the crystal parameters.

Line Indices in a Crystal Lattice

While dealing with a crystal system, it is necessary to refer to the crystal planes, and directions of the straight lines joining the lattice points in a space lattice. A notation system which uses a set of three integers (n_1, n_2 & n_3) is adopted to describe both the positions of planes or directions within the lattice.



Direction of a Line in a Lattice

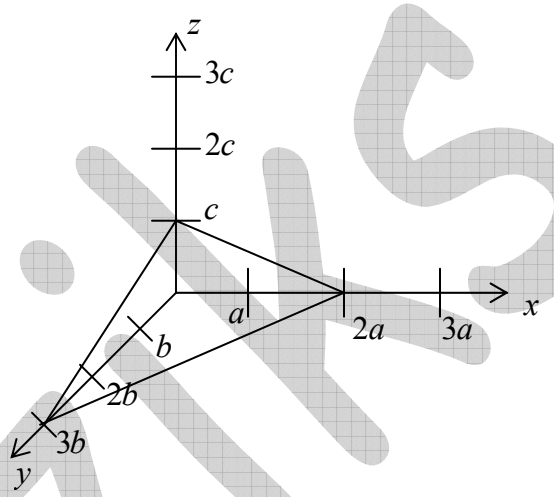
A resultant vector $\vec{R}$ which joins Lattice points A and B can be represented by an Equation.

$\vec{R} = n_1\vec{a} + n_2\vec{b} + n_3\vec{c}$, If $n_1 = n_2 = n_3 = 1$, then $\vec{R} = \vec{a} + \vec{b} + \vec{c}$

1.4 Miller Indices in Crystal Lattice

The crystal lattice may be regarded as made up of an aggregate of a set parallel equidistance planes. For a given lattice, the lattice planes can be chosen in different ways. Miller devised a scheme for indexing lattice planes by integers. These indices are known as the miller indices of the lattice planes.

The steps in the determination of miller indices of a plane are illustrated with the help of figure.



Step:

1) Determination the co-ordinate of the intercepts made by the plane along the three crystallographic axes (x, y, z)

For example:

	x	y	z
	2a	3b	c
	pa	qb	rc
with	p = 2	q = 3	r = 1

2) Express the Intercepts as multiples of the unit cell dimensions, or lattice parameters along axes

i.e.	$\frac{2a}{a}$	$\frac{3b}{b}$	$\frac{c}{c}$
	2	3	1

3) Get the reciprocals of these numbers

1/2	1/3	1/1

4) Reduce these reciprocals to the smallest set of Integer numbers and enclosed them in bracket

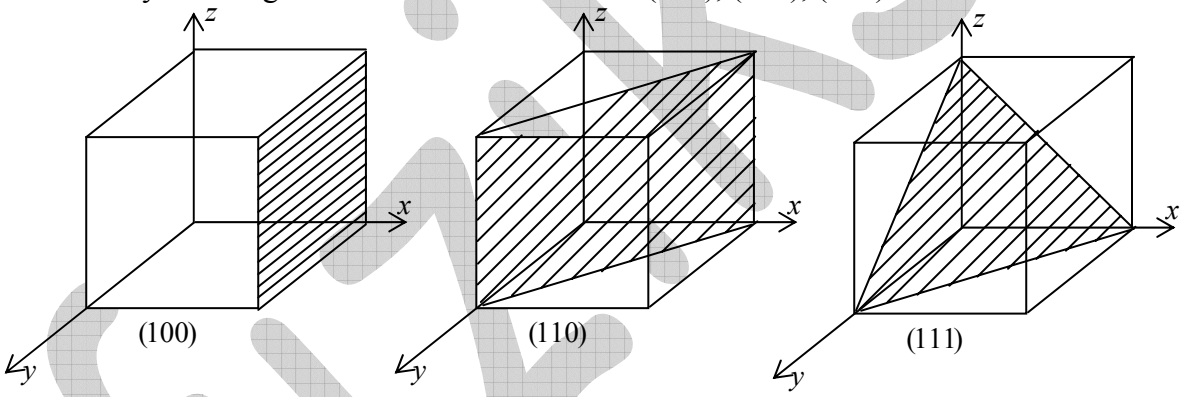
6 x 1/2	6 x 1/3	6 x 1/1
3	2	6

In general it is denoted by (hkl), (326)

Hence, the miller indices are the three smallest possible integers which have the same ratios as the reciprocal of the intercepts of the plane concerned on the three axes.

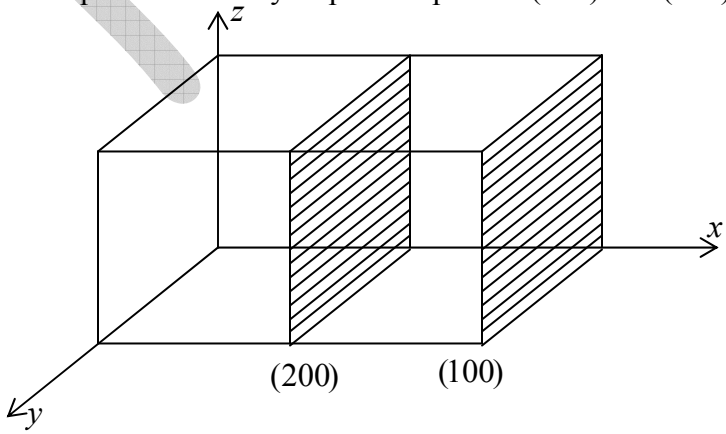
Important Points

(1) A zero miller indices indicate that the plane is parallel to the corresponding crystal axis, eg: (130) means the plane is parallel to the z-axis (001) means the plane is parallel to the x and y-axis. Figure shows the miller indices (100), (110), (111).



2) A negative miller indices shows that the plane $(\bar{h}kl)$ cuts the x-axis on the negative side of the origin, eg $(\bar{1}00)$

3) Miller indices represent a family of parallel planes. (200) and (100) are parallel planes.



4) {} brackets are used to represent all sets of a family of planes. Ex: {100} represents (100), (010), (001) and $(\bar{1}00)$, $(0\bar{1}0)$, $(00\bar{1})$

5) When the integers used in the miller indices contain more than one digit, the indices must be separated by commas for clarity Ex: (3, 11, 13)

Example: Obtain the Miller indices of a plane which intercepts at – a, b/2, 3c in a simple cubic unit cell.

Solution:

i)	x	y	z
	a	b/2	3c
ii)	$\frac{a}{a}$	$\frac{b/2}{b}$	$\frac{3c}{c}$
	1	1/2	3
iii)	1	2	1/3
iv)	3 x 1	3 x 2	3 x 1/3

Miller indices are (361)

Example: A plane makes intercepts of 1, 2 and 3 Å on the crystallograph i.e. axes of an orthorhombic crystal with a : b : c = 3 : 2 : 1. Determine the miller indices of this plane.

Example: The miller indices of a plane in a simple cubic lattice crystal are (123). Get the co-ordinates of the plane

Solution: $x \times 1 = a$ or $x = \frac{a}{1}$

$y \times 2 = b$ or $y = \frac{b}{2}$

$z \times 3 = c$ or $z = \frac{c}{3}$

Thus, the co-ordinates of the plane are *a, b*/2*, c*/3.

Inter-planar Spacing (d_{hkl})

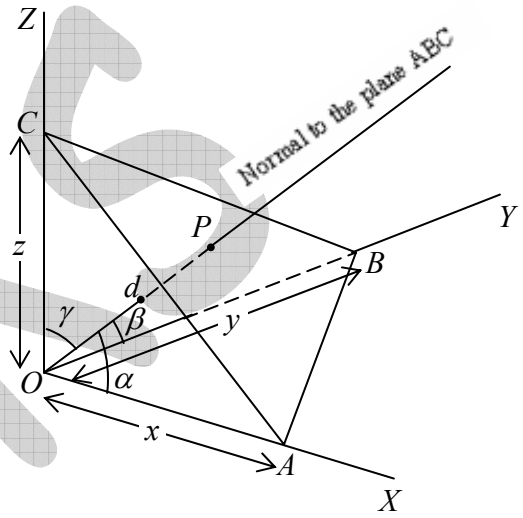
To get the inter-planar distance, consider a plane ABC with Miller indices (hkl). In the reference frame, draw a normal to the plane from the origin. Let OP is the normal to the plane ABC. Let angle POA = α', and POB = β' and POC =γ' be the angles made by the normal to the plane with the x, y and the z directions. OP = d is the interplanar distance

Then, from the figure,

Cos α = ON/OA = d/x

Cos β = ON/OB = d/y

Cos γ = ON/OC = d/z



But from the definition of Millers incise derived in the earlier section,

$h = a/x$ therefore, $x = a/h$

$k = b/y$ therefore, $y=b/k$

$l = c/z$ therefore z= c/l

Writing the values of x, y and z in the above trigonometric equations we get,

$$\text{Cos } \alpha = \frac{OP}{OA} = \frac{d}{a/h}, \qquad \text{Cos } \beta = \frac{OP}{OB} = \frac{d}{b/k}, \qquad \text{Cos } \gamma = \frac{OP}{OC} = \frac{d}{c/l}$$

From solid geometry, $\text{Cos}^2 \alpha + \cos^2 \beta + \text{Cos}^2 \gamma = 1$

Substituting the values of the trigonometric relations we get

$$\frac{h^2 d^2}{a^2} + \frac{k^2 d^2}{b^2} + \frac{l^2 d^2}{c^2} = 1 \Rightarrow d^2 \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2} \right] = 1$$

$$\Rightarrow d_{hkl}^2 = \frac{1}{\left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2} \right]}$$

Then, the inter-planar distance'd' is given by $d_{hkl} = \dfrac{1}{\left[\dfrac{h^2}{a^2} + \dfrac{k^2}{b^2} + \dfrac{l^2}{c^2}\right]^{1/2}}$

case I: Cubic crystal a = b = c $d_{hkl} = \dfrac{a}{\sqrt{h^2 + k^2 + l^2}}$

case II: tetragonal a = b ≠ c $d_{hkl} = \dfrac{1}{\sqrt{\dfrac{h^2 + k^2}{a^2} + \dfrac{l^2}{c^2}}}$

case III: orthorhombic a ≠ b ≠ c $d_{hkl} = \dfrac{1}{\sqrt{\dfrac{h^2}{a^2} + \dfrac{k^2}{b^2} + \dfrac{l^2}{c^2}}}$

fiziks

1.5 Density of Crystal

Density is a macroscopic property. Basically it is the mass per unit volume. In case of crystals, mass of atoms packed in a conventional unit cell per unit volume of the cell gives the density of the crystal.

$$\text{Density of a crystal} = \rho = \frac{Mass\ of\ atoms\ in\ the\ unit\ cell}{Volume\ of\ the\ unit\ cell} = \frac{m}{V}$$

Here m is the mass of atoms packed in the conventional unit cell of the crystal. V is the volume of the unit cell. Mass of atoms packed in the conventional unit cell of the crystal. V is the volume of the unit cell. Mass of an atom in the structure is given by the ratio of the atomic weight or Molecular weight (M) to the Avogadro number (N_A). Mass of atoms contained in the conventional unit cell is then the number of atoms in the unit cell times the mass of an atom. If there are n_{eff} atoms in the conventional unit cell, then, the mass of atoms in the unit cell is given by

$$m = \frac{n_{eff}\ M}{N_A\ a}$$

From density equation, mass of atoms in the unit cell in terms of the density of the crystal is given by $m = \rho V$

$$\rho = \frac{n_{eff}\ M}{N_A\ V}$$

From this equation, the density of the crystal is

$$\boxed{\rho = \frac{n_{eff}\ M}{N_A V}\left(kg/m^3\right)}$$

In case of a cubic lattice the volume of the unit cell $V = a^3$

Therefore $\rho = \frac{n_{eff}\ M}{N_A a^3}\left(kg/m^3\right)$

The lattice constant $a = \left(\frac{n_{eff}\ M}{N_A \rho}\right)^{1/3} A^0$

1.5.1 Packing Fraction

The packing of atoms in a unit cell of the crystal structure of a material is known as atomic packing.

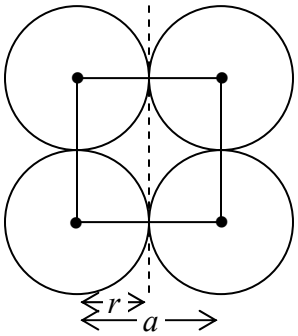
Atomic packing fraction (APF) is defined as the ratio of the volume of the atoms per unit cell to the volume of the unit cell.

$$APF = \frac{\text{volume of atomic per unit cell}}{\text{volume of the unit cell}}$$

(1) Simple Cubic (SC): In a simple cubic structure, the space lattice is cubic. Atoms are placed at all lattice locations. Therefore, an atom at a lattice location will be in contact with all its six nearest neighbors. Thus the co-ordination number for a simple cubic structure is N=6. In a simple cubic structure, the lattice constant is the cube edge of the conventional unit cell. Thus the lattice constant is the distance between the centers of two neighboring atoms. That is the lattice constant a = 2r, where r is the atomic radius. The atoms at the lattice locations are shared by eight unit cells. Each atom at the lattice location of the unit cell contributes (1/8) to the unit cell.

The number of atoms contained in a unit cell in the structure is given by the number of lattice locations in the unit cell multiplied by the contribution of the atom at each location. The effective number of atoms in a unit cell of the simple cubic structure is given by

$$n_{eff} = \frac{1}{8}\times n_c + \frac{1}{2}\times n_f + 1\times n_i \Rightarrow n_{eff} = \frac{1}{8}\times 8 + \frac{1}{2}\times 0 + 1\times 0 = 1$$



and $r = a/2$

$$APF = \frac{\text{n}_{\text{eff}} \times \text{volume of each sphere}}{\text{volume of each cell}}$$

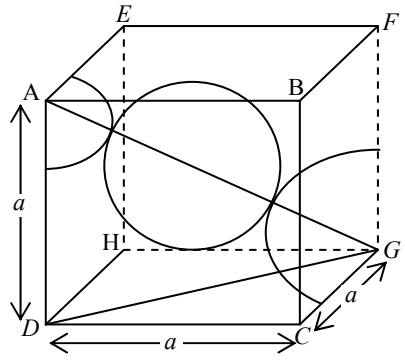
$$APF = \frac{1\times\frac{4\pi}{3}\left(\frac{\text{a}}{2}\right)^3}{\text{a}^3} = \frac{\pi}{6} = 0.52 = 52\%$$

The parameters describing the structure of a simple cubic structure are:

1	Co-ordination Number	N = 6
2	Number of atoms in a unit cell	n = 1
3	Atomic radius	R
4	Lattice constant	r = 2r
5	Volume of a unit cell	$V = a^3$
6	Volume of an atom	$(4/3)\ \pi\ r^3$

(2) Body Centered Cubic (BCC): In body centered cubic structure, the space lattice is cubic. Atoms are placed at all lattice locations and there will be an additional atom at the body center of the cubic unit cell. In this structure, the atoms stacked along the body diagonal of the cubic unit cell are in contact. That is, the atom at the body center of the unit cell will be in contact with all other atoms at the cubic lattice locations of the unit cell. Each atom in the structure will have eight nearest neighbors. Thus, the co-ordination number for a body centered cubic structure is N = 8. In a body centered cubic structure, the lattice constant (a), is the cube edge of the conventional unit cell.

The effective number of atoms in a unit cell of the bcc is given by



$$n_{eff} = \frac{1}{8}\times 8 + \frac{1}{2}\times 0 + 1\times 1 = 2$$

$$n_{eff} = 2 \quad and \quad r = \frac{a\sqrt{3}}{4}$$

$$APF = \frac{2\times\frac{4\pi}{3}\left(\frac{a\sqrt{3}}{4}\right)^3}{a^3} = \frac{\pi\sqrt{3}}{8} = 0.68 = 68\%$$

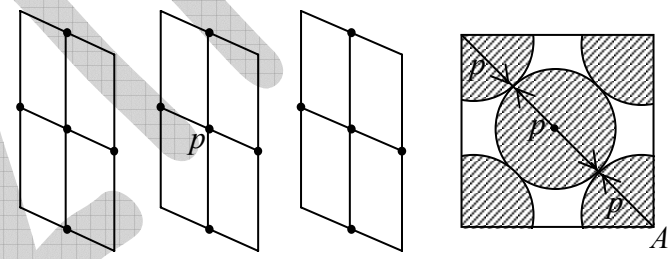
The parameters describing the structure of a body centered cubic structure are :

1	Co-ordination Number	N = 8
2	Number of atoms in a unit cell	n = 2
3	Atomic radius	r
4	Lattice constant	a =(4 / $\sqrt{3}$) r
5	Volume of a unit cell	V = a^3
6	Volume of an atom	a = (4/3) π r^3

(3) Face Centered Cubic (FCC): In a face centered cubic structure, the space lattice is cubic. The conventional unit cell consists of eight small cube lets. Atoms are placed at alternate lattice locations. The atoms stacked along the face diagonal of the cubic unit cell are in contact. The atom at any lattice location of the unit cell will have twelve nearest neighbors. The effective number of atoms in a unit cell of the fcc is given bv

$$n_{eff} = \frac{1}{8}\times 8 + \frac{1}{2}\times 6 + 1\times 0 = 4$$

$$n_{eff} = 4 \qquad and \qquad r = \frac{a\sqrt{2}}{4}$$



$$\text{APF} = \frac{4\times\frac{4\pi}{3}\left(\frac{a\sqrt{2}}{4}\right)^3}{a^3} = \frac{\pi}{3\sqrt{2}} = 0.74 = 74\%$$

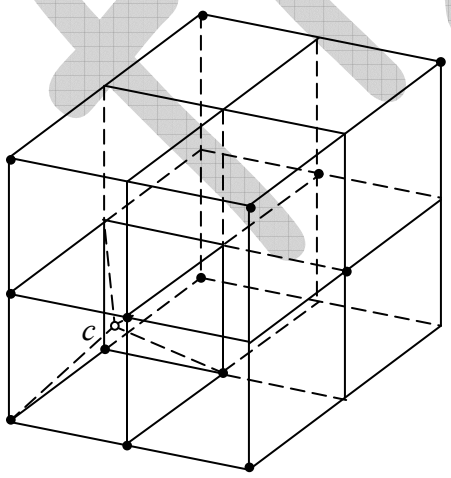
The parameters describing the structure of a face centered cubic structure are :

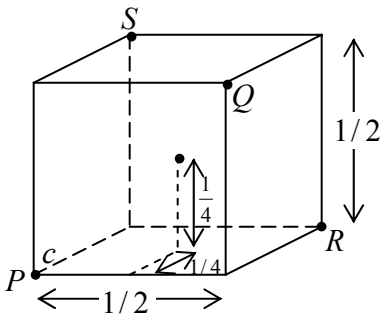
1	Co-ordination Number	N =12
2	Number of atoms in a unit cell	n = 4
3	Atomic radius	r
4	Lattice constant	a =(4 / $\sqrt{2}$) r
5	Volume of a unit cell	V = a^3
6	Volume of an atom	a = (4/3) π r^3

(4) Diamond Cubic (DC): Diamond is the crystalline form of carbon. The structure consists of the periodic stacking of carbon atoms in its crystalline state.

A conventional unit cell of structure consists of eight cubic primitive cells. On the basis of the positions of the carbon atoms in the unit cell they are classified as first kind and second kind only for the sake of convenience. Otherwise they are the same carbon atoms. The carbon atoms placed at the FCC positions of the conventional unit cell are the first kind of carbon atoms. Another set of carbon atoms occupy the body centered positions in the primitive cubic cells in the structure. The second type of carbon atoms are placed at the body centers of the alternate cubic primitive cells. These carbon atoms independently form another FCC structure. Therefore, diamond crystal can be regarded as a structure produced by the penetration of one type of carbon atoms forming FCC structure in to the other FCC structure, formed by the second kind of carbon atoms, through a distance equal to three fourth the body diagonal.

If the locations of the atoms in a diamond structure is seen in the primitive part of the unit cell (one of the eight cubelets), they look like a tetrahedra. In these tetrahedra, one atom at the center is bonded to three others in the primitive cell. Periodic stacking of the tetrahedral structure also produces the diamond crystal.





The distance between the two carbon atoms place on the cube edge of the conventional unit cell is the lattice constant and it is, a = 3.567 Au for diamond crystal. This can be treated as the graphical representation of the crystal structure.

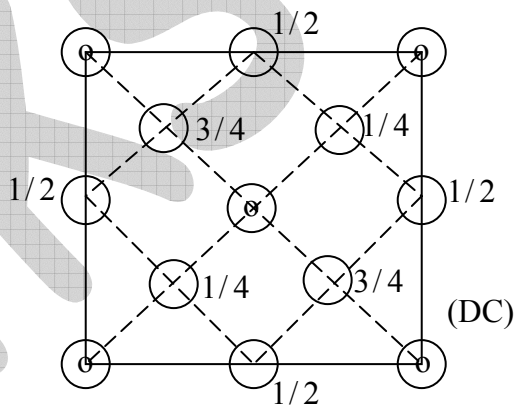
The crystal structure can also be illustrated in terms of the co-ordinates of the atoms in the structure. To write the co-ordinates of the carbon atoms in the structure, an orthogonal co-ordinate system is considered. The origin of the co-ordinate system is fixed at one of the carbon atoms. With this co-ordinate system the co-ordinates of the atoms in the structure are shown above.

The effective number of atoms in a unit cell of the diamond cubic is given by

$$n_{eff} = \frac{1}{8}\times 8 + \frac{1}{2}\times 6 + 1\times 4 = 8$$

$$n_{eff} = 8 \qquad r = \frac{a\sqrt{3}}{8}$$

$$\text{APF} = \frac{8\times\frac{4\pi}{3}\left(\frac{a\sqrt{3}}{8}\right)^3}{a^3} = \frac{\pi\sqrt{3}}{16} = 0.34 = 34\%$$



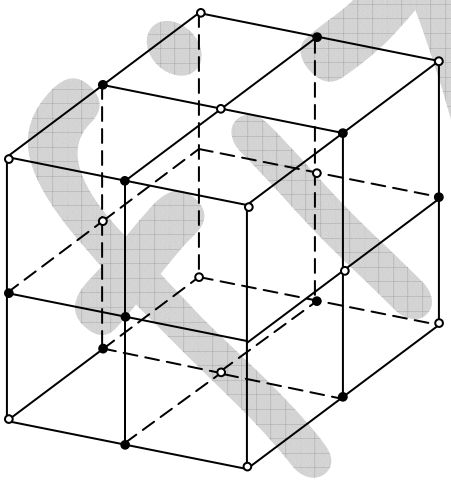
(DC)

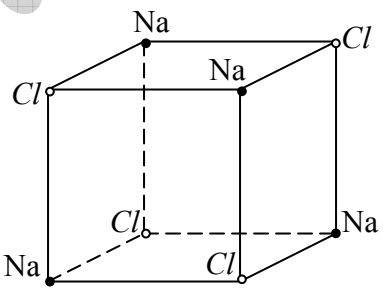
Atoms	Co-ordinates
Carbon atoms of first kind	(0 0 0); (1 0 0); (1 0 1); (0 0 1); (1/2 0 1/2)
	(1/2 1/2 0); (1 1/2 1/2); (1/2 1/2 1); (0 1/2 1/2)
	(1 1 0); (1 1 1); (0 1 1); (0 1 0); (1/2 1 1/2)
Carbon atoms of second kind	(1/4 1/4 1/4); (3/4 3/4 1/4); (1/4 3/4 3/4); (3/4 1/4 3/4)

(5) Crystal Structure of Sodium Chloride

Sodium chloride is an ionic crystal. It is a compound of the alkali halide family. Due to the proximity of the sodium atom with a chlorine atom, the valence electron from the sodium atom is transferred to the chlorine atom. With this transfer of an electron, the sodium atom becomes a cation and the chlorine is converted to an anion. Then, these ions are held by the ionic force and the bond that holds the atoms together is ionic.

A conventional unit cell of the structure consists of eight cubic primitive cells. In the conventional unit cell of the crystal, sodium atoms occupy the FCC positions of a cubic structure. Chlorine atoms are placed at the intermediate positions between the sodium atoms. Chlorine atoms independently from an FCC structure similarly sodium atoms also from an independent FCC structure. A unit cell of a sodium chloride crystal can be regarded as the structure formed by the inter-penetration of sodium FCC lattice with the Chlorine FCC lattice through one half of the lattice constant. This can be treated as the graphical representation of the crystal structure.





The crystal structure can also be illustrated in terms of the co-ordinates of the atoms in the structure. For this, an orthogonal co-ordinate system is considered. The origins of the co-ordinate system can either be at the location of a chlorine atom or at the location of a sodium atom. With the origin at the location of the sodium atom, the co-ordinates of the sodium and chlorine atoms in the crystal structure are shown below. Sodium chloride is an ionic crystal formed by the transfer of the valence electron from sodium atom to the

chlorine atom. Due to this transfer the sodium atom is converted to a cation and the chlorine is converted to an anion. These ions are held by the ionic force and the bond that holds the atoms together is ionic bond. In the unit cell of the compound, chlorine atoms occupy the FCC positions of a cubic structure. Sodium atoms occupy edge center positions.

Atoms	Co-ordinates
Sodium	(0 0 0); (1 0 0); (1 0 1); (0 0 1); (1/2 0 1/2)
	(1/2 1/2 0); (1 1/2 1/2); (1/2 1/2 1); (0 1/2 1/2)
	(1 1 0); (1 1 1); (0 1 1); (0 1 0); (1/2 1 1/2)
Chlorine	(1/2 1/2 0); (1 0 1/2); (1/2 0 1); (0 0 1/2);
	(1 1/2 0); (1 1/2 1); (0 1/2 1); (0 1/2 0); (1/2 1/2 1/2)
	(1/2 1 0); (1 1/2 1/2); (1/2 1 1); (0 1 1/2)

Example: Show that for cubic lattice, the lattice constant a is given by $a = \sqrt[3]{\frac{nM}{N\rho}}$

Example: α – Iron is BCC and has the volume density of 7.86 gm/cm^3. Calculate α value for the atomic radius of an iron atom. The atomic weight of iron is 55.85 gm/mol.

Solution: $a = 2.867$ Å, $r = \frac{\sqrt{3}a}{4} = 1.241$ Å

Information in a Glance

	SC	BCC	FCC	HCP	DC
No. of atoms per unit cell	1	2	4	6	8
Co-ordination no.	6	8	12	12	4
Packing fraction	52%	68%	74%	74%	34%
Atomic radii	$\frac{a}{2}$	$\frac{\sqrt{3}a}{4}$	$\frac{a\sqrt{2}}{4}$	$\frac{a}{2}$	$\frac{a\sqrt{3}}{8}$
Volume	a^3	a^3	a^3	a^3	a^3
Example	Polonium	Na, Li, Cr	Al, Cu, Ag, Pb	Zn	Ge, Si, C

2. *X*-Ray diffraction

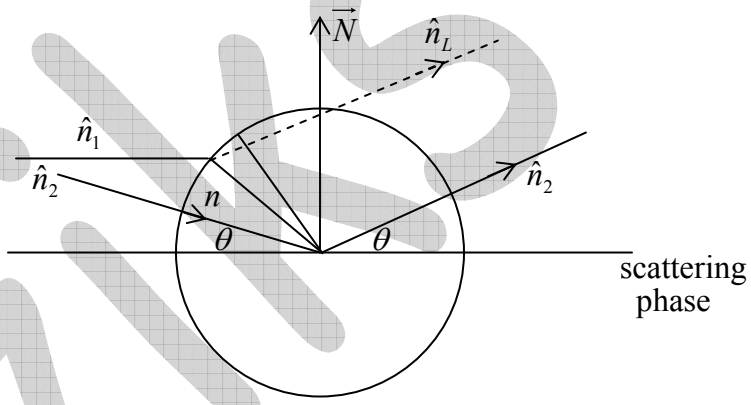
2.1 Structure Factor

The atomic scattering factor or form factor f describes the scattering power of a single atom in relation to the scattering power of a single electron and given by:

$$f = \frac{\text{amplitude of radiation scattered from an atom}}{\text{amplitude of radiation scattered from an electron}}$$

In general f < Z, it approaches Z (atomic no.) in the limit case. The phase difference between the wave scattered from the charge ρ(r) dv and that scattering from electron, in accordance equation is given by

Geometric Structure Factor



The intensity of an x-ray beam diffracted from a crystal not only depends upon the atomic scattering factor of the various atom involved but also on the contents of the unit cell, i.e. on the number. The total scattering amplitude F(h' k' *l*') for the reflection (h' k' *l*') is defined as the ratio of the amplitude of radiation scattered by a single point electron placed at origin for the same wavelength. It is given by:

$$F(h'k'l') = \sum f_j e^{i\phi j} = \sum_j f_j e^{i\left(\frac{2\pi}{\lambda}\right)(r_j \cdot N)}$$

where f_j is the atomic scattering factor for the j^{th} atom $\alpha\phi_j$ is the phase difference if (u_j, v_j, w_j) represents the co-ordinate of j^{th} atom we can write $r_j = u_j a + v_j b + w_j c$, and

$r_j . N = \lambda (u_j h + v_j k + w_j l)$ i.e. $F(hkl) = \sum_i f_j e^{i2\pi(u_j h + v_j k + w_j l)}$

For identical atoms, all the f_j's have the same value *f*. Therefore, equation (i) take a simple form. $F(hkl) = f S$

where $S = \sum_j e^{2\pi i(u_j h + v_j k + w_j l)}$ is called Geometrical Structure Factor.

Now since the intensity of a radiation is proportional to the square of its amplitude, the intensity of diffracted beam may be written by $I = |F|^2 = F^* F$

The structure factor for some simple crystals are calculated below and the intensity of various order of reflections associated with these structures is discussed.

Note: i) We can by find the structure factor allowed reflection by given crystal structure either wise zero.

ii) $e^{i\theta} = \cos\theta + i\sin\theta$ or $e^{i\pi} = \cos\pi + i\sin\pi$

(a) *Simple cubic crystal:* Unit cell contains only one atom at the origin

i.e. $(u, v, w) = (0, 0, 0)$

$\therefore \quad F = fe^{2\pi i(0)} = f$

And therefore intensity which is the square of the amplitude is $F^2 = f^2$.

Thus F^2 is thus independent of h, k & l and the same for all reflection.

$\therefore$ All h, k, l are possible in SC.

(b) *Base centered cell*: It has 2 atoms at 0, 0, 0 & $\frac{1}{2}, \frac{1}{2}, 0$

$\therefore \quad F = f[1 + e^{\pi i(h+k)}]$

And $F = 2f \Rightarrow F^2 = 4f^2$: for h & k unmixed

While $F = 0 \Rightarrow F^2 = 0$: for h & k mixed

Note : The value of l index has no effect on the structure factor

$\therefore$ Present planes are: (111), (112), (113) & (021), (022), (023)

Absent planes are: (011), (012), (013) & (101), (102), (103)

(c) *Body Centered cell*: It has 2 atoms at (0 0 0) & $\left(\frac{1}{2}\frac{1}{2}\frac{1}{2}\right)$

$\therefore \quad F = f[1 + e^{\pi i(h+k+l)}]$

And $F = 2f \Rightarrow F^2 = 4f^2$: when $(h + k + l)$ = Even

While $F = 0 \Rightarrow F^2 = 0$: when $(h + k + l)$ = odd

$\therefore$ Present planes are: (1 1 0), (2 0 0), (2 2 2) etc

Absent planes are: (1 0 0), (1 1 1), (2 1 0) etc

(d) *Face centered cell*: It has 4 atoms at (0 0 0), $\left(\frac{1}{2}\frac{1}{2}0\right)$, $\left(\frac{1}{2}0\frac{1}{2}\right)$ and $\left(0\frac{1}{2}\frac{1}{2}\right)$

$\therefore \quad F = f[1 + e^{\pi i(h+k)} + e^{\pi i(h+l)} + e^{\pi i(k+l)}]$

And $F = 4f \Rightarrow F^2 = 16f^2$: If h, k, l are unmixed

While $F = 0 \Rightarrow F^2 = 0$: If h, k, l are mixed

$\therefore$ Present planes are: (1 1 1), (2 0 0), (2 2 0) etc

Absent planes are: (1 0 0), (1 1 0), (2 1 1) etc.

Simplified Table

Bravais lattice	Reflection possibly Present	Reflection necessarily absent
Simple cubic	All	None
Base centered cubic	h & k unmixed	h & k mixed
Body centered cubic	(h + k + l) even	(h + k + l) odd
Face centered cubic	h, k & l unmixed	h, k & l mixed

Note: (1) The first order reflection from (1 0 0) planes in a bcc crystal is absent, while second order reflections from (1 0 0) plane is present and it is this reflection which appears at the position of the 1st order reflection from (2 0 0) planes.

(2) The ratio of $(h^2 + k^2 + l^2)$ values for all allowed reflection from cubic crystal as obtained from the extinction rules are given as follows.

SC : : 1 : 2 : 3 : 4 : 5 : 6 : 8 : 9 : 10 : 11 : 12

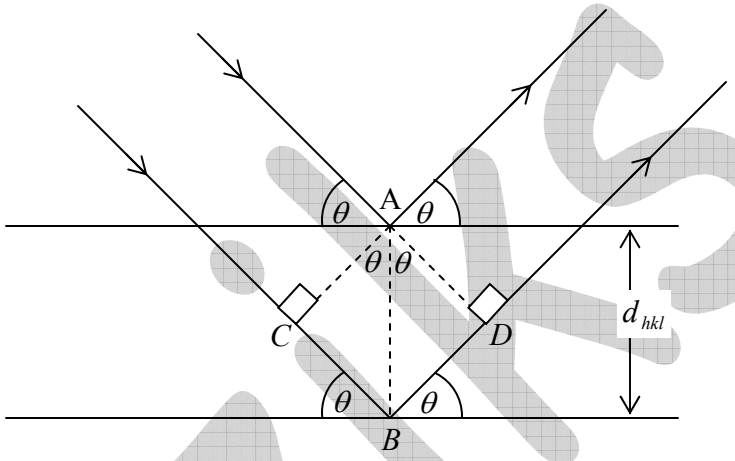
BCC : : 2 : 4 : 6 : 8 : 10 : 12 : 14 : 16 : 18 : 20

FCC : : 3 : 4 : 8 : 11 : 12 : 16 : 19 : 20 :

DC : : 3 : 8 : 11 : 16 : 19 : 20

2.2 Braggs Law

X-rays are a form of electromagnetic radiation and these are used for determining the crystal structures are they have high energy and short wavelengths. The wavelengths of x-rays are of the order of the atomic spacing for solids. When a beam of x-rays impinges on a solid material, a portion of this beam will be scattered in all direction by the electrons associated with each atom or ion that lies within the beam path.



From figure $n\lambda = CB + BD$

In Δ ABC,

$CB = AB \sin\theta = d_{hkl} \sin\theta$

$BD = AD \sin\theta = d_{hkl} \sin\theta$

$\therefore d_{hkl} \sin\theta = n\lambda$

$2d_{hkl} \sin\theta = n\lambda$

This is known as Bragg's law, n is order of reflection. For SC a = b = c

$$\sin\theta = \frac{n\lambda}{2a}\sqrt{h^2 + k^2 + l^2}$$

2.3 Methods of X-ray Diffraction

In x-ray diffraction studies, the probability that the atomic planes with right orientations are exposed to x-rays is increased by adopting there of crystal structure studies method.

1) Law Method

2) Rotating Method

3) Powder Method

1) Laue's Technique

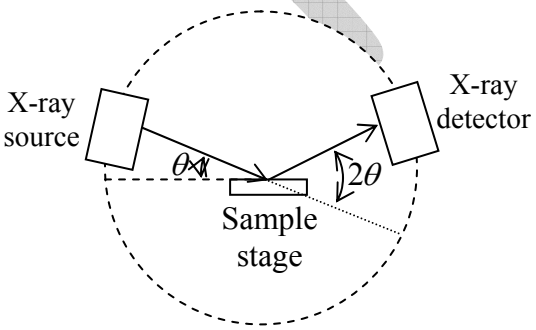
The single crystal is held stationary and a beam of white radiations is inclined on it at a fixed glacing angle θ i.e. θ is fixed while λ is varies different wavelengths present in the white radiations select the appropriate reflecting planes out of the numerous present in the crystal such that the Bragg's condition is satisfied this technique is called the Laue's technique.

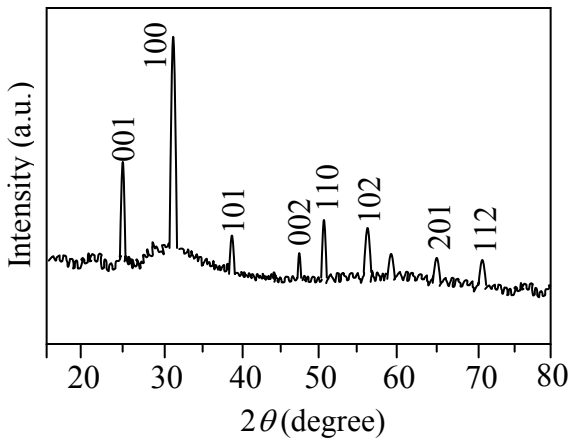
2) Rotating Crystal Method

A single crystal is held in the path of monochromatic radiations and is rotated about an axis i.e. λ is fixed while θ varies. Different sets of parallel atomic planes are exposed to incident radiations for different values of θ and reflections take place from those atomic planes for which d and θ satisfied the Bragg's law. This method is known as the rotating crystal method.

3) Powder Method

Modern x-ray crystal analysis uses an x-ray diffractometer which has a radiation counter to detect the angle and intensity of the diffraction beam.





A recorder automatically plots the intensity of the diffracted beam as the counter moves on a Goniometer circle. Figure shows an x-ray diffraction recorder chart for the intensity of the diffracted beam verses the diffraction angle 2θ for a powdered pure metal specimen.

Characteristics of the Bragg's law:

(i). It is the consequence of the periodicity of the space lattice.

(ii). The Bragg's law does not refer to the arrangement or basis of the atoms associated with the lattice points.

(iii). For a given order n and spacing d, the angle θ decreases with decrease in the wavelength λ.

(iv). The relative intensity of the various orders n of diffraction from a given set of parallel planes is determine by the composition of the basis of the crystal results in the change in d. This in turn changes the phase difference between the interfering waves and hence the resulting amplitude. Being proportional to the square of the amplitude the intensity is therefore affected.

(v). Bragg's reflection occur only for the wavelength $\lambda \leq 2d$. That's why crystal cannot diffract visible rays.

(vi). For X-ray photons,

$$E = h\nu = hc/\lambda \quad \text{or} \quad \lambda(A^0) = \frac{hc}{E} = \frac{12400}{E(eV)}$$

Therefore the energy of X-rays of wavelength $1A^0$ is 12.4keV.

Electron Diffraction by Crystal

According to de-Broglie, a particle of momentum p (= mv) is associated with a wave of wavelength

$$\lambda(A^0) = \frac{h}{P} = \frac{h}{\sqrt{2mE}}$$

Where h is the Planck constant. M is the mass of the electron and E is the energy of the electron. Since m = 9.1×10^{-31}kg.

Therefore $\lambda(A^0) = \frac{12}{\sqrt{E}}$

Thus the energy of electrons of wavelength $1A^0$ is 144eV.

Characteristic of Electron Diffraction

(i). Incident electrons are not only diffracted by the electrons but also by the nuclei of the atoms of the crystal. So the electron diffraction is much more intense than X-ray diffraction.

(ii). An electron beam has very low penetration power as it experiences repulsion from atomic electrons. Therefore this technique is useful for thin films.

Neutron diffraction by Crystal

Using the de-Broglie relation for neutron (mass m=1.675×10^{-27}kg) the associated wavelength

$$\lambda(A^0) = \frac{0.28}{\sqrt{E}}$$

This shows that the energy associated with neutrons of wavelength $1A^0$ is approximately 0.08eV. This energy is much less than of X-rays of the same wavelength.

Characteristic of Neutron Diffraction

(i) Neutrons are scattered only by the nuclei of the atoms of the crystal.

(ii) Scattering cross section does not vary with increasing atomic number.

(iii) Lighter elements such as hydrogen and carbon produces strong neutron scattering which is almost absent in X-ray diffraction.

3. Einstein and Debye Theory of Specific Heat

3.1 Classical Theory

A crystal consists of atoms which are arranged in a periodic manner and are bound togather by strong binding forces.

"In the classical theory it is consumed that each atom of a crystal acts as a three-dimensional harmonic oscillator and all the atoms vibrate independent of one other."

Further a system of N vibrating atoms or N independent three dimensional harmonic oscillators is equivalent to a system of 3N identical and independent one-dimensional Harmonic oscillators.

Assuming that the distribution of oscillators in energy obeys the Maxwell-Boltzmann distribution law the average energy of each harmonic oscillator is given by

According to Equipartition Theory $\langle K.E\rangle = \frac{1}{2}k_BT$

According to Hooks Law $\langle K.E\rangle = \langle P.E\rangle = \frac{1}{2}k_BT$

But $\langle E\rangle = \langle K.E\rangle + \langle P.E\rangle$

$$\Rightarrow \langle E\rangle = \frac{1}{2}k_BT + \frac{1}{2}k_BT = k_BT$$

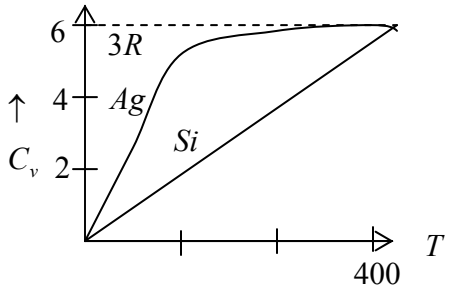
Thus the total vibration energy of the crystal containing N identical atoms or 3N one-dimensional Harmonic oscillator become

$$E = 3\,N < E >$$

$$E = 3Nk_BT$$

Using the definition of specific heat of $C_V = \left(\frac{\partial E}{\partial T}\right)_V$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = 3Nk_B = 3R$$



The above result so that the molar specific heat of all the solids is constant and is independent of temperature and frequency this is called Dulog and Petit's law

3.2 Einstein Theory of Specific Heat

Einstein, in 1911, attempted to resolve the discrepancies of the classical theory of specific heat by applying the Planck's quantum theory. Einstein retained all the assumptions of the classical theory as such except replacing the classical harmonic oscillator by quantum Harmonic oscillator i.e. $\left[\in_n = nh\nu\right] = n\hbar\omega_o \rightarrow \in_n = \left(n+\frac{1}{2}\right)h\nu = \left(n+\frac{1}{2}\right)\hbar\omega_o$

The salient features of the Einstein's theory are listed as:

1) A crystal consist of atoms which may be regarded as identical and independent harmonic oscillator

2) A solid consisting of N atoms is equivalent to 3 N one-dimensional harmonic oscillator

3) All the oscillators vibrate with the same natural frequency due to the identical environment of each.

4) The oscillators are quantum oscillators and have discrete energy

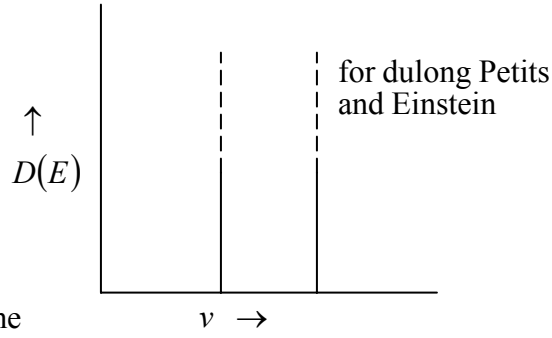
5) Any number of oscillators may be present in the same quantum state.

6) The atomic oscillators form an assembly of system which are distinguishable or identifiable due to their location at separate and distinct lattice state and hence obey the Maxwell-Boltzmann distribution of energy.

To calculate the average energy of an oscillator, we replace integration by summation in expression for the m-B distribution of energy and obtain.

$$\langle E\rangle = \frac{\sum_{n=0}^{\infty} \in_n e^{-\beta E_n}}{\sum_{n=0}^{\infty} e^{-\beta E_n}} = \bar{E} \quad where \in_n = \left(n+\frac{1}{2}\right)\hbar\omega_o, \quad \beta = \frac{1}{k_B T}$$

$$\langle E\rangle = \frac{1}{2}\hbar\omega_o + \frac{\hbar\omega_o}{\left(e^{\beta\hbar\omega_o}-1\right)}$$



The expression for the internal energy of the crystal become

$$E = 3N\langle E\rangle = \frac{3}{2}N\hbar\omega_o + \frac{3N\hbar\omega_o}{e^{\beta\hbar\omega_o}-1}$$

$$\therefore C_v = \left(\frac{\partial E}{\partial T}\right)_V = 3NK_B \left(\frac{\hbar\omega_o}{k_B T}\right)^2 \frac{e^{\frac{\hbar\omega_o}{k_B T}}}{\left(e^{\beta\hbar\omega_o} - 1\right)^2}$$

Then, $C_V = 3Nk_B \left(\frac{\theta_E}{T}\right)^2 = \frac{e^{\theta_E/T}}{\left(e^{\theta_E/T} - 1\right)^2}$ (let Einstein temperature $\theta_E = \frac{\hbar\omega_o}{k_B}$)

Case-I:

High temperature behavior $k_B T >> \hbar\omega_0$ or $T >> \theta_E$

$$e^{\frac{\hbar\omega_o}{k_B T}} - 1 \cong 1 + \frac{\hbar\omega_o}{k_B T} - 1 = \frac{\hbar\omega_o}{k_B T} \Rightarrow C_V = 3Nk_B \left(\frac{\hbar\omega_o}{k_B T}\right)^2 \frac{\left(1 + \frac{\hbar\omega_o}{k_B T}\right)}{\left(\frac{\hbar\omega_o}{k_B T}\right)^2}$$

$$C_V = 3Nk_B = 3R \qquad \frac{\hbar\omega_o}{k_B T} \to 0 \quad \text{for large T}$$

which is the Dulong and Petits law as obtain from classical theory.

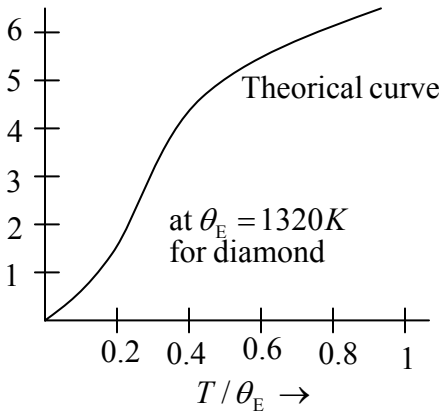
Case-II:

Low temperature behavior $k_B T << \hbar\omega_0$ or $T << \theta_E$

$$e^{\frac{\hbar\omega_o}{k_B T}} - 1 = e^{\frac{\hbar\omega_o}{k_B T}}$$

$$\Rightarrow C_V = 3Nk_B \left(\frac{\theta_E}{T}\right)^2 e^{-\theta_E/T}$$

Thus, for $T << \theta_E$ the heat capacity is proportional to $e^{-\theta_E/T}$ which is the dominating factor. But experimentally it is found to vary as T^3 for most of the solid.



Thus Einstein theory failed to explain actual variation of specific heat.

Debye Theory of Specific Heat

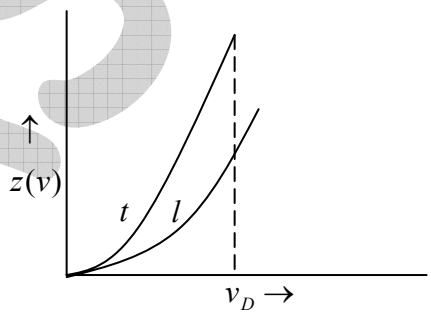
In this model, the vibrational motion of the crystal as a whole was considered to be equivalent to the vibrational motion of system of complete harmonic oscillator which can propagate a range of frequency rather than a single frequency. Debye proposed that crystal can propagate elastic waves of wave lengths ranging from low frequency of (sound wave) to high frequencies corresponding to infrared absorption. This means that a crystal can be a number of modes of vibration. The number of vibration modes per unit frequency range is called density of modes $z(v)$.

Thus, the number of possible modes of vibration is

$$\left(\lambda_n = \frac{2L}{n},\ dx = \frac{2L}{v_s}dv\right)$$

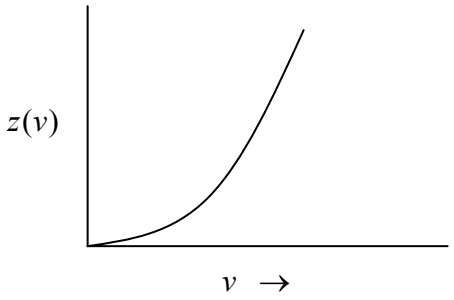
$$z(v)\,dv = \frac{1}{8}\left(4\pi R^2 dR\right) = \frac{1}{8}\times 4\pi \frac{4L^2v^2}{v_s^2}\frac{2L}{v_s}dv \qquad \because R^2 = \frac{4L^2v^2}{v_s^2}$$

$$z(v)\,dv = \left(\frac{4\pi v}{v_s^3}\right)v^2 dv \qquad v = \frac{4}{3}\pi R^3,\ dx = 4\pi R^2 dR$$



In general, the elastic waves propagating in solid are of two types, transverse waves and longitudinal wave

$$z(v)\,dv = 4\pi v\left(\frac{2}{v_t^3} + \frac{1}{v_l^3}\right)v^2 dv$$



For the total number of vibrational modes with frequencies ranging from zero to v_D. We can write

$$\int_0^{v_D} z(v)\,dv = 3N$$

$$\text{or} \int_0^{v_D} 4\pi v\left(\frac{2}{v_t^3} + \frac{1}{v_l^3}\right)v^2 dv = 3N \quad \text{or} \quad 4\pi v\left(\frac{2}{v_t^3} + \frac{1}{v_l^3}\right)\frac{v_D^3}{3} = 3N \qquad \text{(i)}$$

We can associate a Harmonic oscillator of the same frequency with each vibrational mode. Thus, the vibrational energy of the crystal is given by

$E = \int_0^{v_D} \bar{E}z(v)\,dv$ (ii) where $\bar{\epsilon} = \dfrac{hv}{\left(e^{\frac{hv}{k_BT}} - 1\right)}$ (Plank's theory)

Using equation (i) and (ii) $E = 4\pi hv\left(\dfrac{2}{v_t^3} + \dfrac{1}{v_l^3}\right)\int_0^{v_D} \dfrac{v^3 dv}{\left(e^{\frac{hv}{k_BT}} - 1\right)} \Rightarrow E = \dfrac{9Nh}{v_D^3}\int_0^{v_D} \dfrac{v^3 dv}{\left(e^{\frac{hv}{k_BT}} - 1\right)}$

Putting $\dfrac{hv}{k_BT} = x$ and $\dfrac{hv_D}{k_BT} = x_m$, $v = \dfrac{xk_BT}{h}$, $dv = \dfrac{k_BT}{h}dx$

$$E = \frac{9Nh}{v_D^3}\left(\frac{k_BT}{h}\right)^4 \int_0^x \frac{x^3 dx}{\left(e^x - 1\right)} = 9Nk_BT\left(\frac{k_BT}{hv_D}\right)^3 \int_0^{x_m} \frac{x^3 dx}{\left(e^x - 1\right)}$$

Let Debye temperature is $\theta_D = \dfrac{hv_D}{k_B}$, $x_m = \dfrac{\theta_D}{T}$ $\Rightarrow E = 9Nk_BT\left(\dfrac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} \dfrac{x^3 dx}{\left(e^x - 1\right)}$

For more general $E = 3NdkT\left(\dfrac{T}{\theta_D}\right)^d \int_0^{\theta_D/T} \dfrac{x^d dx}{\left(e^x - 1\right)}$

$d = 1$ for $1D$, $d = 2$ for $2D$, $d = 3$ for $3D$

We will discuss for 3 dimension

Case-I: High temperature $T >> \theta_D$ $e^x - 1 \approx x$

$$E = 9Nk_BT\left(\frac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} \frac{x^3 dx}{\left(e^x - 1\right)} \Rightarrow E = 3Nk_BT$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = 3Nk_B = 3R \Rightarrow C_V = 3R$$

Thus, at high temperatures, the Debye's theory also obeys the Dulong and Petit law as obeyed by classical theory and the Einstein theory.

Case-II: Low temperature

For $T << \theta_D$, $x_m = \theta_D / T \to \infty$

$$E = 9Nk_BT\left(\frac{T}{\theta_D}\right)^3 \int_0^\infty \frac{x^3 dx}{\left(e^x - 1\right)} = 9Nk_BT\left(\frac{T}{\theta_D}\right)^3 \frac{\pi^4}{15} \ldots\ldots \int_0^\infty \frac{x^3 dx}{\left(e^x - 1\right)} = \frac{\pi^{n+1}}{1:3:5:\left(2n-1\right)}$$

$$E = \frac{3}{5}\pi^4 Nk_B \frac{T^4}{\theta_D} \qquad \Rightarrow E \propto T^4$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = \frac{12}{5}\pi^4 Nk_B \left(\frac{T}{\theta_D}\right)^3$$

$$C_V = \frac{12}{5}\pi^4 R\left(\frac{T}{\theta_D}\right)^3 \qquad \Rightarrow C_V \propto T^3$$

Thus, at very low temperature, the specific heat is proportional to T^3. This is called the debye T^3 law.

Note: The heat capacity dependency on temperature in 3D, 2D & 1D as follows

$$\left.\begin{aligned} C_V &\propto T^3 \to 3D \\ &\propto T^2 \to 2D \\ &\propto T \to 1D \end{aligned}\right\} \text{ at low temperature}$$

4. Free Electron Theory of Metals

The classical, Einstein, and Debye theories of specific heats of solids apply with equal degrees of success to both metals and nonmetals, which is strange because they ignore the presence of free electrons in metals.

In a typical metal each atom contributes one electron to the common "electron gas," so in 1 kilomole of the metal there are N_0 free electrons. If these electrons behave like the molecules of an ideal gas, each would have $\frac{3}{2}k_BT$ of kinetic energy on the average. The metal would then have

$$E_e = \frac{3}{2}N_0k_BT = \frac{3}{2}RT$$

of internal energy per kilomole due to the electrons. The molar specific heat due to the electrons should therefore be

$$c_{Ve} = \left(\frac{\partial E_e}{\partial T}\right)_V = \frac{3}{2}R$$

and the total specific heat of the metal should be

$$c_V = 3R + \frac{3}{2}R = \frac{9}{2}R$$

at high temperature where a classical analysis is valid. Actually, of course, the Dulong-Petit value of $3R$ holds at high temperatures, from which we conclude that the free electrons do not in fact contribute to the specific heat.

Electrons are fermions and obey Fermi-Dirac statistics with average energies $\bar{\epsilon} = \frac{1}{2}kT$ per degree of freedom at "high" temperature, how high is high enough for classical behaviour is not necessarily the same for the two kinds of systems in a metal.

The distribution function that gives the average occupancy of a state of energy ϵ in a system of fermions is $f_{FD}(\epsilon) = \frac{1}{e^{(\epsilon-\epsilon_F)/kT}+1}$ (i)

where $f_{FD}(\epsilon)$ is average occupancy per state.

What we also need is an expression for $g(\in)d\in$, the number of quantum states available to electrons with energies between $\in$ and $\in+d\in$.

The number of standing waves in a cubical cavity L on a side is

$$g(f)dj=\pi j^2dj \qquad \text{(ii)}$$

where $j=2L/\lambda$ and λ is its de Broglie wavelength of $\lambda=\frac{h}{p}$. Electrons in a metal have nonrelativistic velocities.

$$j=\frac{2L}{\lambda}=\frac{2Lp}{h}=\frac{2L\sqrt{2m\in}}{h} \qquad dj=\frac{L}{h}\sqrt{\frac{2m}{\in}}d\in$$

Using these expressions for j and dj in equation (ii) gives

$$g(\in)d\in=\frac{8\sqrt{2}\pi L^3m^{3/2}}{h^3}\sqrt{\in}\,d\in$$

As in the case of standing waves in a cavity the exact shape of the metal sample does not matter, so we can substitute its volume V for L^3 to give

$$g(\in)d\in=\frac{8\sqrt{2}\pi Vm^{3/2}}{h^3}\sqrt{\in}\,d\in \qquad \text{(iii)}$$

where $g(\in)d\in$ is the number of energy states in energy range $\in$ and $\in+d\in$.

Fermi Energy

We can find Fermi energy $\in_F$ by filling up the energy states in the metal sample at $T=0$ with the N free electrons it contains in order of increasing energy starting from $\in=0$. The highest state to be filled will then have the energy $\in=\in_F$ by definition. The number of electrons that can have the same energy $\in$ is equal to the number of state that have this energy, since each state is limited to one electron. Hence

$$N=\int_0^{\in_F}g(\in)=\frac{8\sqrt{2}\pi Vm^{3/2}}{h^3}\int_0^{\in_F}\sqrt{\in}d\in=\frac{16\sqrt{2}\pi Vm^{3/2}}{3h^3}\in_F^{3/2}$$

and so $$\in_F=\frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3} \qquad \text{(iv)}$$

The number of electrons in an electron gas that have energies between $\in$ and $\in+d\in$

$$n(\in)d\in = g(\in)f(\in)d\in = \frac{\left(8\sqrt{2}\pi Vm^{3/2}/h^3\right)\sqrt{\in}d\in}{e^{(\in-\in_f)/hT}+1} \qquad \text{(v)}$$

If we express the numerator of equation (v) in terms of the Fermi energy $\in_F$ we get

$$n(\in)d\in = \frac{\left(\frac{3N}{2}\right)\in_F^{\frac{-3}{2}}\sqrt{\in}d\in}{e^{(\in-\in_F)/hT}+1} \qquad \text{(vi)}$$

It is interesting to determine the average electron energy at $0K$. To do this, we first find the total energy E_0 at $0K$, which is $E_0 = \int_0^{\in_F} \in n(\in)d\in$ since at $T=0K$ all the electrons have energies less than or equal to the Fermi energy $\in_F$,

$$E_0 = \frac{3N}{2}\in_F^{\frac{-3}{2}}\int_0^{\in_F}\in^{\frac{3}{2}}d\in = \frac{3}{5}N\in_F \qquad \because e^{(\in-\in_F)/kT} = e^{-\infty} = 0$$

The average electron energy $\bar{\in}_0$ is this total energy divided by the number N of electrons present, which gives $\bar{\in}_0 = \frac{3}{5}\in_F$ (vii)

Since Fermi energies for metals are usually several electron volts, the average electron energy in them at $0K$ will also be of this order of magnitude. The temperature of an ideal gas whose molecules have an average kinetic energy of $1\,eV$ is $11,600\,K$. If free electrons behaved classically, a sample of copper would have to be at a temperature of about $50,000K$ for its electrons to have the same average energy they actually have at $0K$.

The failure of the free electrons in a metal to contribute appreciably to its specific heat follows directly from their energy distribution-those within about kT of the Fermi energy-are because the states above them are already filled. It is unlikely that an electron with, say, an energy $\in$ that is $0.5eV$ below $\in_F$ can leapfrog the filled states above it to the nearest vacant state when kT at room temperature is $0.025\,eV$ and even at $500\,K$ is only $0.043\,eV$.

The specific heat of the electron gas in a metal is given by $c_{Ve} = \frac{\pi^2}{2}\left(\frac{kT}{\in_F}\right)R$

The coefficient of R is very much smaller than the classical figure of $\frac{3}{2}$. The dominance of the atomic specific heat c_V in a metal over the electronic specific heat is pronounced over a wide temperature range. However, at very low temperature c_{Ve} becomes significant because c_V is then approximately proportional to T^3 whereas c_{Ve} is proportional to T. At very high temperature c_V has leveled out at about $3R$ while c_{Ve} has continued to increase, and the contribution of c_{Ve} to the total specific heat is then detectable.

Example: Find the Fermi energy in copper on the assumption that each copper atom contributites one free electron to the electron gas. (This is a reasonable assumption sice, a copper atom has a single $4s$ electron outside closed inner shells). The density of copper is $8.94\times10^3\ kg/m^3$ and its atomic mass is $63.5\ u$.

Solution: The electron density $\frac{N}{V}$ in copper is equal to the number of copper atoms per unit volume. Since $1u = 1.66\times10^{-27}\ kg$,

$$\frac{N}{V} = \frac{\text{atoms}}{\text{m}^3} = \frac{\text{mass/m}^3}{\text{mass/atom}} = \frac{8.94\times10^3\ kg/m^3}{(63.5\ u)\times(1.66\times10^{-27}\ kg/u)}$$

$$= 8.48\times10^{28}\ \text{atoms/m}^3 = 8.48\times10^{28}\ \text{electrons/m}^3$$

The corresponding Fermi energy is,

$$\in_F = \frac{(6.63\times10^{-34}J.s)^2}{(2)(9.11\times10^{-31}\ \text{kg/electron})}\left[\frac{(3)(8.48\times10^{28}\text{electrons/m}^3)}{8\pi}\right]^{\frac{2}{3}}$$

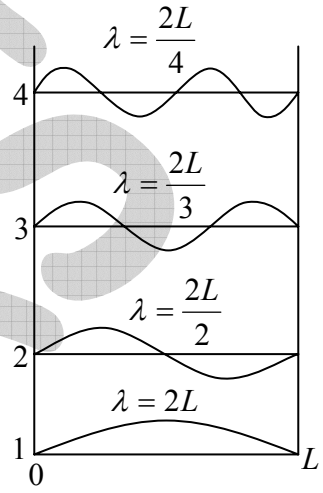
$$= 1.13\times10^{-18}J = 7.04\ eV$$

4.3 Density of States

The density of states of a system describes the number of states per interval of energy at each energy level that are available to be occupied.

Density of states in 1-Dimension

Consider a particle of mass m confined to a line of length L. The energy levels are labeled according to the quantum number n which gives the number of half wavelength in the wavefunction.



For n^{th} mole the wavelength is $\lambda_n = \frac{2L}{n}$.

This can be expressed in term of wave vector $K = \frac{2\pi}{\lambda}$

$$\therefore K_n = \frac{n\pi}{L} \qquad n = 1, 2, 3, ...$$

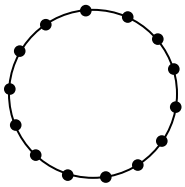
The number of K - states in interval K and $K + dK$ is

$$g(K)\,dK = \frac{(K + dK) - K}{\frac{\pi}{L}} = \frac{L}{\pi} dK \Rightarrow g(K)\,dK = \frac{L}{\pi} dK$$

The density of state is $\rho(K) = \frac{g(K)\,dK}{dK} = \frac{L}{\pi}$

Second Method

The wave function of trapped in one dimensional unbounded medium of periodic boundary condition with period $\psi(x) = Ae^{ikx}$



Since wave function is periodic $\therefore \quad \psi(x) = \psi(x + L)$

$$\Rightarrow \qquad Ae^{ikx} = Ae^{ik(x+L)} \Rightarrow \qquad e^{ikL} = 1$$

$$\Rightarrow \qquad e^{ikL} = e^{i2n\pi} \qquad \text{where } n = 0, \pm 1, \pm 2, \pm 3....$$

$$\therefore \quad K = \pm \frac{2n\pi}{L} \qquad \text{where } n = 0, 1, 2, 3....$$

The number of K-states in range K and $K+dK$ is

$$g(K)dK = 2\times\frac{(K+dK)-K}{\left(\frac{2\pi}{L}\right)} \Rightarrow g(K)dK = \frac{L}{\pi}dK$$

The density of state is $\rho(K) = \frac{g(K)dK}{dK} = \frac{L}{\pi}$

Density of States in 2-Dimension

The wave function of particle trapped in two dimensional unbounded medium of periodic boundary conditions with period of L in both x and y dimensions is

$$\psi(x,y) = Ae^{i\vec{k}.\vec{r}} = Ae^{i\left(K_x x+K_y y\right)} = Ae^{lK_x x}e^{iK_y y} = \psi(x)\psi(y)$$

where $\vec{K} = K_x\hat{i} + K_y\hat{j}$, $\vec{r} = x\hat{i} + y\hat{j}$ and $\psi(x) = A_1 e^{ik_x}$ and $\psi(y) = A_2 e^{ik_y}$.

Since wave function is periodic in x & y direction

$\therefore\ \psi(x) = \psi(x+L)$ and $\psi(y) = \psi(y+L)$

This gives $K_x = \pm\frac{2n\pi}{L}$ and $K_y = \pm\frac{2n\pi}{L}$, $n = 0,1,2,3....$

The possible values of K - states in 2-dimensional K-space can be represented as

The number of K-states in range K and $K+dK$ is

$$g(K)dK = \frac{\pi(K+dK)^2 - \pi K^2}{\left(\frac{2\pi}{L}\right)^2}$$

$$= \left(\frac{L}{2\pi}\right)^2\left[\pi\left(K^2 + dK + 2KdK - K^2\right)\right]$$

$$g(K)dK = \left(\frac{L}{2\pi}\right)^2 2\pi KdK$$

The density of state is

$$\rho(K) = \frac{g(K)dK}{dK} = \left(\frac{L}{2\pi}\right)^2 2\pi K$$

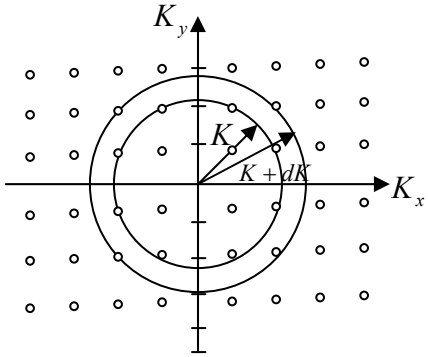


Figure: Draw two circles of radius $K+dK$ with dK of extremely small width

Density of States in 3-Dimension

The wavefunction of particle trapped in 3-dimensional unbounded medium of periodic boundary conditions with period L is

$$\psi(x,y,z) = Ae^{i\vec{k}.\vec{r}} = Ae^{i(K_x x + K_y y + K_z z)} = \psi(x)\psi(y)\psi(z)$$

where $K = K_x\hat{i} + K_y\hat{j} + K_z\hat{k}$, $r = x\hat{i} + y\hat{j} + 3\hat{k}$, $\psi(x) = A_1 e^{ik_x}$, $\psi(y) = A_2 e^{ik_y}$ and $\psi(z) = A_3 e^{ik_z}$

For periodicity

$$\psi(x) = \psi(x+L),\ \psi(y) = \psi(y+L)\ \ \&\psi(z) = \psi(z+L)$$

This gives

$$K_x = \pm\frac{2n\pi}{L};\qquad n = 0,1,2,3...$$

$$K_y = \pm\frac{2n\pi}{L};\qquad n = 0,1,2,3....$$

$$\&\qquad K_z = \pm\frac{2n\pi}{L};\quad n = 0,1,2,3....$$

There values can be represented as in figure.

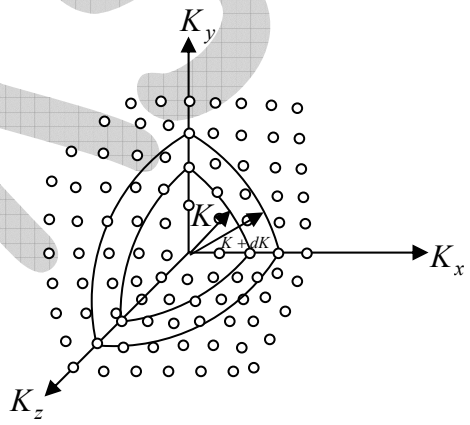


Fig: Draw two spheres of radius K and $K+dK$ with dK of extremely small width

The number of K-states in range K and $K+dK$ is

$$g(K)dK = \frac{\frac{4\pi}{3}(K+dK)^3 - \frac{4\pi}{3}K^3}{\left(\frac{2\pi}{L}\right)^3}$$

$$g(K)dK = \left(\frac{L}{2\pi}\right)^3 \frac{4\pi}{3}\left[K^3 + dK^3 + 3K^2dK + 3KdK^2 - K^3\right]$$

$$g(K)dK \cong \left(\frac{L}{2\pi}\right)^3 4\pi K^2 dK$$

The density of state is $\rho(K)dK = \frac{g(K)dK}{dK} \Rightarrow \rho(K)dK = \left(\frac{L}{2\pi}\right)^3 4\pi K^2$

5. Origin of Energy Bands

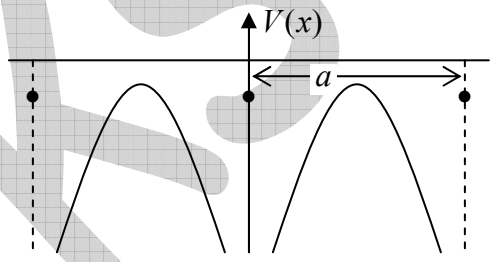
The failure of the free electron model is due to the over simplified assumption that a conduction electron in a metal experiences a constant or zero potential due to the ion cores and hence is free to move about crystal.

Now the periodic potential described below forms the basis of the band Theory of solids. The behavior of an electron in this potential is describe by constructing the electron wave functions using one-electron approximates.

As we shall discuss later, the motion of an electron in a periodic potential yields the following results.

(a) There exist allowed energy bands separated by forbidden region or band gap.

(b) The electronic energy function E(K) is periodic in the wave vector K



In the free electron Theory E varies with K

$$E_K = \frac{\hbar^2 k^2}{2m}$$

The Block Theorem

The 1-D Schrödinger equation for an electron moving in a constant potential V_0 is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0. \qquad \text{The solution } \psi(x) = e^{\pm ikx}$$

For periodic potential with period equal to the lattice constant a we have

$$\psi(x) = e^{\pm ikx}\, U_k(x) \quad \text{where } U_k = U_k\,(x + a)$$

Note: Let g(x) and f(x) be two real and independent solution to the second order differential The general equation can be written as $\psi(x) = A\,f(x) + B\,g(x)$

while $f(x)\frac{dg(x)}{dx} - g(x)\frac{df(x)}{dx}$ = constant

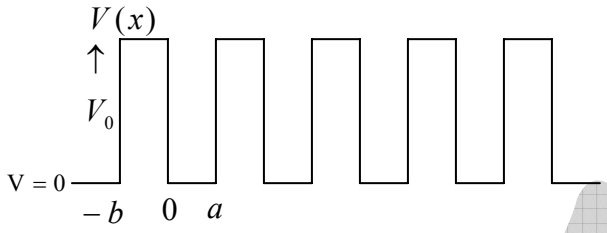
In three dimensions, the block Theorem is expressed as

$$\psi_k(r) = e^{ik\cdot r}u_k(r), \quad \psi_k(r) = e^{ikr}$$

Thus the wave function becomes the one of a free electron.

The Kronig-Penney Model

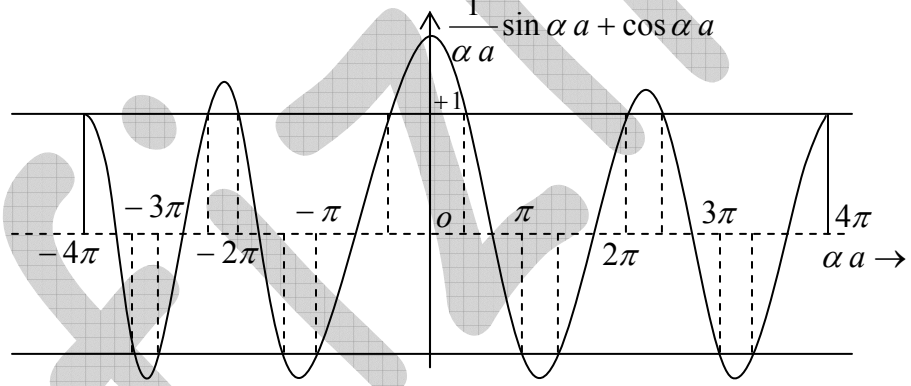
This model illustrates the behaviour of electrons in a periodic potential by assuming a relatively simple one-dimensional model of periodic potential as shown figure.



For this potential write down the Schrödinger wave equation and its general solution with taking potential constant finally. We get

$$P \cdot \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka \qquad \text{where } P = \frac{mV_0 ba}{\hbar^2}$$

which is a measure of the area $V_0 b$ of the potential barrier. Thus increasing P has the physical meaning of bonding an electron more strongly to a particular potential well



We know that $E = \frac{\alpha^2 \hbar^2}{2m} = \frac{\pi^2 \hbar^2}{2ma^2} n^2$

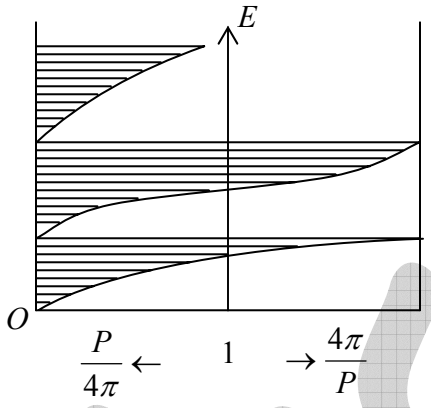
It may be noted that since α^2 is proportional to the energy E the abscissa is a measure of the energy. The following conclusion may be drawn from figure.

(i) The energy spectrum of the electrons consists of alternate regions of allowed energy bands (solid lines on abscissa) and forbidden energy band (broken lines)

(ii) The width of the allowed energy bands increases with αa or the energy

(iii) The width of particular allowed energy band decreases with increase in value of P.

Figure- Allowed (shaded) and forbidden (open) energy ranges as a function of P



Energy Verses Wave-Vector relationship

The energy E is also an even periodic function of k with period of 2π/a i.e k = ± nπ/a

$$dn = \frac{l}{2\pi} dk$$

Velocity is $v = \frac{1}{\hbar}\frac{dE}{dk}$.

5.1 Concept Effective Mass and Holes

In one dimension, an electron with wave-vector k has group velocity

$$v = \frac{d\omega}{dk} = \frac{1}{\hbar}\frac{dE}{dk} \qquad \text{(i)}$$

If an electric field ε acts on the electron, then in time δt. It will do work

$$\delta E = \text{force times distance} = -e\varepsilon v\delta t \qquad \text{(ii)}$$

But $$\delta E = \frac{dE}{dk}\delta k = \hbar v \delta t . \qquad \text{(iii)}$$

So, comparing equation (ii) with (iii), we have

$$\delta k = -\frac{e\varepsilon}{\hbar}\delta t , \quad \text{or} \quad \hbar\frac{dk}{dt} = -e\varepsilon \qquad \text{(iv)}$$

In terms of force F, $\hbar\frac{dk}{dt} = F$

Generalising to three dimensions: $v = \frac{1}{\hbar}\nabla_k E$

where $\nabla_k = \hat{i}\frac{d}{dk_x} + \hat{j}\frac{d}{dk_y} + \hat{k}\frac{d}{dk_z}$ and $\hbar\frac{dk}{dt} = F$

From equation (i) $v = \frac{d\omega}{dk} = \frac{1}{\hbar}\frac{dE}{dk}$,

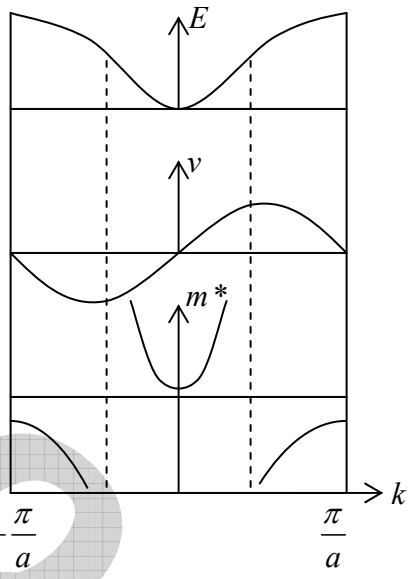
Differentiating with respect to time $\frac{dv}{dt} = \frac{1}{\hbar}\frac{d^2E}{dkdt} = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt}$

But from equation (iv), $\hbar\frac{dk}{dt} = F$

So $\frac{dv}{dt} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}F$

But from Newton's equation we expect $\frac{dv}{dt} = \frac{1}{m}F$

which leads us to define an effective mass $\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}$



That is

The dynamics of electrons is modified by the crystal potential;

The effective mass depends on the curvature of the bands;

Flat bands have large effective masses;

Near the bottom of a band, m^* is positive, near the top of a band, m^* is negative.

Hole Concept

Hole wavevector: The total k of a full band is zero: if we remove an electron with wavevector k_e the total k of the band is $k_h + k_e = 0 \Rightarrow k_h = -k_e$

Hole energy: Take the energy zero to be the top of the valence band. The lower the electron energy, the more energy it takes to remove it; thus

$$E_h(k_h) = -E_e(k_e)$$

But bands are usually symmetric, $E(k) = E(-k)$

So $E_h(k_h) = E_h(-k_h) = -E_e(k_e)$

Hole velocity: In three dimensions $v_h = \frac{1}{\hbar}\nabla_{k_h} E_h$

But $k_h = -k_e$ So $\nabla_{k_h} = -\nabla_{k_e}$

and so $v_h = -\frac{1}{\hbar}\nabla_{k_e}(-E_e) = v_e$

The group velocity of the hole is the same as that of the electron.

Hole effective mass: The curvature of E is just the negative of the curvature of $-E$,

So $m_h^* = -m_e^*$

Note that this has the pleasant effect that if the electron effective mass is negative, as it is at the top of the band, the equivalent hole has a positive effective mass.

Hole dynamics

We know that $\hbar\frac{dk_e}{dt} = -e(\varepsilon + v_e \times B)$,

Substituting $k_h = -k_e$ and $v_h = v_e$ gives

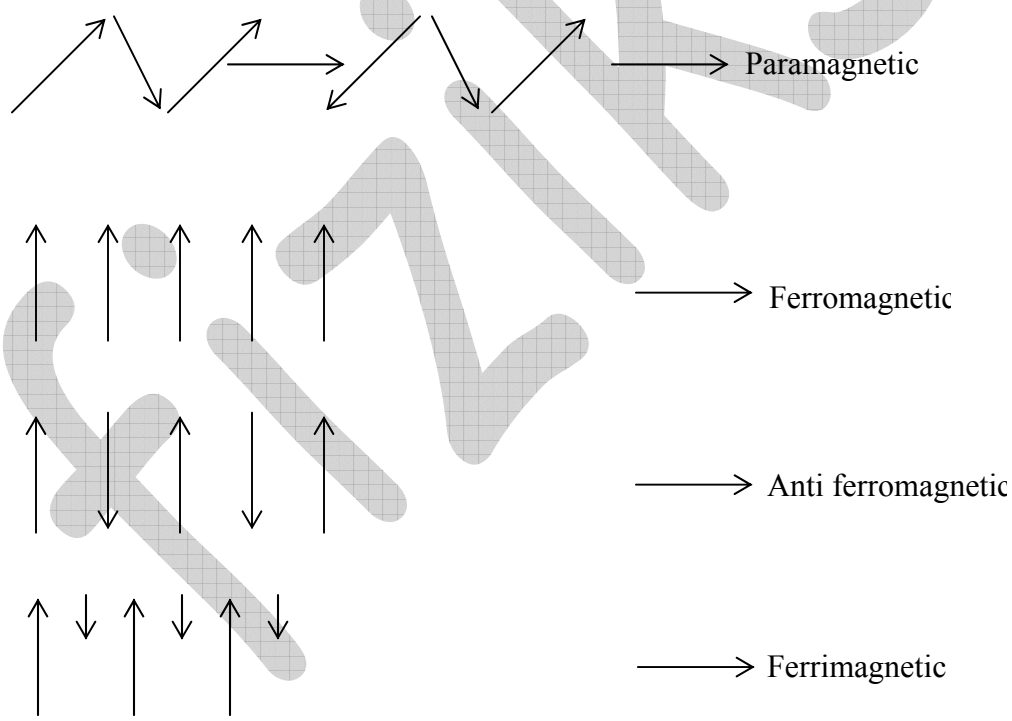
$$\hbar\frac{dk_h}{dt} = e(\varepsilon + v_h \times B)$$

Exactly the equation of motion for a particle of positive charge.

Under an electric field, electrons and holes acquire drift velocities in opposite directions, but both give electric current in the direction of the field.

6. Elementary Ideas about Dia-, Para- and Ferromagnetism

Materials which look permanent magnetic dipole are called diamagnetic if there are permanent magnetic dipoles associated with the atoms in a material such a material may be paramagnetic ferromagnetic, anti-ferromagnetic, depending on the interaction between the individual dipoles. Thus if the interaction between the atomic permanent dipole moment is zero (or negligible), a material will be paramagnetic. If the dipoles interact in such a manner that they tend to line up in parallel, the material will be ferromagnetic, if the neighbouring dipoles tend to line up so that they are anti-parallel the material is anti-ferromagnetic or ferromagnetic, depending on the magnitudes of the dipoles on the two "sub lattice" as indicated schematically for a one-dimensional model in figure shows:



In case of antiferromagnetic configuration that net magnetic moment is zero, because the spins of neigbhouring dipoles are equal and opposite whereas in ferrimagnetic the spins of neighbouring dipoles are opposite and unequal in magnitude, thus in this case there may be relatively large net magnetization.

The magnetic materials can also be classified on the basis of the behaviour when these materials are placed in a magnetic field.

Magnetic Properties

There are two fields to consider:

The magnetic field $\vec{H}$ which is generated by currents according to Ampere's law. $\vec{H}$ is measured in Am^{-1} (Oersteds in old units)

The magnetic induction, or magnetic flux density, $\vec{B}$ which gives the energy of a dipole in a field, $\vec{E} = -\vec{m} \cdot \vec{B}$ and the torque experienced by a dipole moment $\vec{m}$ as $\vec{G} = \vec{m} \times \vec{B}$. $\vec{B}$ is measured in $Wb\,m^{-2}$ or T (Gauss in old units).

In free space, $\vec{B} = \mu_0 \vec{H}$

In a meterial $\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right) = \mu_0 \mu_r \vec{H}$

where μ_r is the relative permeability, χ is the magnetic susceptibility, which is a dimensionless quantity

Note, through that χ is sometimes tabulated as the molar susceptibility $\chi_m = V_m \chi$

where V_m is the volume occupted by one mole, or as the mass susceptibility $\chi_g = \frac{\chi}{\rho}$,

where ρ is the density.

M, the magnetisation, is the dipole moment per unit volume.

$$M = \chi H$$

In general μ_r (and hence χ) will depend on position and will be tensors (so that B is not necessarily parrallel to H)

6.1 Diamagnetism

Classically, we have Lenz's law, which states that the action of a magnetic field on the orbital motion of an electron causes a back-emf which opposes the magnetic field which causes it.

Frankly, this is an unsatisfactory explanation, but we cannot do better until we have studied the inclusion of magnetic fields into quantum mechanics using magnetic vector potentials.

Imagine an electron in an atom as a charge e moving clockwise in the x-y plane in a circle of radius a, area A, with angular velocity ω.

This is equivalent to a current $I = \text{charge/time} = e\omega/(2\pi)$,

so there is a magnetic moment $\mu = IA = e\omega a^2/2$

The electron is kept in this orbit by a central force $F = m_e\omega^2 a$

Now if a flux density B is applied in the z - direction there will be a Lorentz force giving an additional force along a radius $\Delta F = evB = e\omega aB$

If we assume the charge keeps moving in a circle of the same radius it will have a new angular velocity ω',

$m_e\omega'^2 a = F - \Delta F$ so $m_e\omega'^2 a = m_e\omega^2 a - e\omega aB$, or $\omega'^2 - \omega^2 = -\dfrac{e\omega B}{m_e}$

If there change in frequency is small we have $\omega'^2 - \omega^2 \approx 2\omega\Delta\omega$ where $\Delta\omega = \omega' - \omega$.

Thus $\Delta\omega = -\dfrac{eB}{2m_e}$ where $\dfrac{eB}{2m_0}$ is called the Larmor frequency.

Substituting back into $\mu = IA = e\omega a^2/2$,

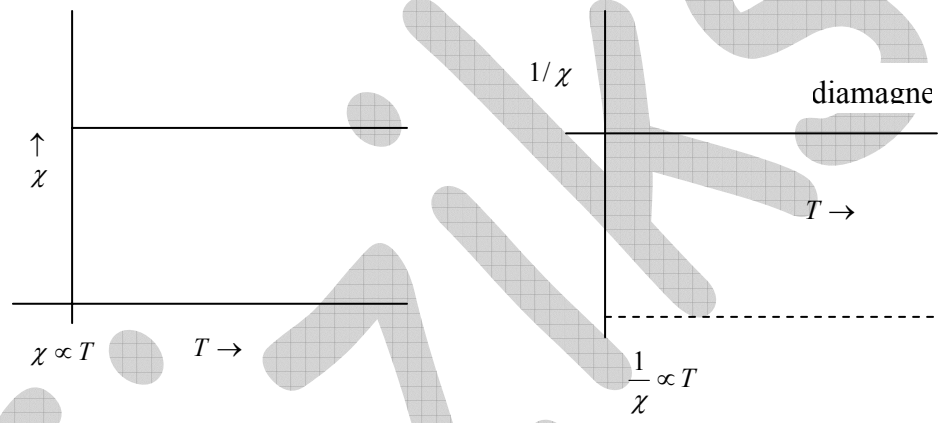
we find a change in magnetic moment $\Delta\mu = -\dfrac{e^2a^2}{4m_e}B$.

Recall that a was the radius of a ring of current perpendicular to the field: if we average over a spherical atom

$$a^2 = \langle x^2 \rangle + \langle y^2 \rangle = \frac{2}{3}\left[\langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle\right] = \frac{2}{3}\langle r^2 \rangle, \qquad \text{so} \qquad \Delta\mu = \frac{e^2 \langle r^2 \rangle}{6m_e} B .$$

If we have n atoms per volume, each with p electrons in the outer shells, the magnetisation will be

$$M = np\Delta\mu \qquad \text{and} \qquad \chi = \frac{M}{H} = \mu_0 \frac{M}{B} = -\frac{\mu_0 npe^2 \langle r^2 \rangle}{6m_e}$$



Range: $-1 \le \chi \le 0 \approx -10^{-5}\,(-ve)$

Note: $\chi = 0$ only for vacuum.

Diamagnetic susceptibility:

- Negative
- Typically -10^{-6} to -10^{-5}
- Independent of temperature
- Always present, even when there are no permanent dipole moments on the atoms.

6.2 Paramagnetism

Paramagnetism occurs when the material contains permanent magnetic moments. If the magnetic moments do not interact with each other, they will be randomly arranged in the absence of a magnetic field.

When a field is applied, there is a balance between the internal energy trying to arrange the moments parallel to the field and entropy trying to randomize them.

The magnetic moments arise from electrons, but if we they are localized at atomic sites we can regard them as distinguishable, and use Boltzmann statistics.

Paramagnetism of spin - $\frac{1}{2}$ ions:

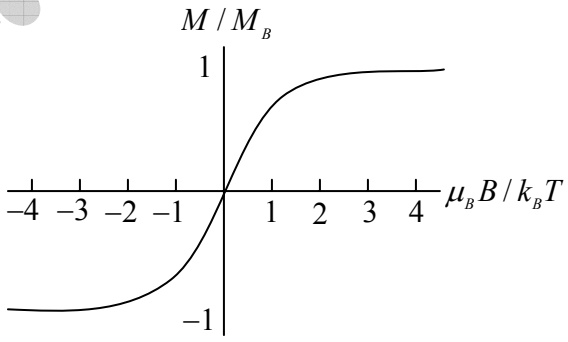
The spin is either up or down relative to the field, and so the magnetic moment is either $+\mu B$ or $-\mu B$, where $\mu B = \frac{e\hbar}{2m_e} = 9.274\times10^{-24}\ Am^2$.

The corresponding energies in a flux density B are $-\mu_B B$ and $\mu_B B$, so the average magnetic moment per atom is $\langle\mu\rangle = \frac{\mu_B e^{\mu_B B/k_B T} - \mu_B e^{-\mu_B B/k_B T}}{e^{\mu_B B/k_B T} + e^{-\mu_B B/k_B T}} = \mu_B \tan\left(\frac{\mu_B B}{k_B T}\right)$

For small z, $\tanh z \approx z$, so for small fields or high temperature $\langle\mu\rangle \approx \frac{\mu_B^2 B}{k_B T}$

If there are n atoms per volume, then, $\chi = \frac{n\mu_0\mu_B^2}{k_B T}$

Clearly, though, for low T or large B the magnetic moment per atom saturates, as it must, as the largest magnetization possible saturation magnetization has all the spins aligned fully, $M_B = n\mu_B$



Genral J ionic paramagnetism

An atomic angular momentum J made of spin S and orbital angular momentum quantum number L, will have a magnetic moment $g_J \mu_B J$, where gJ is the Lande

g - factor $\quad g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$

The average atomic magnetic moment will be $\langle \mu \rangle = \frac{\sum_{m=-J}^{J} m g_J \mu_B e^{mx}}{\sum_{m=-J}^{J} e^{mx}} \qquad \because x = \frac{g_J \mu_B}{k_B T}$

If we assume that T is large and / or B is small, we can expand the exponential, giving

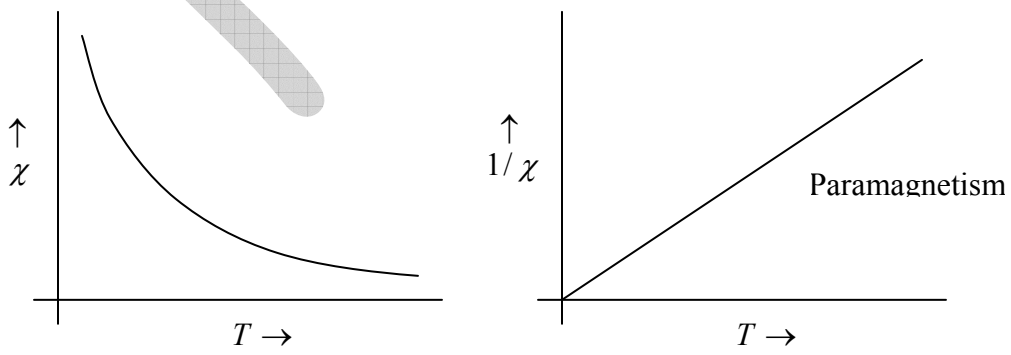
$$\langle \mu \rangle \approx g_J \mu_B \frac{\sum_{m=-J}^{J} m(1+mx)}{\sum_{m=-J}^{J} (1+mx)}$$

Note that $\sum_{m=-J}^{J} J = 2J+1$, $\sum_{m=-J}^{J} m = 0$, $\sum_{m=-J}^{J} m^2 = \frac{1}{3} J(J+1)(2J+1)$

Then $\langle \mu \rangle \approx g_J \mu_B \frac{xJ(J+1)(2J+1)}{3(2J+1)} = \frac{g_J^2 \mu_B^2 J(J+1)}{3k_B T}$

This leads to susceptibility $\quad x = \frac{\mu_0 n_J^2 \mu_B^2 J(J+1)}{3k_B T}$

This is curie's Law, often written $\chi = \frac{C}{T}$ where C is curie's constant.



6.3 Ferromagnetism

Ferromagnetic materials are spontaneously magnetized even in the absence of external magnetic field. All ferromagnetic material becomes paramagnetic above a temperature called Curie temperature T_c; Ferromagnetic is the property of materials to be strongly attracted to a magnetic field and to become a powerful magnet. The source of ferromagnetism namely.

1. Electrons given ferromagnetism must originate from partly filled shells in the free atoms, in order that the energy bands formed from these shells have vacant quantum state available for occupation when the electrons align their spins.
2. The density of state must be high so that the increase in kinetic energy due to the promotion of electrons when these align their spins, shall be smaller than the decrease in energy due to the exchange interaction.

For ferromagnetics the Curie temperature T_C and the constant θ in the Curie-Weiss law are nearly identical. A small difference exists, however, because the transition from ferromagnetism to paramagnetism is gradual.

The most important ferromagnetic materials are Fe, CO, Ni, Gd the ferromagnetic properties of the transition elements Fe, CO and Ni are due to the spins of the inner unpaired electrons. In Fe, CO and Ni the unpaired inner $3d$ electrons are responsible for the ferromagnetism the spins of the $3d$ electrons of adjacent atoms align in a parallel direction by a phenomenon called spontaneous magnetization Ms. The parallel alignment of atomic magnetic domains. The parallel alignment of the magnetic dipoles of atoms of Fe, CO and Ni is due to the creator energy known as exchange interaction.

Using Heitler-London theory of chemical bonding, it can be shown that the total energy of a system of two atoms contains an exchange energy term given by $V_{ij} - 2J_e S_i S_j$.

Where S_i and S_i represent the spins of the two atoms and Je is the exchange integral which is assumed to be the same for any pair of atoms.

Consider the Curie temperature; for $T > T_c$ the spontaneous magnetization is zero and an external field will be have to be applied to produce some magnetization. This field should, however, be weak enough to avoid the saturation state. In such a state, we find

equation $$\chi = \frac{gJ_{\mu B}B_{eff}}{k_1 T} = \frac{gJ_{\mu B}}{kT}\left(B + \lambda M\right) \qquad \text{---------(i)}$$

while $$B_J(\chi) = \frac{2J+1}{2J}\coth h\left(\frac{2JH}{2J}\right)\chi - \frac{1}{2J}\coth h\left(\frac{\chi}{2J}\right)$$

and $B_{eff} = B + B_E = B + \lambda M$ where $M = NgJ_{\mu B}B_J(\chi)$ ----------(ii)

for that $\chi << 1$, $$B_J(\chi) \cong \left(\frac{J+1}{3J}\right)\chi$$

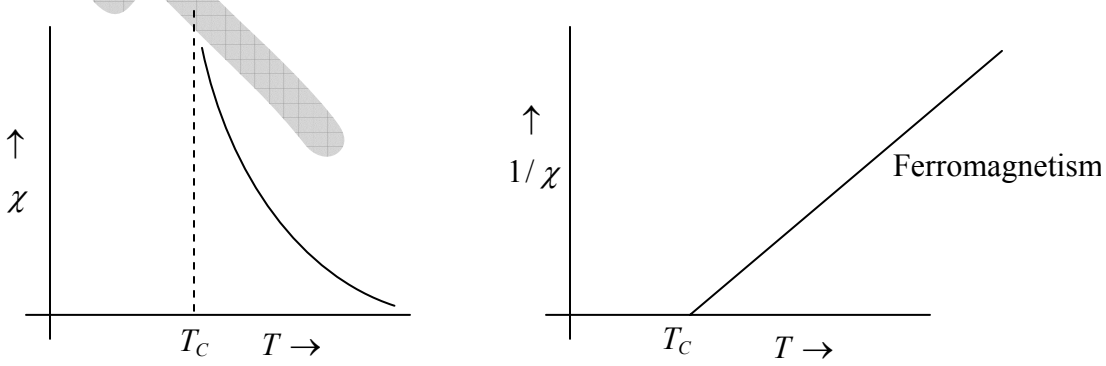
Therefore, the expression (ii) become $M = N_g \mu_B (J+1)\frac{\chi}{3}$

$$\chi = \frac{gJ_B(B+\lambda M)}{kT} \qquad \Rightarrow M = \frac{Ng^2\mu_B^2 J(J+1)}{3kT}(B+\lambda M)$$

which gives $$\chi = \frac{\mu_0 M}{B} = \frac{\mu_0 T_C / \lambda}{T - T_C} = \frac{C}{T - T_C}$$

$$\chi = \frac{C}{T - T_C} \qquad C = \frac{\mu_0 T_C}{\lambda}, \qquad T_C = \frac{\lambda N g^2 \mu_B^2 J(J+1)}{3k}$$

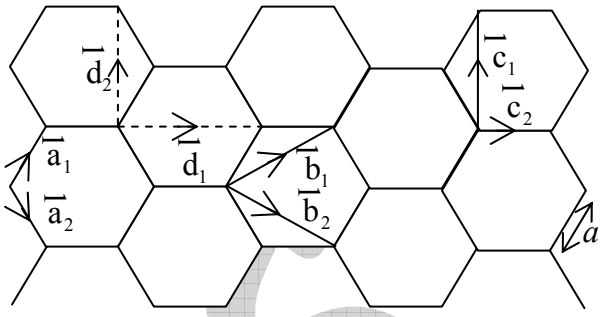
This expression is known as curie-weiss law. It is satisfactory explain the susceptibility and temperature dependence in paramagnetic region.



Range $\chi >> 1 \approx$ very large and $+$ve.

MCQ (Multiple Choice Questions)

Q1. The two dimensional lattice of graphene is an arrangement of Carbon atoms forming a honeycomb lattice of lattice spacing a, as shown below. The Carbon atoms occupy the vertices.



The Wigner-Seitz cell has an area of

(a) $6\sqrt{3}a^2$ (b) $\frac{3\sqrt{3}}{2}a^2$

(c) $2a^2$ (d) $\frac{3\sqrt{3}}{4}a^2$

Q2. A metal with body centered cubic (bcc) structure show the first (i.e. smallest angle) diffraction peak at a Bragg angle of $\theta = 30^o$. The wavelength of X-ray used is 2.1 Å. The volume of the PRIMITIVE unit cell of the metal is

(a) $26.2\left(A^0\right)^3$ (b) $13.1\left(A^0\right)^3$ (c) $9.3\left(A^0\right)^3$ (d) $4.6\left(A^0\right)^3$

Q3. A simple cubic crystal with lattice parameter a_c undergoes transition into a tetragonal structure with lattice parameters $a_t = b_t = \sqrt{2}a_c$ and $c_t = 2a_c$, below a certain temperature. The ratio of the interplanar spacing of (1 0 1) planes for the cubic and the tetragonal structure is

(a) $\sqrt{\frac{1}{6}}$ (b) $\frac{1}{6}$ (c) $\sqrt{\frac{3}{8}}$ (d) $\frac{3}{8}$

Q4. Sodium metal transform from BCC to HCP structure at about 23K. The lattice constant in cubic phase is $4.23A^0$ and assume that the density remains unchanged during the transition and the ratio c/a = $\sqrt{8/3}$. The lattice constant in the hexagonal phase is

(a) a = $2.99A^0$ and c = $4.88A^0$. (b) a = $3.66A^0$ and c = $5.98A^0$

(c) a = $1.49A^0$ and c = $2.44A^0$ (d) a = $1.83A^0$ and c = $2.98A^0$

Q5. The crystal structure of Zinc Sulphide (ZnS) is:

(a) HCP with two atoms basis of (0 0 0) and $\frac{a}{4}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(b) HCP with two atom basis of (0 0 0) and $\frac{a}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(c) FCC with two atoms basis of (0 0 0) and $\frac{a}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(d) FCC with two atom basis of (0 0 0) and $\frac{a}{4}\left(\hat{i}+\hat{j}+\hat{k}\right)$

Q6. Consider *X*-ray diffraction from a crystal with a face-centered-cubic (*fcc*) lattice. The lattice plane for which there is NO diffraction peak is

(a) (2, 1, 2) (b) (1, 1, 1) (c) (2, 0, 0) (d) (3, 1, 1)

Q7. A X-ray of wavelength $1.7A^0$ falls on the CsCl crystal. The atomic scattering factor of Cs and Cl are related as $f_{cs} = 3f_{cl}$. The ratio of the intensity scattered from (100) plane to (110) plane is

(a) $I_1 : I_2 = 3 : 2$ (b) $I_1 : I_2 = 2 : 3$

(c) $I_1 : I_2 = 1 : 4$ (d) $I_1 : I_2 = 4 : 1$

Q8. A narrow electron beam of electrons, accelerated under a potential difference, incident on a crystal whose grating space is 0.3nm.If the 1^{st} diffraction ring is produced at an angle 5.8^0 from the incident beam, then the momentum of the electron in $Kgms^{-1}$ is

(a) 10.8×10^{-26} (b) 10.8×10^{-25}

(c) 10.8×10^{-24} (d) 10.8×10^{-23}

Q9. The Einstein frequency in a cesium solid is $1.724\times10^{12}\ Hz$. The Einstein temperature of assume is

(a) 325 K (b) 165.4 K (c) 82.7 K (d) 330.8 K

Q10. If classical theory is applicable to solid then the thermal energy of 1g - mole of a material at 300 K is

(a) 7470 J (b) 2490 J (c) 4980 J (d) 9960 J

Q11. and inferatemic distance are of the order of $a = 3\times10^{-10}\ m$. The Debye temperature is [Assume cut-off frequency equal to debye frequencies

(a) 120 K (b) 240 K (c) 190 K (d) 360 K

Q12. The Debye temperature of Na metal is 150 K. The specific heat at 10 K is (in $J\ mole^{-1}\ K^{-1}$)

(a) 0.574 (b) 0.287 (c) 0.861 (d) 1.722

Q13. At a very low temperatures, the specific heat of rock salt varies with temperature as $C_V = A\left(\frac{T}{\theta_D}\right)^3$ where $A = 464$ Cal mol^{-1} K^{-1} and $\theta_D = 281K$. The amount of heat (in calorie) required to raise the temperature of $n = 2$ moles rock salt from 10 to 50 K is

(a) 31.6 (b) 94.8 (c) 126.4 (d) 63.2

Q14. The lattice specific heat of KCl at $5K$ is $3.8\times10^{-2}\ J\,mol^{-1}\,K^{-1}$

(a) 2.43×10^{-1} (b) 2.43×10^{-2} (c) 2.43×10^{-3} (d) 2.49×10^{-4}

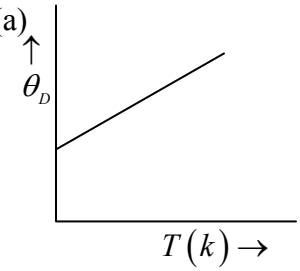
Q15. At low temperature, the Debye temperature θ_D for $NaCl$ and KCl which have the same crystal structure are 330 K and 220 K respectively. The lattice specific heat of Kcl at $5K$ is $3.8\times10^{-2}\ J\,mol^{-1}\,K^{-1}$

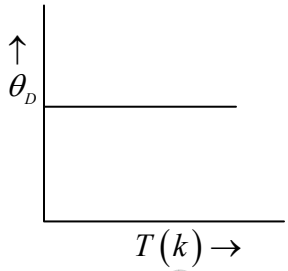
(a) 0.09 (b) 0.18 (c) 0.03 (d) 0.29

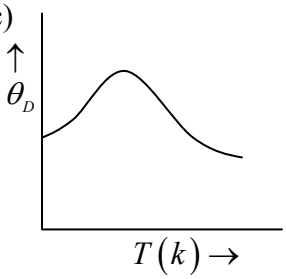
Q16. In the Debye model for a three dimensional crystal the internal energy U at low temperature is given by

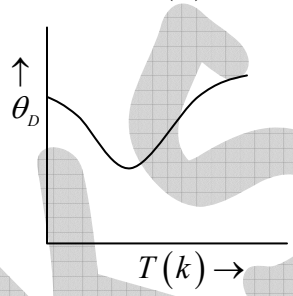
(a) $U \propto T$ (b) $U \propto T^2$ (c) $U \propto T^3$ (d) $U \propto T^4$

Q17. The experiment curve for θ_D as T is

(a)


(b)


(c)


(d)


Q18. Which is incorrect as a Debye -formula?

(a) $C_V = 464.4\left(\frac{T}{\theta_D}\right)^3 \; Cal/mol-K$

(b) $C_V = 1944\left(\frac{T}{\theta_D}\right)^3 \; J/mol-K$

(c) $C_V = \frac{12}{5}\pi^4 R\left(\frac{T}{\theta_D}\right)^3$

(d) $C_V = \frac{5}{12}\pi^4 R\left(\frac{T}{\theta_D}\right)^3$

Q19. Debye theory of specific heat is valid at

(a) room temperature

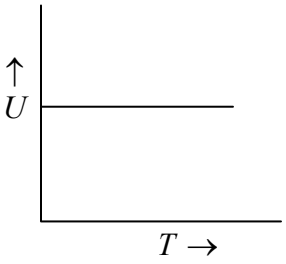
(b) low temperature

(c) intermediate temperature

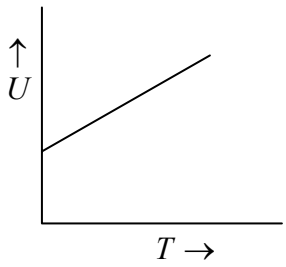
(d) all temperature

Q20. Internal energy U vs T graph for Debye model is

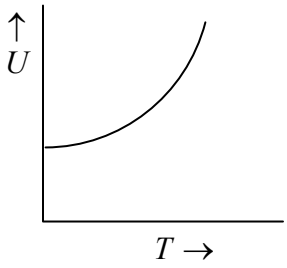
(a)



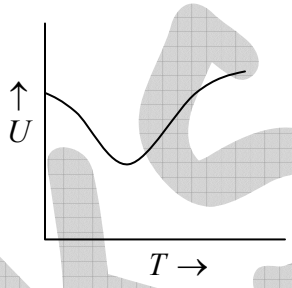
(b)



(c)



(d)



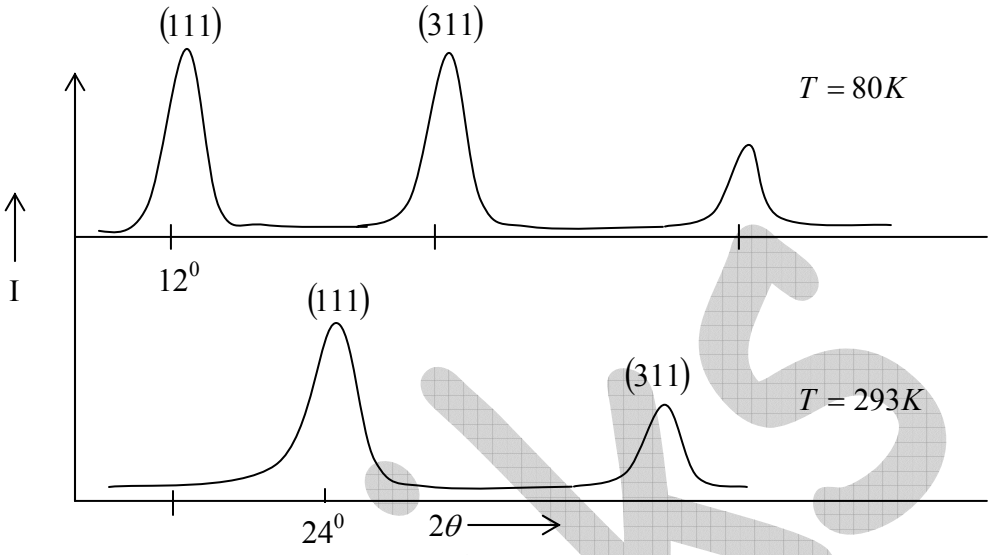
Q21. Iron atoms crystallize in BCC structure with a lattice constant a = 2.86 A^0 . The saturation magnetization of iron is 1.74 x 10^6 Am^{-1}. The value of magnetic dipole moment of an Fe atom in term of Bohr magneton (μ_B) is

(a) 3.1 μ_B (b) 1.1 μ_B (c) 0.45 μ_B (d) 2.2 μ_B

Q22. The paramagnetic susceptibility of Fermi free electron gas with electron concentration n = 4.7 x 10^{28} m^{-3} at T = 300 K is

(a) 2.0 x 10^{-4} (b) 2.0 x 10^{-5} (c) 2.0 x 10^{-6} (d) 2.0 x 10^{-7}

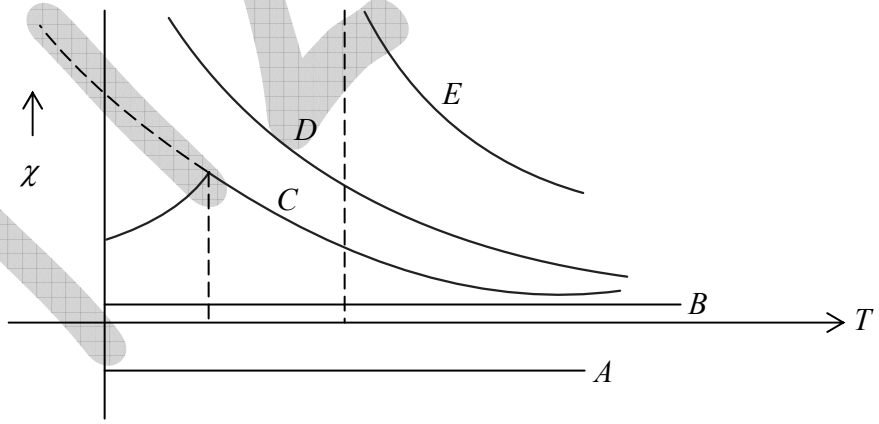
Q23. Following figure displays neutron diffraction data for scattering from a crystal of MnO, which have the same structure as NaCl.



If the lattice constant at 293 K is a = 4.43 A^0, then approximate lattice constant at 80 K is

(a) 4.43 A^0 (b) 2.2 A^0 (c) 8.8 A^0 (d) 11.12 A^0

Q24. Following graph shows the variation of magnetic susceptibility (χ) for different type of magnetic material



Which of the following represent the correct combination?

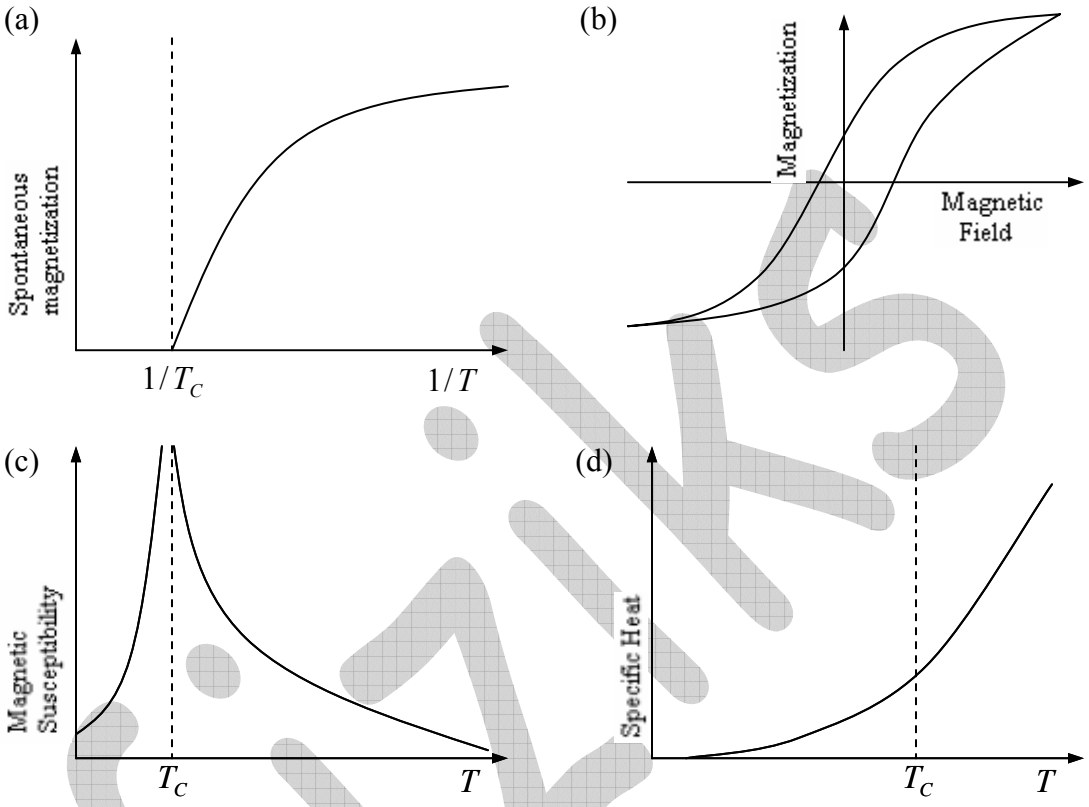
(a) A – Metal, B – Diamagnet, C-Antiferromagnet, D – Ferromagnet and E-Paramagnetic

(b) A – Diamgnet, B – Antiferromagnet, C-Metal, D – Paramagnet, and Ferromagnet.

(c) A – Diamgnet, B – Metal, C- Antiferromagnet, D–Paramagnet, and E - Ferromagnet.

(d) A – Diamgnet, B – Metal, C- Antiferromagnet, D–Paramagnet, and E - Ferromagnet.

Q25. In an experiment involving a ferromagnetic medium, the following observations were made. Which one of the plots does NOT correctly represent the property of the medium? (T_C is the Curie temperature)



Q26. The temperature (T) dependence of magnetic susceptibility (χ) of a ferromagnetic substance with a Curie temperature (T_c) is given by

(a) $\frac{C}{T-T_c}$, for $T < T_c$

(b) $\frac{C}{T-T_c}$, for $T > T_c$

(c) $\frac{C}{T+T_c}$, for $T > T_c$

(d) $\frac{C}{T+T_c}$, for all temperatures

where C is constant .

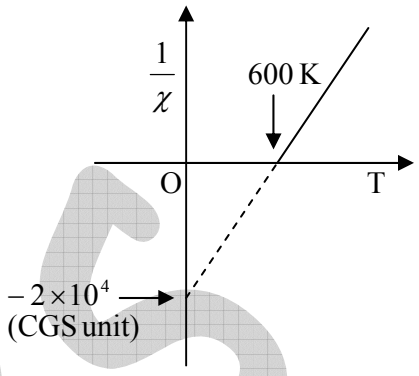
Q27. Inverse susceptibility ($1/\chi$) as a function of temperature, T for a material undergoing paramagnetic to ferromagnetic transition is given in the figure, where O is the origin. The values of the Curie constant, C, and the Weiss molecular field constant, λ, in CGS units, are

(a) $C = 5\times10^{-5}, \lambda = 3\times10^{-2}$

(b) $C = 3\times10^{-2}, \lambda = 5\times10^{-5}$

(c) $C = 3\times10^{-2}, \lambda = 2\times10^{4}$

(d) $C = 2\times10^{4}, \lambda = 3\times10^{-2}$



Q28. The value of magnetic field at which the number of dipoles with m_j = - ½ become twice than that of dipoles with m_j = + ½ at T = 2K is (μ_B = 9.27 x 10^{-24} Am^2)

(a) 2.0 T (b) 2.1 T (c)1.03 T (d) 0.5 T

Q29. A system of spins (J = S = ½) is placed in a magnetic field H = 5 x 10^4 A-m. The fraction of spins parallel to the field at T = 1 K

(a) 52 % (b) 62 % (c) 75 % (d) 95 %

Q30. The Fermi energy of one dimensional free electron gas confined in length L at absolute zero is

(a) $\frac{\hbar^2}{2m}\left(\frac{N\pi}{L}\right)^2$ (b) $\frac{\hbar^2}{2m}\left(\frac{N\pi}{2L}\right)^2$

(c) $\frac{\hbar^2}{2m}\left(\frac{2N\pi}{L}\right)^2$ (d) $\frac{\hbar^2}{2m}\left(\frac{2N\pi}{L}\right)^2$

Q31. Consider a two dimensional electron gas of N electrons of mass m each in a system of size L x L. The Fermi energy of the system of T = 0 K is

(a) $\frac{\hbar^2\pi^2}{2m}\left(\frac{N}{L^2}\right)$ (b) $\frac{\hbar^2\pi^2}{2m}\left(\frac{N}{2L^2}\right)$

(c) $\frac{\hbar^2\pi^2}{2m}\left(\frac{2N}{L^2}\right)$ (d) $\frac{\hbar^2\pi^2}{2m}\left(\frac{N}{\pi L^2}\right)$

Q32. Consider a three dimensional gas of N electrons of mass m each in a system of size L x L x L. The Fermi energy of the system at T = 0 K is

(a) $\frac{\hbar^2}{2m}\left(\frac{3\pi N}{V}\right)^{2/3}$ (b) $\frac{\hbar^2}{2m}\left(\frac{\pi^2 N}{V}\right)^{2/3}$

(c) $\frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$ (d) $\frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{2V}\right)^{2/3}$

Q33. The total ground state kinetic energy of N free electrons gas in one, two and three dimension system is respectively (Fermi energy is E_F)

(a) $\frac{1}{2}NE_F; \frac{1}{3}NE_F$ and $\frac{3}{5}NE_F$ (b) $\frac{2}{3}NE_F; \frac{1}{3}NE_F$ and $\frac{5}{3}NE_F$

(c) $\frac{1}{3}NE_F; \frac{1}{2}NE_F$ and $\frac{3}{5}NE_F$ (d) $\frac{1}{3}NE_F; \frac{3}{2}NE_F$ and $\frac{3}{5}NE_F$

Q34. Sodium atoms crystallic in BCC metal. The atomic radius of sodium is 1.86 Å. The Fermi energy of sodium at 0K is

(a) 5.11 eV (b) 6.01 eV (c) 3.11 eV (d) 4.21 eV

Q35. The density of state at a Fermi level for N electrons gas in one, two and three dimension respective (E_F is the Fermi energy)

(a) $\frac{N}{2E_F}; \frac{N}{E_F}$ and $\frac{3}{2}\frac{N}{E_F}$ (b) $\frac{N}{4E_F}; \frac{N}{E_F}$ and $\frac{3}{2}\frac{N}{E_F}$

(c) $\frac{N}{E_F}; \frac{2N}{E_F}$ and $\frac{3}{}\frac{N}{E_F}$ (d) $\frac{N}{4E_F}; \frac{N}{2E_F}$ and $\frac{3}{2}\frac{N}{E_F}$

Q36. The radius of the Fermi sphere of free electrons in a monovalent one dimensional metal with the unit cell dimension of a is

(a) $\frac{\pi N}{a}$ (b) $\frac{\pi N}{2a}$ (c) $\frac{2\pi N}{a}$ (d) $\frac{\pi N}{3a}$

Q37. The radius of the Fermi sphere of free electrons in a monovalent two dimensional metal with the unit cell dimension of a is

(a) $\sqrt{\frac{\pi N}{a^2}}$ (b) $\sqrt{\frac{\pi N}{2a^2}}$ (c) $\sqrt{\frac{2\pi N}{a^2}}$ (d) $\sqrt{\frac{2\pi N}{a^2}}$

Q38. The radius of the Fermi sphere of free electrons in a monovalent three dimensional metal with the unit cell dimension of a is

(a) $\left(\frac{3\pi^2 N}{a^3}\right)^{1/3}$ (b) $\left(\frac{2\pi^2 N}{a^3}\right)^{1/3}$ (c) $\left(\frac{\pi^2 N}{a^3}\right)^{1/3}$ (d) $\left(\frac{3\pi N}{a^3}\right)^{1/3}$

Q39. The velocity of non-relativistic free electrons in one-dimensional metal, at $T=0$ is (density of electrons is n)

(a) $\frac{\pi\hbar}{m}n$ (a) $\frac{2\pi\hbar}{m}n$ (a) $\frac{\pi\hbar}{3m}n$ (a) $\frac{\pi\hbar}{2m}n$

Q40. The velocity of non-relativistic free electrons in two-dimensional metal, at $T=0$ is (density of electrons is n)

(a) $\frac{\hbar}{m}(2\pi n)^{1/2}$ (b) $\frac{\hbar}{2m}(\pi n)^{1/2}$ (c) $\frac{\hbar}{2m}(2\pi n)^{1/2}$ (d) $\frac{\hbar}{2m}(3\pi n)^{1/2}$

Q41. The velocity of non-relativistic free electrons in three-dimensional metal, at $T=0$ is (density of electrons is n)

(a) $\frac{\hbar}{2m}(3\pi^2 n)^{1/3}$ (b) $\frac{\hbar}{2m}(\pi^2 n)^{1/3}$ (c) $\frac{\hbar}{m}(\pi^2 n)^{1/3}$ (d) $\frac{\hbar}{m}(3\pi^2 n)^{1/3}$

Q42. Fermi energy of copper is 7.0 eV. The electronic heat capacity of one gram mole of Cu at 300 K is (in J $mol^{-1}K^{-1}$)

(a) 1.516 (b) 0.1516

(c) 0.01516 (d) 0.001516

Q43. If the debye temperature of Cu is 343K and Fermi energy 7.0 eV. The temperature of which the electronic and lattice specific heat of Cu is equal, is

(a) 4.2K (b) 9.1K (c) 2.1K (d) 3.24K

Q44. The energy wave vector dispersion relation for one dimensional crystal of lattice constant a using the tight binding model is $E(k) = E_v - \beta - 2\gamma\cos(ka)$. The bottom of the band is located at $k=0$ and is equal to $\mathbf{E_0}$ where $\mathbf{E_0 = E_v - \beta - 2\gamma}$. The effective mass of electron at the bottom of the band is

(a) $\frac{\hbar^2}{2\gamma a^2}$ (b) $\frac{\hbar^2}{\gamma a^2}$ (c) $\frac{\hbar^2}{4\gamma a^2}$ (d) $\frac{\hbar^2}{3\gamma a^2}$

Q45. The energy wave vector dispersion relation for one dimensional crystal of lattice constant a using the tight binding model is $E(k) = E_v - \beta - \gamma\cos(ka)$. The effective mass of electron at the top of the band is

(a) $-\frac{\hbar^2}{3\gamma a^2}$ (b) $-\frac{\hbar^2}{\gamma a^2}$ (c) $-\frac{\hbar^2}{4\gamma a^2}$ (d) $-\frac{\hbar^2}{2\gamma a^2}$

Q46. The energy of an electron in a band as a function of its wave vector k is given by $E(k) = E_0 - 2B(\cos k_x a + \cos k_y a + \cos k_z a)$, where E_0 & B and a are constants. The effective mass of the electron near the top of the band is

(a) $-\frac{2\hbar^2}{3Ba^2}$ (b) $-\frac{\hbar^2}{3Ba^2}$ (c) $-\frac{\hbar^2}{2Ba^2}$ (d) $-\frac{\hbar^2}{Ba^2}$

Q47. A two dimensional metal has one atom of valency in a simple rectangular primitive cell a = $2A^0$; b = $4A^0$. The radius of the free electron Fermi sphere, in cm^{-1} is.

(a) 1.756×10^{-10} (b) 1.756×10^{-8} (c) 0.88×10^{-10} (d) 0.88×10^{-12}

Q48. If C_v and C_p is the heat capacity at constant volume and constant pressure. Which of the following statement is not true about the heat capacity of solid?

(i) For harmonic crystal, $C_p \neq C_v$

(ii) For anharmonic crystal, $C_v = C_p$

(a) only (i) is true while (ii) is false (b) only (ii) is true while one is false

(c) both (i) and (ii) are true (d) neither (i) nor (ii) is true

Q49. The variation The lattice specific heat C C of a crystalline solid can be obtained using the Dulong Petit model, Einstein model and Debye model. At low temperature $h\omega >> k_B T$, which one of the following statements is true (a and A are constants)?

(a) Dulong Petit: $C \propto \exp\left(\frac{-a}{T}\right)$; Einstein: C=constant; Debey: $C \propto \left(\frac{T}{A}\right)^3$

(b) Dulong Petit: C =constant; Einstein: $C \propto \left(\frac{T}{A}\right)^3$; Debey: $C \propto \exp\left(\frac{-a}{T}\right)$

(c) Dulong Petit: C =constant; Einstein: $C \propto \exp\frac{e^{-a/T}}{T^2}$; Debey: $C \propto \left(\frac{T}{A}\right)^3$

(d) Dulong Petit: $C \propto \left(\frac{T}{A}\right)^3$; Einstein: $C \propto \exp\frac{e^{-a/T}}{T^2}$; Debey: C = constant

Q50. A plane makes inter of $5\mathring{A}, 4\mathring{A}$ and $3\mathring{A}$ on the crystallographic axes of orthorhombic crystal whose lattice parameter are in ratio of $a:b:c = 1:2:3$. The Miller indices of this plane is

(a) $(5\ \ 2\ \ 1)$ (b) $(2\ \ 5\ \ 10)$ (c) $(1\ \ 2\ \ 5)$ (d) $(5\ \ 4\ \ 3)$

Q51. The index of line common to the planes (111) and (110) is

(a) $[110]$ (b) $[\bar{1}10]$ (c) $[10\bar{1}]$ (d) $[\bar{1}01]$

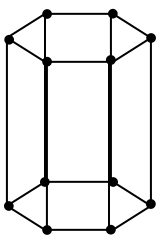
Q52. The packing fraction of simple Hexagonal structure is

(a) 72%

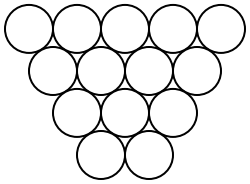
(b) 74%

(c) 40%

(d) 92%



Q53. Two dimensional discs of each radius r is arranged in closely packed arrangement as shown below. The packing fraction of this structure is



(a) $\frac{\pi}{3\sqrt{3}}$ (b) $\frac{\pi}{\sqrt{3}}$

(c) $\frac{\pi}{2\sqrt{3}}$ (d) $\frac{\pi}{4\sqrt{3}}$

Q54. An X- ray diffraction experiment is carried out on a crystalline solid having FCC structure at room temperature. The undergoes a phase transition and shows orthorhombic structure with small decrees in its unit cell length as compared to the FCC unit cell length. As a result (135) line of the XRD pattern corresponding to the FCC system

(a) will split into 5 (b) will split into 6

(c) will split into 8 (d) will split into 9

Q55. The density of α-Iron is 7860 kg/m^3 and atomic weight is 55.85 amu. The radius of atoms in α-iron, which has a BCC structure is

(a) 2.87 $\overset{o}{A}$ (b) 1.24 $\overset{o}{A}$ (c) 2.48 $\overset{o}{A}$ (d) 4.9 $\overset{o}{A}$

Q56. For a cubic crystal the diffraction line from the planes with $h^2+k^2+l^2=8$ is observed at an angle of diffraction 10.23^o. It only one line is observed at an angle lower than this, the crystal structure is

(a) simple cubic (b) BCC

(c) FCC (d) diamond cubic

Q57. Consider the planes with indices (100) and (001); the lattice is FCC, and these Miller indices refers to the conventional unit cell. The primitive cell of the FCC is Trigonal. The indices of these planes when referred to the primitive axes of FCC is

(a) $(101)\&(011)$ (b) $(011)\&(110)$

(c) $(101)\&(110)$ (d) $(101)\&(111)$

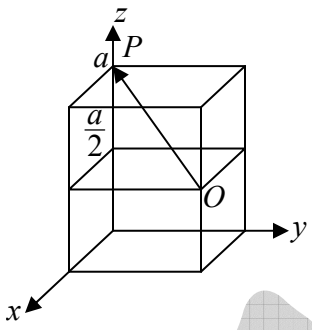
Q58. In crystallographic notation the $\overrightarrow{OP}$ in the cubic cell in the figure is

(a) $[221]$

(b) $[1\bar{2}\bar{2}]$

(c) $[1\bar{2}\bar{1}]$

(d) $[\bar{2}\bar{2}1]$



Q59. A beam of X-rays is incident on a sodium chloride (lattice constant $= 0.282\ nm$). The first-order Bragg reflection is observed at a glancing angle of 8^o. At what angle will be the 2nd order Bragg reflection occur?

(a) 16.16^o (b) 18.2^o (c) 15.1^o (d) 12.12^o

Q60. Electrons are accelerated under $344V$ and then reflected from a crystal. The first reflection maxima occur when the glancing angle is 60^o. The interplanar spacing of the crystal is

(a) $0.285\ \mathring{A}$ (b) $1.14\ \mathring{A}$ (c) $0.76\ \mathring{A}$ (d) $0.38\ \mathring{A}$

NAT (Numerical Answer Type)

Q61. The valence band of a simple cubic metal has formed $E = Ak^2 + B$ where $A = 10^{-38}\ Jm^2$. The value of $\frac{m^*}{m} = ...$

Q62. Assuming the free electron theory, the Fermi energy of Aluminium (FCC) with $a = 4A^o$ is $.....eV$

Q63. In a one-dimensional lattice of interatomic separation of $2.3\overset{0}{A}$, the maximum wavelength of the electron that gets energy reflected is $.....\overset{0}{A}$

Q64. The Fermi energy of metal is $2.1\ eV$, the electron concentration (number of electrons per unit volume) is $.....\times 10^{28}/m^3$

Q65. Fermi energy of Cu metal is 7.05eV, the electronic specific heat of Cu at $600\ K$ is $.....J\left(k\,mol\right)^{-1}K^{-1}$

Q66. The temperature at which there is one percent probability that a state with on energy $0.2\,eV$ above the Fermi energy will be occupied by an electron is $.....K$

Q67. The Fermi energy of silver is $5.51eV$. The average energy of the free electrons in silver at $0K$ is……

Q68. Each copper atom contributes one free electron to the electron gas. The density of copper is $8.94\times 10^3\ kg/m^3$ and its atomic mass is $63.5\ amu$. The Fermi energy in copper is …….. eV

Q69. If the Debye's temperature of a metal is $450\ K$, the Debye's frequency is …. GHz

Q70. The Debye temperature of carbon (diamond) is $1850\,K$. The specific heat per k mol for diamond at $20\ K$ is $.....J\left(k\,mol\right)^{-1}K^{-1}$

Q71. If the classical Theory of lattice specific heat is valid, the thermal energy of one mol of copper at the Debye temperature is $.....J$ (where $\theta_D = 340\ K$)

Q72. The Debye temperature for aluminium as obtained from specific heat measurement is $375\ K$, the Debye frequency for aluminium is $\ldots\ldots \times 10^{12}\ Hz$

Q73. A system of oscillators at temperature $300\ K$ has a certain number of them carrying an energy $E = 10^{-21}\ J$ and a certain number carrying twice this energy. The ratio of the number of oscillators in these two energy states is ……

Q74. A metal with atomic weight $55.85\ amu$ has lattice parameter $0.29\ nm$ and density of $7870\ kg/m^3$. The number of atoms per unit cell is …..

Q75. Copper has FCC structure and the atomic radius is $0.1278\ nm$. The interplanar spacing for (321) plane is …… $\mathring{A}$

Q76. Zinc has hcp structure. The height of the unit cell is $0.494\ nm$. The nearest neighbour distance is $0.27\ nm$. The atomic weight of zinc is 65.37. The density of zinc is $\ldots\ldots\ldots kg/m^3$

Q77. The lattice constant of the unit cell of α-iron is $0.287\ nm$. The number of atoms in plane (100) is $\ldots\ldots \times 10^{10}\ atoms/mm^2$. [Note: - α-iron has bcc structure]

Q78. In a crystal whose lattice primitives are $1.2\ \mathring{A}, 1.8\ \mathring{A}$ and $2.0\ \mathring{A}$ a plane (231) cuts an intercept of $1.2\ \mathring{A}$ on the x-axis. The corresponding intercepts on the z-axes is …. $\mathring{A}$

Q79. In a tetragonal lattice $a = b = 2.5\ \mathring{A},\ c = 1.8\ \mathring{A}$. The lattice spacing between (111) planes is ….. $\mathring{A}$

Q80. X- Ray of wavelength $0.71\ \mathring{A}$ incident on a simple cubic crystal $\left(a = 2.814\ \mathring{A}\right)$. The glancing angle of the (110) plane corresponding to the second-order diffraction maximum is ……..degree.

Q81. The wavelength of X-ray is $0.1537\ nm$. This radiations when diffracted from (111) planes of a crystal with FCC structure, corresponds to a Bragg angle of 19.2^{o}. The lattice constant of FCC crystal is …..….. $\mathring{A}$

Q82. Electrons are accelerated by $844V$ and are reflected from a crystal. The refection maximum occurs when the glancing angle is 58^{o}. The interplanar spacing of the crystal is …….. $\mathring{A}$

Q83. The ratio of second to first nearest neighbour distance in BCC crystal is ……

fiziks

MSQ (Multiple Select Questions)

Q84. Which of the following statements are correct for the Debye specific heat

(a) In one dimension C_V varies as T

(b) In two dimension C_V varies as T^2

(c) In three dimension C_V varies as T^3

(d)In two dimension C_V is constant when $T > \theta_D$

Q85. The classical value of molar lattice specific heat of solid in different dimensions is

(a) In $1D$, $C_V = \frac{1}{2}R$

(b) In $1D$, $C_V = R$

(c) In $2D$, $C_V = 2R$

(d) $2D, C_V = \frac{3}{2}R$

Q86. Which of the following statement are correct from experimental facts about heat capacity of solid?

(a) At low temperature, the heat capacity decreases sharply as T^3

(b) At low temperature, the heat capacity decreases as T

(c) If metal become super conduction, the decrease is even faster

(d) In magnetic solids, the heat capacity increase near the curie temperature

Q87. Which of the following statements are correct for the limitation of the debye theory of lattice specific heat?

(a) Debye's model is valid for long wavelengths only

(b) Debye's temperature θ_D depends on temperature

(c) Debye theory takes into account the interaction among the atoms

(d) The total number of vibrational modes are $3N$

Q88. Which of the following statements are correct for Einstein model of lattice heat capacity

(a) A crystal consists of dependent harmonic oscillators

(b) All the oscillators vibrate with the same natural frequency

(c) The oscillators are quantum oscillators and have discrete energy

(d) Any number of oscillators may be present in the same quantum state

Q89. Which of the following processes increases the resistivity of a metal

(a) The metal is heated (b) The metal is purified

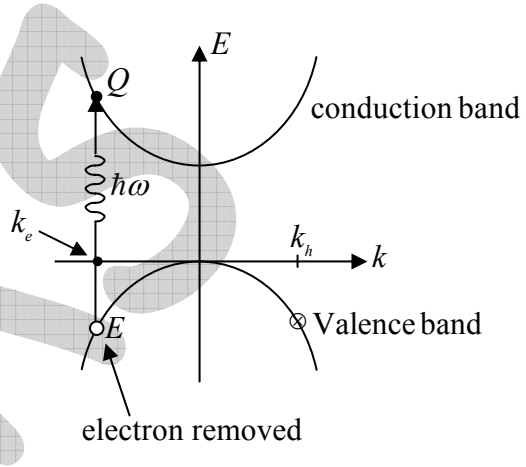
(c) The metal is cooled (d) Imperfections are introduced into the metal

Q90. The effective mass of an electron in a semiconductor is

(a) negative near the top of the band (b) positive near the bottom of the band

(c) zero near the centre of the band (d) Infinite near the top of the band

Q91. Which of the following correctly represent the relation between electron and holes for given condition and valence band



(a) $k_h = -k_e$

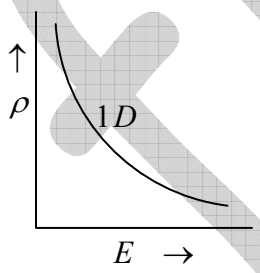
(b) $v_h = -v_e$

(c) $m_h = -m_e$

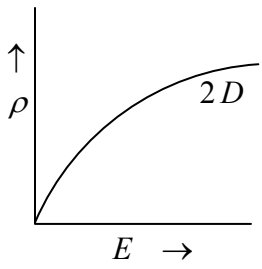
(d) $E_h = -E_e$

Q92. Which of the following correctly represent the variation of density of electron in different dimensions

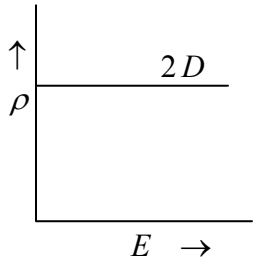
(a)



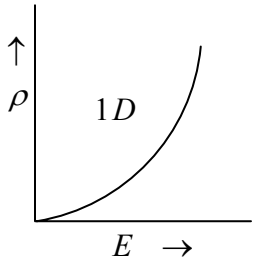
(b)



(c)



(d)



Q93. Which of the following correctly represent the expression of Fermi energy

(a) In one dimension $E_F = \frac{\hbar^2}{2m}\left(\frac{\pi N}{L}\right)^2$

(b) In one dimension $E_F = \frac{\hbar^2}{2m}\left(\frac{\pi N}{2L}\right)^2$

(c) In two dimension $E_F = \frac{\hbar^2}{2m}\left(\frac{\pi N}{L^2}\right)$

(d) In two dimension $E_F = \frac{\hbar^2}{2m}\left(\frac{2\pi N}{L^2}\right)$

Q94. Which of the following statements are correct expression of density of state for relativistic electrons

(a) In 3 dimension $\rho \propto E^{\frac{1}{2}}$ (b) In 3 dimension $\rho \propto E^2$

(c) In 2 dimension $\rho \propto E$ (d) In 2 dimension $\rho \propto E^o$

Q95. Which of the following is the correct expression for relativistics electron in one dimension

(a) Density of state is $\frac{4L}{\hbar c^2}$

(b) Fermi energy is $E_F = \frac{hc}{4}\left(\frac{N}{L}\right)$

(c) Fermi temperature is $T_F = \frac{\hbar c}{4k}\left(\frac{N}{L}\right)$

(d) Fermi wave vector is $k_F = \sqrt{\frac{\pi mc}{\hbar}.\frac{N}{L}}$

Solutions (MCQ)

Ans. 1: (d)

Solution: Area of Wigner-Seitz cell $= \dfrac{\text{Area of normal cell}}{N_{eff}}$

Where N_{eff} = effective number of lattice paints $= 6 \times \dfrac{1}{3} = 2$

Area of normal cell $= \dfrac{3\sqrt{3}}{2}a^2 \therefore$ Area of wigner-Seitz cell $= \dfrac{3\sqrt{3}}{2}a^2/2 = \dfrac{3\sqrt{3}}{4}a^2$

Ans. 2: (b)

Solution: According to Bragg's law $2d\sin\theta = \lambda$ where $d = \dfrac{a}{\sqrt{h^2+k^2+l^2}}$

For BCC structure the first diffraction peak appear for (110) plane.

$\therefore \qquad d = \dfrac{a}{\sqrt{2}}$

Now $\dfrac{2a}{\sqrt{2}}\sin 30^0 = \lambda \Rightarrow \sqrt{2}a\sin 30^0 = 2.1A^0 \Rightarrow \sqrt{2}\,a \times \dfrac{1}{2} = 2.1A^0 \Rightarrow a = \sqrt{2} \times 2.1A^0$

$\therefore a = 2.97A^0$

The volume primitive unit cell of BCC is $= \dfrac{a^3}{2} = \dfrac{26.2}{2}\left(A^0\right)^3 = 13.1\left(A^0\right)^3$

Ans. 3: (c)

Solution: For Cubic Lattice

$$d_c = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{a_c}{\sqrt{2}}$$

For Tetragonal lattice

$$d_t = \frac{1}{\sqrt{\dfrac{h^2+k^2}{a^2}+\dfrac{l^2}{c^2}}} = \frac{2}{\sqrt{3}}a_c$$

Therefore the ratio is $\dfrac{d_c}{d_t} = \sqrt{\dfrac{3}{8}}$

Ans. 4: (b)

Solution: For BCC, $4r = \sqrt{3}a$ $\therefore r = 1.83A^0$

For HCP, $2r = a$ $\therefore a = 3.66\ A^0$ and $c = 5.98A^0$

Ans. 5: (d)

Solution: ZnS has a Diamond structure whose primitive cell contains two atoms one is of Zn and other is of S placed at (000) and (1/4 1/4 1/4)

Ans. 6: (a)

Solution: In FCC there are 4 atoms at (0 0 0), $\left(\frac{1}{2}\frac{1}{2}0\right)$, $\left(\frac{1}{2}0\frac{1}{2}\right)$ and $\left(0\frac{1}{2}\frac{1}{2}\right)$

Thus the crystal structure factor is

$\therefore \quad F = f[1 + e^{\pi i(h+k)} + e^{\pi i(h+l)} + e^{\pi i(k+l)}]$

And $F = 4f \Rightarrow F^2 = 16f^2$: If h, k, l are unmixed

While $F = 0 \Rightarrow F^2 = 0$: If h, k, l are mixed

$\therefore$ Present planes are: (1 1 1), (2 0 0), (1 1 1)

Absent plane is: (2 1 2)

Ans. 7: (c)

Solution: The structure factor F is defined as

$$F = f\sum e^{2\pi i(hu_n + kv_n + lw_n)}$$

Where (h k l) are miller indices and (u_n, v_n, w_n) are position of the atoms.

In CsCl crystal, the unit cell consist 8 Cs atoms at the corner and one Cl atom at the body center. Unit cell has 2 effective number of atoms, one Cs atom at (0 0 0) and one Cl atom at (1/2 1/2 1/2).

$$\therefore F = f_{cl}e^{\pi i(h+k+l)} + f_{cs}$$

For (100) plane $F_1 = f_{cs} - f_{cl}$ and Intensity $I_1 = (f_{cs} - f_{cl})^2$

For (110) plane $F_2 = f_{cs} + f_{cl}$ and Intensity $I_2 = (f_{cs} + f_{cl})^2$

Given $f_{cs} = 3f_{cl}$

Therefore the ratio of intensity is $\frac{I_1}{I_2} = \frac{(f_{cs} - f_{cl})^2}{(f_{cs} + f_{cl})^2} = \frac{1}{4}$

Ans. 8: (c)

Solution: According to Bragg's law $2d\sin\theta = n\lambda$

Where d is the interplanar spacing, θ is angle of diffraction and λ is wavelength of incident electrons beam.

According to De-Broglie relation $\lambda = h/p$ where p is the momentum

Combining these two equations we get

$2d\sin\theta = nh/p$ or $p = nh/2d\sin\theta$

Given $n = 1$, $d = 0.3nm$ and $\theta = 5.8^0$

Thus

$$p = \frac{1 \times 6.625 \times 10^{-34}}{2 \times 0.3 \times 10^{-9} \times \sin(5.8)} = 10.9 \times 10^{-24} kgms^{-1}$$

Ans. 9: (c)

Solution: The Einstein temperature is

$$\theta_E = \frac{h\nu_E}{k_B}, \text{ where } \nu_E = 1.724\times10^{12}\ Hz,\ h = 6.62\times10^{-34}\ J.S\ \&\ k_B = 1.38\times10^{-23}\ J/K$$

$$\therefore \theta_E = \frac{\left(6.62\times10^{-34}\ J.S\right)\left(1.724\times10^{12}\ Hz\right)}{1.38\times10^{-23}\ J/K} \Rightarrow \theta_E = 82.7\ K$$

Ans. 10: (a)

Solution: According to the belong petit formula the thermal energy of a mole is

$$F = 3RT = 3\times\left(8.3\ J/mol\,K\right)\times\left(300\ K\right) = 7470\ J/mole$$

Ans. 11: (b)

Solution: $\nu_D = \nu_{\max} = \frac{v}{\lambda_{\min}} = \frac{\theta}{2a}$

$$\theta_D = \frac{h\nu_D}{k} = \frac{h\nu}{2ka} = \frac{\left(6.62\times10^{-34} J.S\right)\left(3000\ m/\sec\right)}{2\times\left(1.38\times10^{-23}\ J/k\right)\left(3\times10^{-10}\ m\right)} = \frac{6.62}{2.76}\times10^2$$

$$\therefore\quad \theta_D = 240\ K$$

Ans. 12: (a)

Solution: The molar specific heat is $C_V = \frac{12}{5}\pi^4 R\left(\frac{T}{\theta_D}\right)^3$

where $T = 10\ K,\ \theta_D = 150\ K,\ R = 8.314\ J/\text{mole-K}$

$$C_V = \frac{12}{5}\times\pi^2\times\left(8.314\ J\,\text{mol}^{-1}\ K^{-1}\right)\left(\frac{10}{150}\right)^3 = 1936.4\left(\frac{10}{150}\right)^3 = \frac{1936.4}{3375} = 0.574\ J\,\text{mol}^{-1}\ K^{-1}$$

Ans. 13: (d)

Solution: Heat required is

$$\theta = n\int_{10}^{50} C_V dT = \frac{2K}{\theta_D^3}\int_{10}^{50} T^3 dT = \frac{464\times 2}{(281)^3}\left[\frac{T^4}{4}\right]_{10}^{50} = \frac{116}{(281)^3}\left[50^4 - 10^4\right]$$

$$= \frac{116\times 2}{(281)^3}\times 10^4\left(625-1\right) = \frac{232\times 624\times 10^4}{22.19\times 10^6} = \frac{232\times 624}{22.9} = 63.2\ cal$$

Ans. 14: (c)

Solution: $C_V = A\left(\frac{T}{\theta_D}\right)^3,\qquad 3.8\times 10^{-2} = A\left(\frac{5}{\theta_D}\right)^3 \qquad (1)$

Assume at $T = 2K$ the specific heat is $x \quad \therefore \quad x = A\left(\frac{2}{\theta_D}\right)^3 \qquad (2)$

Dividing equation (1) from equation (2) we get $\frac{x}{9.8\times 10^{-2}} = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$

$$x = \frac{8}{125}\times 3.8\times 10^{-2} \Rightarrow x = 2.43\times 10^{-3}\ J\,mol^{-1}K^{-1}$$

Ans. 15: (a)

Solution: For KCl: $3.8\times10^{-2} = A\left(\frac{5}{220}\right)^3$ and For $NaCl$ $x = A\left(\frac{10}{330}\right)^3 \because C_V = A\left(\frac{T}{\theta_D}\right)^3$

$$\therefore \quad \frac{x}{3.8\times10^{-2}} = \left(\frac{10}{330}\right)^7 \times \left(\frac{220}{5}\right)^3 = 2^3\left(\frac{2}{3}\right)^3 \quad \therefore \quad x = \frac{2^6}{27}\times 3.8\times10^{-2} = \frac{64\times3.8}{27}\times10^{-2}$$

$$x = 0.09\ J\ mol^{-1}\ K^{-1}$$

Ans. 16: (d)

Solution: The internal energy is

$$E = 9Nk_B\left(\frac{T}{\theta_D}\right)^3 \times \frac{\pi^4}{15} = \frac{3}{5}\pi^4 Nk_B\frac{T^4}{\theta_D^3} \qquad \therefore \quad E \propto T^4$$

Ans. 17: (d)

Solution: According to Debye model, the Debye temperature is independent of temperature. But experimental fact reveals that θ_D first decreases and reach to minimum and than started increasing with temperature. Thus option (d) is correct

Ans. 18: (d)

Solution: The specific heat is $C_V = \frac{12}{5}\pi^4 R\left(\frac{T}{\theta_D}\right)^3$. Thus option (c) is correct.

If we put $R = 8.314\ J\ mol^{-1}\ K^{-1}$

$$\therefore \quad C_V = \frac{12}{5}\times(3.14)^4 \times\left(8.314\ J\ mol^{-1}\ K^{-1}\right)\left(\frac{T}{\theta_D}\right)^3$$

$C_V = 1944\left(\frac{T}{\theta_D}\right)^3\ J/mol.K$. Thus option (b) is correct.

Also $1\ Cal = 4.18\ J$

$\therefore \quad C_V = 464.4\left(\frac{T}{\theta_D}\right)^3\ Cal/mol.K$. Thus option (a) is also correct.

Ans. 19: (b)

Solution: At high temperature i.e. $T >> \theta_D$, $C_V = 3R$

Thus Debye theory is valid at low temperature

Ans. 20: (c)

Solution: The internal energy is $U = \frac{3}{5}\pi^4 N k_B \frac{T^4}{\theta_D^3}$

$\therefore \quad U \propto T^4$

Thus graph (c) is correctly represent.

Ans. 21: (d)

Solution: The number density of Fe crystal is = $\frac{n_{eff}}{a^3} \Rightarrow n = \frac{2}{\left(2.86\times10^{-10}\right)^3} = 8.55\times10^{28}\, m^{-3}$

Here n_{eff} = effective number of atoms in BCC lattice.

The saturation magnetization (M_S) is $M_s = n\mu$

$\therefore \qquad \mu = \frac{M_S}{n} = \frac{1.74\times10^6\, Am^{-1}}{8.55\times10^{28}\, m^{-3}} = 2.035\times10^{-23}\, Am^2$

while $\mu_B = 9.27\times10^{-24}\, Am^2 \qquad \therefore \mu = 2.2\mu_B$

Ans. 22: (b)

Solution: The paramagnetic susceptibility of Fermi free electron gas is $\chi = \frac{3\mu_0 n\mu_B^2}{2E_F}$

where $E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3} = \frac{\left(1.054\times10^{-34}\right)^2}{2\times9.1\times10^{-31}}\left(3\pi^2\times4.7\times10^{28}\right)^{2/3} = 7.6 \text{ x } 10^{-19} = 4.75 \text{ eV}$

$\therefore \chi = \frac{3\times4\pi\times10^{-7}\times4.7\times10^{28}\times\left(9.27\times10^{-24}\right)^2}{7.6\times10^{-19}} \Rightarrow \chi = 2.0 \text{ x } 10^{-5}$

Ans. 23: (c)

Solution: MnO has FCC lattice structure, thus the interplaner spacing is $\frac{a}{\sqrt{h^2+k^2+l^2}}$

At 293 K : The lattice constant is a = 4.43 Å

The peak (iii) appears at $2\theta = 24^0$

Thus $\theta = 12^o$ and $d = \frac{a}{\sqrt{3}}$

At 80K,

Let a' is the lattice parameter

Thus $d' = \frac{a'}{\sqrt{3}}$ for (iii) and $2\theta' = 12^0$ $\therefore \theta' = 6^0$

now from Bragg's law $2d \sin\theta = n\lambda$

this gives $2d \sin\theta = 2d' \sin\theta' \Rightarrow a' = \frac{a\sin\theta}{\sin\theta'} = 4.43 \times \frac{\sin 12^o}{\sin 6^o} = 8.81\,\overset{o}{A}$

Ans. 24: (d)

Ans. 25: (c)

Ans. 26: (b)

Ans. 27: (c)

Solution: $\frac{1}{\chi} = \frac{T - T_c}{C}$ and $T_c = \lambda C$. Here $T_c = 600K$ and $\frac{1}{\chi} = -2 \times 10^4$

Thus $C = 3 \times 10^{-2}$ and $\lambda = 2 \times 10^4$

Ans. 28: (c)

Solution: Since $\frac{N_1}{N_2} = \exp\left(\frac{2\mu_B B}{kt}\right) \Rightarrow \ln\left(\frac{N_1}{N_2}\right) = \frac{2\mu_B B}{kT} \Rightarrow B = \frac{kT}{2\mu_B}\ln\left(\frac{N_1}{N_2}\right)$

where $N_1\left(m_j = \frac{1}{2}\right) = 2N2\left(m_j = +\frac{1}{2}\right)$

$\therefore$ $B = \frac{1.38\times10^{-23}\times 2}{2\times 9.27\times10^{-24}}\ln(2) \Rightarrow B = 1.03T$

Ans. 29: (a)

Solution: The field density $B = \mu_o H = 0.063$ T

Thus $\frac{N_1}{N_2} = \exp\left(\frac{2\mu_B B}{kT}\right) = \exp\left(\frac{2\times 9.27\times 10^{-24}\times 0.063}{1.38\times 10^{-23}\times 1}\right)$

$= \exp(0.0846) = 1.083 \Rightarrow \frac{N_2}{N_1} = 0.919 \Rightarrow 1+\frac{N_2}{N_1} = 1.9188 \Rightarrow \frac{N_1}{N_1+N_2} = 0.52$

Thus the percentage of dipoles parallel to the field is

$\frac{N_1}{N}\times 100 = \frac{N_1}{N_1+N_2}\times 100 = 52\%$

Ans. 30: (b)

Solution: If the electron gas confined in one dimension rod of length L, then number of possible states in between k and k + dk is $g(k)dk = \frac{L}{\pi}dk$

Since, $E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}}$ $\quad \therefore dk = \left(\frac{2m}{\hbar^2}\right)^{1/2}\cdot\frac{1}{2}E^{-1/2}dE$

$\therefore g(E)dE = \frac{L}{\pi}\times\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{1}{2}E^{-1/2}dE$

Since, electrons have spin ±1/2. So multiply above equation with 2.

$$g(E)dE = \frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}E^{-1/2}dE$$

At $T = 0^0$ K.

Total number of electron between E = 0 and E_F is

$N = \int_0^{E_F} F(E)g(E)dE \Rightarrow N = \frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}\int_0^{E_F} E^{-1/2}dE$ (Since F(E) = 1 at T=0K)

$= \frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{E_F^{1/2}}{1/2} = \frac{2L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}E_F^{1/2}$

$\therefore \quad E_F = \frac{\hbar^2\pi^2}{2m}\left(\frac{N}{2L}\right)^2 = \frac{\hbar^2}{2m}\left(\frac{\pi N}{2L}\right)^2$

Ans. 31: (d)

Solution: For two dimensional gas, the number of possible k- states in between k and k + dk is

$$g(k)dk = \left(\frac{L}{2\pi}\right)^2 2\pi\, kdk \;\; = 2\left(\frac{L}{2\pi}\right)^2 2\pi\, kdk$$

It is multiply by 2 for electron gas.

Since $k^2 = \frac{2mE}{\hbar^2}$ $\quad \therefore 2kdk = \frac{2m}{\hbar^2}dE \therefore g(E)dE = 2\left(\frac{L}{2\pi}\right)^2 \pi \frac{2m}{\hbar^2}dE$

The total number of electrons of electrons at T = 0K is

$$N \;\; = \;\; 2\pi \cdot \frac{2m}{\hbar^2}\left(\frac{L}{2\pi}\right)^2 \int_0^{E_F} dE \;\; = 2\pi \cdot \frac{2m}{\hbar^2} \cdot \frac{L^2}{4\pi^2} E_F$$

$$E_F = \frac{\hbar^2\pi^2}{2m}\left(\frac{2N}{\pi L^2}\right) = \frac{\hbar^2}{2m}\left(\frac{2\pi N}{L^2}\right)$$

Ans. 32: (c)

Solution: For three dimensional gas

$$g\left(k\right)dk = \left(\frac{L}{2\pi}\right)^3 4\pi k^2 dk = 2\times\left(\frac{L}{2\pi}\right)^3 4\pi k^2 dk$$

since $k^2 = \frac{2mE}{\hbar^2}$ and $dk = \left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{1}{2}E^{-1/2}dE$

$$\therefore g\left(E\right)dE = 2\times\left(\frac{L}{2\pi}\right)^3 4\pi \frac{2mE}{\hbar^2}\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{1}{2}E^{-1/2}dE = \left(\frac{L}{2\pi}\right)^3 4\pi\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}dE$$

Total number of electrons at T=0K is

$$N = \left(\frac{L}{2\pi}\right)^3 \cdot 4\pi\left(\frac{2m^2}{\hbar}\right)^{3/2}\int_0^{E_F} E^{1/2}dE = \left(\frac{L}{2\pi}\right)^3 \cdot 4\pi\left(\frac{2m^2}{\hbar}\right)^{3/2} \times \frac{2}{3}$$

This gives $E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$

Ans. 33: (c)

Ans.34: (c)

Solution: For BCC lattice, $a = \frac{4r}{\sqrt{3}} = 4.295$ Å

Volume of the unit cell, $V = a^3 = 7.926 \times 10^{-29}$ m^3

Thus Fermi energy $E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} = 3.11$ eV

Ans. 35: (a)

Solution: **For 1D:**

density of state at Fermi level is $\rho(E_F) = \left(\frac{L}{\pi}\right)\left(\frac{2m}{\hbar^2}\right)^{1/2} E_F^{-1/2}$

and total numbers of electrons $N = \frac{2L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} E_F^{1/2}$

These two equations gives $\rho(E_F) = \frac{N}{2E_F}$

For 2D:

$\rho(E_F) = 2\pi\left(\frac{L}{2\pi}\right)^2 \frac{2m}{\hbar^2}$ while $N = 2\pi\left(\frac{L}{2\pi}\right)^2 \frac{2m}{\hbar^2} E_F$

Thus $\rho(E_F) = \frac{N}{E_F}$

For 3D:

$\rho(E_F) = 4\pi\left(\frac{L}{2\pi}\right)^3 \left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{1/2}$ and $N = 4\pi\left(\frac{L}{2\pi}\right)^3 \left(\frac{2m}{\hbar^2}\right)^{3/2} \frac{2}{3} E_F^{1/2}$

Thus $\rho(E_F) = \frac{3}{2}\frac{N}{E_F}$

Ans. 36: (b)

Solution: The Fermi radius is defined as $k_F = \sqrt{\frac{2mE_F}{\hbar^2}}$

The Fermi energy in 1D is $E_F = \frac{\hbar^2}{2m}\left(\frac{\pi N}{2a}\right)^2$

Above two equation gives $k_F = \frac{\pi N}{2a}$

Ans. 37: (c)

Solution: The Fermi radius is defined as $k_F = \sqrt{\frac{2mE_F}{\hbar^2}}$

The Fermi energy in 2D is $E_F = \frac{\hbar^2}{2m}\left(\frac{2\pi N}{a^2}\right)$

Above two equation gives $k_F = \sqrt{\frac{2\pi N}{a^2}}$

Ans. 38: (a)

Solution: The Fermi radius is defined as $k_F = \sqrt{\frac{2mE_F}{\hbar^2}}$

The Fermi energy in 3D is $E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{a^3}\right)^{2/3}$

Above two equation gives $k_F = \left(\frac{3\pi^2 N}{a^3}\right)^{1/3}$

Ans. 39: (d)

Solution: Fermi energy in one dimension is $E_F = \frac{\hbar^2}{2m}\left(\frac{\pi N}{2a}\right)^2$

Fermi velocity of electrons will be

$$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2}{m}\frac{\hbar^2}{2m}\left(\frac{\pi n}{2}\right)} = \frac{\pi\hbar n}{2m}$$

Ans. 40: (a)

Solution: Fermi energy in one dimension is $E_F = \frac{\hbar^2}{2m}\left(\frac{2\pi N}{a^2}\right)$

Fermi velocity of electrons will be $v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2}{m}\frac{\hbar^2}{2m}2\pi n} = \frac{\hbar}{m}\sqrt{2\pi n}$

Ans. 41: (d)

Solution: Fermi energy in one dimension is $E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{a^3}\right)^{2/3} = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$

Fermi velocity of electrons will be $v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2}{m}\frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}} = \frac{\hbar}{m}\left(3\pi^2 n\right)^{1/3}$

Ans. 42: (b)

Solution: The electronic heat capacity is $C_e = \gamma T$

Where γ = electronic heat capacity constant = $\frac{\pi^2 N_A K_B^2}{2E_F}$

$$= \frac{\pi^2 \times 6.023\times10^{23} \times \left(1.38\times10^{-23} Jk^{-1}\right)^2}{2\times7.0\times1.6\times10^{-19}} = 0.505 \text{ mJmol}^{-1}\text{k}^{-2}$$

$\therefore$ at T = 300 K, $C_e = 0.505 \times 10^{-3} \times 300 = 15.16 \times 10^{-2}$ J mol^{-1}K^{-1}

Ans. 43: (d)

Solution: The lattice specific heat $C_v = AT^3$

where $A = \frac{12\pi^4}{5} \times \frac{N_A k_B}{Q_D^3} = \frac{12\pi^4}{5} \times \frac{6.023\times10^{23} \times 1.38\times10^{-23}}{(343)^3}$

$A = 4.815 \times 10^{-5}$ Jmol^{-1}k^{-4}

The electronic heat capacity $C_e = \gamma T$

When Cv = Ce $\Rightarrow \gamma T = AT^3$

$$\therefore T^2 = \frac{\gamma}{A} \quad or \quad T = \sqrt{\frac{\gamma}{A}} = \sqrt{\frac{0.505\times10^{-3} Jmol^{-1}K^{-2}}{4.815\times10^{-5} Jmol^{-1}K^{-4}}} = \sqrt{10.49}$$

T = 3.239 K

Ans. 44: (a)

Solution: The dispersion relation is $E(k) = E_v - \beta - 2\gamma\cos(ka)$

where $\frac{dE}{dk} = 2\gamma a\sin(ka) \quad \Rightarrow \quad \frac{d^2E}{dk^2} = 2\gamma a^2\cos(ka)$

The effective mass of the electron is $m^* = \frac{\hbar^2}{d^2E/dk^2} = \frac{\hbar^2}{2\gamma a^2\cos(ka)}$

The effective mass of electron at the bottom of the band ($k = 0$) is $m^* = \frac{\hbar^2}{2\gamma a^2}$

Ans. 45: (b)

Solution: The dispersion relation is $E(k) = E_v - \beta - \gamma\cos(ka)$

where $\frac{dE}{dk} = \gamma a\sin(ka) \quad \Rightarrow \quad \frac{d^2E}{dk^2} = \gamma a^2\cos(ka)$

The effective mass of the electron is $m^* = \frac{\hbar^2}{d^2E/dk^2} = \frac{\hbar^2}{\gamma a^2\cos(ka)}$

The effective mass of electron at the top of the band ($k = \pm\pi/a$) is $m^* = -\frac{\hbar^2}{\gamma a^2}$

Ans. 46: (c)

Solution: Define new variable, $k_x' = k_x - \pi/a$, $k_y' = k_y - \pi/a$ and $k_z' = k_z - \pi/a$

The dispersion relation becomes

$$E(k) = E_0 - 2B\left(\cos k_x a + \cos k_y a + \cos k_z a\right) = E_0 - 2B\left(\cos\left(k_x' a + \pi\right) + \cos\left(k_y' a + \pi\right) + \cos\left(k_z' a + \pi\right)\right)$$

$$E(k) = E_0 + 2B\left(\cos k_x' a + \cos k_y' a + \cos k_z' a\right)$$

Near the top of the band the $k_x \to \pi/a, \quad k_y \to \pi/a, \quad k_z \to \pi/a$

The new variable becomes $k_x' \to 0 \quad k_y' \to 0, \quad k_z' \to 0$

$$\cos k_x' a \approx 1 - \frac{1}{2}\left(k_x' a\right)^2, \quad \cos k_y' a \approx 1 - \frac{1}{2}\left(k_y' a\right)^2, \quad \cos k_z' a \approx 1 - \frac{1}{2}\left(k_z' a\right)^2$$

$$E(k) = E_0 + 2B\left(\cos k_x' a + \cos k_y' a + \cos k_z' a\right) = E_0 + 2B\left(1 - \frac{1}{2}\left(k_x' a\right)^2 + 1 - \frac{1}{2}\left(k_y' a\right)^2 + 1 - \frac{1}{2}\left(k_z' a\right)^2\right)$$

$$= E_0 + 2B\left(3 - \frac{1}{2}a^2\left(k_x' + k_y' + k_z'\right)^2\right) = E_0 + 6B - Ba^2k^2$$

Effective mass of the electron is $m^* = \dfrac{\hbar^2}{d^2E/dk^2} = -\dfrac{\hbar^2}{2Ba^2}$

Ans. 47: (b)

Solution: Fermi radius is

$$k = \sqrt{k_x^2 + k_y^2} = \sqrt{\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}} = \sqrt{\frac{\pi^2}{a^2} + \frac{\pi^2}{4a^2}} = \frac{\pi}{2a}\sqrt{5}$$

$$k = \sqrt{k_x^2 + k_y^2} = \sqrt{\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}} = \sqrt{\frac{\pi^2}{a^2} + \frac{\pi^2}{4a^2}} = \frac{\pi}{2a}\sqrt{5} = 1.756 \times 10^{10} m^{-1} = 1.756 \times 10^{8} cm^{-1}$$

Ans. 48: (d)

Solution: The form of potential energy $V(x)$ for a crystal including the anharmonic behavior is $V(x) = cx^2 - gx^3 - fx^4$ The 1st term i.e. cx^2 is harmonic term while 2nd and 3rd terms (gx^3 and fx^4) are anharmonic terms.

The difference between the C_v and C_p is proportional to the square of the temperature coefficient of linear thermal expansion □. Which is defined as, $\alpha = \frac{3g}{4rc^2}k_B$. Where r is equilibrium inter-atomic distance and g is anharmonic constant.

Thus $C_p - C_V \propto \alpha^2$

For harmonic crystal $g = 0$, so $C_p = C_v$

While for anharmonic crystal $g \neq 0$ $g \neq 0$, so $C_p \neq C_v$

Ans. 49: (c)

Ans. 50: (b)

Solution: (i) Intercepts are $x = 5\overset{\circ}{A}, y = 4\overset{\circ}{A}, z = 3\overset{\circ}{A}$

Whereas lattice constants are $a = 1\overset{\circ}{A}, b = 2\overset{\circ}{A}$, & $c = 3\overset{\circ}{A}$

(ii) $\therefore \quad \frac{x}{a}, \frac{y}{b}, \frac{z}{c} = \frac{5\overset{\circ}{A}}{1\overset{\circ}{A}}, \frac{4\overset{\circ}{A}}{2\overset{\circ}{A}}, \frac{3\overset{\circ}{A}}{3\overset{\circ}{A}} = 5, 2, 1$

(iii) Reciprocal of these number are $\frac{1}{5}, \frac{1}{2}, 1$

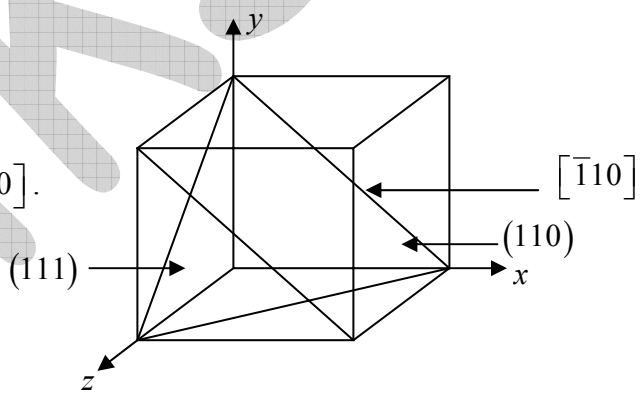
(iv) Multiply with LCM , $10 \times \left(\frac{1}{5}, \frac{1}{2}, 1\right) = 2, 5, 10$

$\therefore$ Miller Index is $(2 \quad 5 \quad 10)$

Ans. 51: (b)

Solution: The plane $(110)\&(110)$ can be drawn in unit as

The crossed line common to $(111)\&(110)$ is $[\bar{1}10]$ or $[1\bar{1}0]$.



Ans. 52: (c)

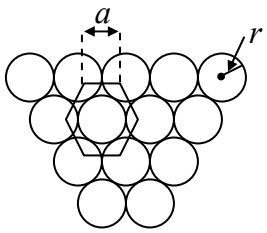
Solution: Packing fraction $P.F. = \frac{n_{eff} \times \frac{4\pi}{3} r^3}{V}$

Where $n_{eff} = \frac{1}{6} n_c + \frac{1}{2} n_f + 1 n_i = \frac{1}{6} \times 12 + \frac{1}{2} \times 0 + 1 \times 0 = 2$, $\quad V = 6 \times \frac{\sqrt{3}}{4} a^3 = \frac{3\sqrt{3}}{2} a^3$ and $a = 2r$

$$\therefore P.F. = \frac{2\pi \times \frac{4\pi}{3} \left(\frac{a}{2}\right)^3}{\frac{3\sqrt{3}}{2} a^3} = 2 \times \frac{4\pi}{3} \times \frac{a^3}{8} \times \frac{2}{3\sqrt{3} a^3} = \frac{2\pi}{a\sqrt{3}} = 0.40 (= 40\%)$$

Ans. 53: (c)

Solution: For Hexagonal unit cell $P.F. = \dfrac{n_{eff} \times \pi r^2}{A}$



where $n_{eff} = \dfrac{1}{3}\pi_c + \dfrac{1}{2}n_f + 1 \times n_i = \dfrac{1}{3} \times 6 + \dfrac{1}{2} \times 0 + 1 \times 1 = 3$

$$A = 6 \times \frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}}{2}a^2 \text{ and } a = 2r \;\; \therefore P.F = \frac{3 \times \pi\left(\frac{a}{2}\right)^2}{\frac{3\sqrt{3}}{2}a^2} = 3\pi \times \frac{a^2}{4} \times \frac{2}{3\sqrt{3}a^2} = \frac{\pi}{2\sqrt{3}}$$

Ans. 54: (b)

Solution: According to Bragg's law

$$2d\sin\theta = \lambda \quad \Rightarrow \quad \sin\theta = \frac{\lambda}{2d} \qquad \text{where } d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}$$

the different combination of (135) gives different 'd' values and hence different angle θ

the possible combination of miller indices are $(135),(153),(315),(351),(531),\&(513)$.

Thus (135) will split into 6 peaks

Ans. 55: (b)

Solution: $\rho = \dfrac{n_{eff} \times M}{N_A \times a^3} \Rightarrow a = \left[\dfrac{n_{eff} \times M}{N_A \times \rho}\right]^{\frac{1}{3}}$

where $M = 55.85\ kg$, $N_A = 6.023 \times 10^{26}$, $N_{eff} = 2$ and $\rho = 7860\ kg/m^3$

$$\therefore \quad a = \left[\frac{2 \times (55.85\ kg)}{(6.023 \times 10^{26})(7860\, kg/m^3)}\right]^{\frac{1}{3}} = \left[2.36 \times 10^{-29}\right]^{\frac{1}{3}} = 2.87 \times 10^{-10}\ m$$

For bcc structure $r = \dfrac{\sqrt{3}}{4}a = \dfrac{\sqrt{3}}{4} \times 2.87 \times 10^{-10}\ m = 1.24\ \mathring{A}$

Ans. 56: (d)

Solution: The possible values of $h^2+k^2+l^2$ for different lattice is

For SC: $h^2+k^2+l^2=1:2:3:4:5:6:8:9:10$

For BCC: $h^2+k^2+l^2=2:4:6:8:10:12$

For FCC: $h^2+k^2+l^2=3:4:8:11:12$

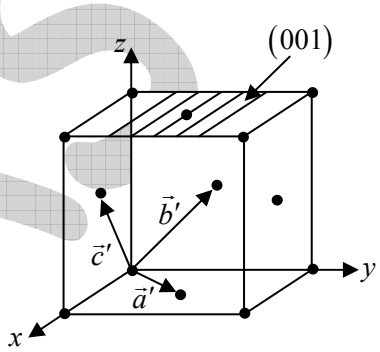
For DC: $h^2+k^2+l^2=3:8:11$

Thus in diamond cubic, only one line is present below 8

Ans. 57: (a)

Solution:

The plane (001) with respect to primitive axes of FCC $(\vec{a},\vec{b}\ \&\ \vec{c})$ is (011) whereas plane (100) w.r.t. primitive axes is (101)



Ans. 58: (d)

Solution: Final co-ordinates $0,0,1$

Initial co-ordinates $=1,1,\frac{1}{2}$

Subtracting these two gives $-1,-1,\frac{1}{2}$ $\therefore$ Indices is $[\bar{2}\bar{2}1]$

Ans. 59: (a)

Solution: From the Bragg's law

$2d\sin\theta=n\lambda$

For $n=1$, $\lambda=2d\sin\theta=2\times0.282\times\sin 8^{o}=0.0785\ nm$

For $n=2$

$\therefore\ \sin\theta=\frac{n\lambda}{2d}=\frac{2\times0.0785\times10^{-9}\ m}{2\times0.282\times10^{-9}\ m}=0.278\Rightarrow\theta=16.16^{o}$

Ans. 60: (d)

Solution: The wavelength of the wave associated with electron is

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{150}{V}}\left[\mathring{A}\right] = \sqrt{\frac{150}{344}}\,\mathring{A} = 0.66\,\mathring{A}$$

From Bragg's law $2d\sin\theta = n\lambda$

For $n = 1,\ \theta = 60^{o}$

$$d = \frac{n\lambda}{2\sin\theta} = \frac{1\times 0,66\times 10^{-10}\,m}{2\times\sin 60^{o}} = \frac{0.66\times 10^{-10}}{2\times 0.866} = 0.38\,\mathring{A}$$

NAT

Ans. 61: 0.604

Solution: $E = Ak^2 + B \Rightarrow \frac{\partial E}{\partial k} = 2Ak$ and $\frac{\partial^2 E}{\partial k^2} = 2A$. Now, $m^* = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k^2}}$

$$\frac{m^*}{m} = \frac{\hbar^2}{m\left(\frac{\partial^2 E}{\partial k^2}\right)} = \frac{\left(1.05\times 10^{-34}\ J.S\right)^2}{\left(9.1\times 10^{-31}\ kg\right)(2A)} = \frac{1.10\times 10^{-68}}{2\times 9.1\times 10^{-31}\times 10^{-38}} = \frac{1.10}{18.2}\times 10 = \frac{11.0}{18.2} \quad \therefore \quad \frac{m^*}{m} = 0.604$$

Ans. 62: 5.69

Solution: The Fermi radius is $K_F = \left(3\pi^2 n\right)^{\frac{1}{3}} = \left(\frac{3\pi^2\times 4}{a^3}\right)^{\frac{1}{3}} = \frac{\left(12\pi^2\right)^{\frac{1}{3}}}{a}$

$$E_F = \frac{\hbar^2 K_F^2}{2m} = \frac{\hbar^2}{2m}\left(12\pi^2\right)^{\frac{2}{3}}\frac{1}{a^2} = \frac{\left(1.05\times 10^{-34}\,J.S\right)^2}{2\times\left(9.1\times 10^{-3}\ kg\right)}\left(12\pi^2\right)^{\frac{2}{3}}\times\frac{1}{\left(4\times 10^{-10}\,m\right)^2}\times\frac{1}{1.6\times 10^{-19}}eV$$

$$= \frac{24.1\times 10}{18.2\times 16\times 1.6}\times\frac{10^{-68}}{10^{-70}}eV = \frac{2631.9}{45.9} = 5.69\ eV$$

Ans. 63: 4.6

Solution: The Bragg's law is $2d\sin\theta = \lambda$ where $d = \dfrac{a}{\sqrt{\hbar^2 + k^2 + l^2}}$

For maximum wavelength, $\theta = 90^o$ $\therefore$ $\lambda = 2a$ where $a = 2.3\times10^{-10}\ m$

$\therefore$ $\lambda = 4.6\ \mathring{A}$

Ans.64: 1.38

Solution: $E_F = \left(\dfrac{h^2}{2m}\right)\left(\dfrac{3n}{8\pi}\right)^{\frac{2}{3}} \Rightarrow n = \left(\dfrac{\pi}{3}\right)\left(\dfrac{8m}{h^2}\right)^{\frac{3}{2}} E_F^{\frac{3}{2}}$

where $E_F = 2.1\ eV = 2.1\times1.6\times10^{-19}\ J$, $h = 6.02\times10^{-34}\ J.S$, $m = 9.1\times10^{-31}\ kg$

$$n = \left(\frac{\pi}{3}\right)\left(\frac{8\times9.1\times10^{-31}\ kg}{\left(6.62\times10^{-34}\ J.S\right)}\right)^{\frac{3}{2}}\left(2.1\times1.6\times10^{-19}\right)^{\frac{3}{2}} \Rightarrow n = 1.38\times10^{28}/m^3$$

Ans. 65: 2.995

Solution: $Ce = \dfrac{\pi^2 k^2 N_A}{2E_F}T$

Where $E_F = 7.05\times1.6\times1.6\times10^{-19}\ J$, $K = 1.38\times10^{-23}\ J/K$, $T = 800\,K$

and $N_A = 6.02\times10^{26}$

$$Ce = \frac{n^2\times\left(1.38\times10^{-23}\ J/K\right)^2\left(6.02\times10^{26}\right)}{2\times7.05\times1.6\times10^{-19}\ J}\times600 \Rightarrow Ce = 2.995\ J\left(kmol\right)^{-1}K^{-1}$$

Ans. 66: 505

Solution: $F(E) = \dfrac{1}{e^{(E-E_F)/kT}+1} \Rightarrow \quad \dfrac{1}{100} = \dfrac{1}{e^{0.2/kT}+1} \Rightarrow \quad 0.01 = \dfrac{1}{e^{0.2/kT}+1}$

$$\Rightarrow \quad 0.01e^{0.2/kT} + 0.01 = 1 \Rightarrow e^{0.2/kT} = \frac{1-0.01}{0.01} = 99 \ \therefore \ \frac{0.2\,eV}{kT} = 2.303\log_{10}99$$

$$T = \frac{0.2\,eV}{2.303\ K\log_{10}99} = \frac{0.2\times1.6\times10^{-19}}{2.303\times1.995\times1.38\times10^{-23}} = \frac{3,2\times10^4}{634} = 505\ K \qquad \Rightarrow T = 505\ K$$

Ans. 67: 3.306

Solution: The average energy at $0K$ is given by $E_0 = \frac{3}{5}E_F = \frac{3}{5}\times 5.51\ eV \Rightarrow E_0 = 3.306\ eV$

Ans.68: 7.04

Solution: The electron density $n = \frac{N}{V}$ is $n = Z\frac{\rho N_A}{M}$.

where $Z = 1,\ \rho = 8.94\times10^3\ kg/m^3,\ N_A = 6.073\times10^{26}\ atoms/k\,mol\ \&\ M = 63.5\ kg$

$$\therefore\quad n = 1\times\frac{\left(8.94\times10^3\ kg/m^3\right)\left(6.023\times10^{26}\right)}{63.5\,kg}$$

$$E_F = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{\frac{2}{3}}\therefore\quad E_F = \frac{\left(6.63\times10^{-34}\ J.S\right)^2}{(2)\left(9.1\times10^{-31}\,kg\right)}\left(\frac{3\times8.48\times10^{28}}{8\pi}\right)^{\frac{2}{3}} = 1.13\times10^{-18}\ J = 7.04\ eV$$

Ans. 69: 9372.2

Solution: The Debye temperature θ_D is defined as $\theta_D = \frac{\hbar v_D}{k_B}$ where v_D is Debye frequency

$$\therefore\quad v_D = \frac{\theta_D k_B}{\hbar}\ \text{where}\ \theta_D = 450\ K,\ k_B = 1.38\times10^{-23}\ JK^{-1}\ \text{and}\ h = 6.626\times10^{-34}\ J.S$$

$$\therefore\quad v_D = \frac{(450\ K)\left(1.38\times10^{-23}\ J/K\right)}{6.626\times10^{-34}\ J.S} = \frac{6.21\times10^{-21}}{6.626\times10^{-34}}$$

$$v_D = 9.372\times10^{12}\ Hz \Rightarrow v_D = 9372.2\times10^9\ Hz \Rightarrow v_D = 9372.2\ GHz$$

Ans. 70: 2.456

Solution: The specific heat is $C_V = \frac{12}{5}\pi^4 R\left(\frac{T}{\theta_D}\right)^3$

where $R = 8.314\ J\,mol^{-1}\,K^{-1},\ \theta_D = 1850\ K$ and $T = 20\ K$

$$\therefore\quad C_V = \frac{12}{5}\pi^4\times\left(8.314\ J\,mol^{-1}\,K^{-1}\right)\left(\frac{20\ K}{1850\ K}\right)^3$$

$$= 1943.66\times1.2635\times10^{-6} = 2.4558\times10^{-3}\ J\,mol^{-1}\ K^{-1} = 2.456\ J\,kmol^{-1}\ K^{-1}$$

Ans. 71: 4239

Solution: The thermal every is $\langle E\rangle = \frac{3}{2}k_BT \Rightarrow \langle E\rangle = \frac{3}{2}k_B\theta_D \because T = \theta_D$

For on mole of copper $E = \frac{3}{2}N_Ak_B\theta_D$

where $\theta_D = 340\,K$, $N_A = 6.023\times10^{23}\,\frac{\text{atoms}}{\text{mol}}$ & $k_B = 1.38\times10^{-23}\ J\,K^{-2}$

$\therefore \quad E = \frac{3}{2}\left(6.023\times10^{23}\right)\left(1.38\times^{-23}\ \text{J/K}\right)\left(340\ K\right) \Rightarrow E = 4238.99 = 4239\ \text{J}$

Ans. 72: 7.8

Solution: Relation between Debye temperature and Debye frequency is $\theta_D = \frac{h\nu_D}{k_B} \Rightarrow \nu_D = \frac{\theta_Dk_B}{h}$

where $\theta_D = 375\ K$, $k_B = 1.38\times10^{-23}\ J/K$ & $h = 6.62\times10^{-34}\ J.S$

$\therefore \quad \theta_D = \frac{\left(375\ K\right)\left(1.38\times10^{-23}\ J/K\right)}{6.62\times10^{-34}\ J.S} \Rightarrow \nu_D = 7.8\times10^{12}\ Hz$

Ans. 73: 1.27

Solution: $\frac{N_2}{N_1} = e^{\frac{-(E_2-E_1)}{kT}} = e^{\frac{-\Delta E}{kT}}$ where $kT = \frac{1.38\times10^{-23}\times300}{1.6\times10^{-19}} = 0.0259\ eV$

and $\Delta E = 10^{-21}\ J = \frac{10^{-21}}{1.6\times10^{-19}} = 0.0063\ eV$

now $\frac{N_1}{N_2} = e^{\frac{\Delta E}{kT}} \quad \therefore \quad \frac{N_1}{N_2} = e^{\left(\frac{0.0063}{0.0259}\right)} = 1.27 \Rightarrow \frac{N_1}{N_2} = 1.27$

Ans. 74: 2

Solution: $\rho = \frac{n_{eff}\times M}{N_A\times a^3} \quad \Rightarrow \quad n_{eff} = \frac{\rho N_Aa^3}{M}$

Where $\rho = 7870\ kg/m^3$, $N_A = 6.023\times10^{26}$, $a = 2.9\times10^{-10}\ m$ and $M = 55.85\ kg$

$\therefore \quad n_{eff} = \frac{\left(7870\ kg/m^3\right)\left(6.023\times10^{26}\right)\left(2.9\times10^{-10}\ m\right)^3}{55.85} = 2$

Ans. 75: 0.966

Solution: The relation between a and r for FCC is $\sqrt{2}a = 4r \Rightarrow a = \frac{4r}{\sqrt{2}} = 2\sqrt{2}r$

$$\because d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}} \Rightarrow d_{hkl} = \frac{2\sqrt{2}r}{\sqrt{h^2+k^2+l^2}}$$

where $r = 1.278\times10^{-10}\ m, \quad (hkl) = (321)$

$$\therefore d_{321} = \frac{2\sqrt{2}\left(1.278\times10^{-10}\ m\right)}{\sqrt{3^2+2^2+1^1}} = \frac{3.615\times10^{-10}}{\sqrt{14}} = 0.966\times10^{-10}\ m = 0.966\ \mathring{A}$$

Ans. 76: 6960

Solution: density is $\rho = \frac{n_{eff}\times M}{N_A \times V}$

For hcp, $n_{eff} = 6$ and $V = \frac{3\sqrt{3}}{2}a^2c$, where $a = 2.7\times10^{-10}\ m$ and $c = 4.94\times10^{-10}\ m$

$$V = \frac{3\sqrt{3}}{2}\times\left(2.7\times10^{-10}\ m\right)^2\left(4.94\times10^{-10}\ m\right) = 9.36\times10^{-29}\ m^3$$

Thus $\rho = \frac{6\times(65.37\ kg)}{\left(6.023\times10^{26}\right)\left(9.36\times10^{-29}\ m^3\right)} = \frac{392.22}{0.05635} = 6960\ kg/m^3$

Ans. 77: 1200

Solution: The number of atoms in plane (hkl) is $n = \frac{n_{eff}}{A}$

where n_{eff} = no of atoms in plane (hkl) and A = Area of plane (hkl)

In BCC for (100) plane $n_{eff} = 1$ and $A = a^2 \quad \therefore \quad N = \frac{1}{a^2}$

where $a = 2.87\times10^{-10}\ m$

$$\therefore \quad N = \frac{1}{\left(2.87\times10^{-10}\ m\right)} = \frac{1}{\left(2.87\times10^{-7}\ mm\right)^2} = 1.2\times10^{13} = 1200\times10^{10}\ atoms/mm^2$$

Ans. 78: 4

Solution: Let p, q and r be the intercepts on the x, y and z-axis respectively. Then from the law of rational indices

$$p:q:r=\frac{a}{h}:\frac{b}{k}:\frac{c}{l}=\frac{1.2}{2}:\frac{1.8}{3}:\frac{2.0}{1}=0.6:0.6:2.0 \Rightarrow \frac{1.2}{q}=1 \Rightarrow r=1.2\,\mathring{A}$$

Similarly $\frac{q}{r}=\frac{0.6}{2} \Rightarrow \frac{1.2}{r}=\frac{0.6}{2} \Rightarrow r=\frac{2.4}{0.6} \Rightarrow r=4\,\mathring{A}$

$\therefore$ The intercept on z-axes is $4\,\mathring{A}$

Ans. 79: 1.26

Solution: $d_{hkl}=\frac{1}{\sqrt{\frac{h^2+k^2}{a^2}+\frac{l^2}{c^2}}} \therefore \quad d_{111}=\frac{1}{\sqrt{\frac{1^2+1^2}{(2.5)^2}+\frac{1^2}{(1.8)^2}}}=\left[\frac{2}{6.25}+\frac{1}{3.24}\right]^{\frac{-1}{2}}$

$$d_{111}=\left[0.63\right]^{\frac{-1}{2}}=\left[\frac{100}{63}\right]^{\frac{1}{2}}=1.26\,\mathring{A}$$

Ans. 80: 21

Solution: $d_{hkl}=\frac{a}{\sqrt{h^2+k^2+l^2}} \quad \therefore \quad d_{110}=\frac{2.814\,\mathring{A}}{\sqrt{1^2+1^2+0}}=\frac{2.814}{\sqrt{2}}\,\mathring{A}$

From Bragg's law: $2d_{110}\sin\theta=n\lambda \qquad \therefore \quad \sin\theta=\frac{n\lambda}{2d_{110}}$

where $n=2,\ \lambda=0.71\,\mathring{A},\ d_{110}=\frac{2.814}{\sqrt{2}}\,\mathring{A}$

$$\therefore \quad \sin\theta=\frac{2\times 0.710\,\mathring{A}}{2\times\frac{2.814}{\sqrt{2}}\,\mathring{A}}=0.357 \Rightarrow \quad \theta=\sin^{-1}\left(0.357\right)=21^{o}$$

Ans. 81: 4.05

Solution: From the Bragg's law $2d\sin\theta = n\lambda \Rightarrow \quad d = \dfrac{n\lambda}{2\sin\theta}$

$$\therefore \quad d = \frac{1\times 1.537\times 10^{-10}\ m}{2\times \sin\left(19.2^{o}\right)} = 2.336\times 10^{-10}\ m$$

The lattice constant is

$$a = d\sqrt{h^2+k^2+l^2} = \left(2.336\times 10^{-10}\ m\right)\sqrt{1^1+1^2+1^2} = 4.046\times 10^{-10}\ m = 4.046\ \mathring{A}$$

Ans. 82: 0.25

Solution: The wavelength of the wave associated with electrons

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \sqrt{\frac{150}{V}}\ \mathring{A} = \sqrt{\frac{150}{844}}\ \mathring{A} = 0.42\ \mathring{A}$$

$$\therefore \quad 2d\sin\theta = n\lambda \quad \Rightarrow d = \frac{n\lambda}{2\sin\theta}$$

For $n = 1 \therefore \quad d = \dfrac{0.42\times 10^{-10}\ m}{2\times \sin 58^{o}} = 0.25\times 10^{-10}\ m \Rightarrow d = 0.25\ \mathring{A}$

Ans. 83: 0.866

Solution: The first nearest neighbour distance is $= a$

The second nearest neighbour distance is $= \dfrac{\sqrt{3}a}{2}$

$$\therefore \quad \frac{\text{second nearest neighbour distance}}{\text{First nearest neighbour distance}} = \frac{\frac{\sqrt{3}}{2}a}{a} = \frac{\sqrt{3}}{2} = 0.866$$

MSQ

Ans. 84: (a), (b), (c) and (d)

Solution: All the statements are correct. The specific heat varies as T, T^2 and T^3 in $1D, 2D$ and $3D$ whereas when temperature is greater than Debye temperature $(T >> \theta_D)$ the specific heat of solid is $C_V = R$

Ans. 85: (b) and (c)

Solution: In one dimension, average energy per oscillating particle is $\langle E \rangle = \langle K.E \rangle + \langle P.E \rangle$

where $\langle K.E \rangle = \langle P.E \rangle = \frac{1}{2} k_B T \qquad \therefore \quad \langle E \rangle = k_B T$

For one mole of substance: $E = N_A \langle E \rangle \Rightarrow E = N_A k_B T = RT$

The specific heat is $C_V = \frac{dE}{dT} = R$

In 2-Dimension: $\langle K.E \rangle = \langle P.E \rangle = k_B T \qquad \therefore \quad \langle E \rangle = 2k_B T$

For 1-mole of substance $E = N_A \langle E \rangle = 2N_A k_B T = 2RT$

The specific heat is $C_V = \frac{dE}{dT} = 2R$

Thus correct options are (b) & (c) only

Ans. 86: (a), (b), (c) and (d)

Solution: (a) In insulator, the specific heat in $3D$ is $C_V = \frac{12}{4} \pi^2 R \left(\frac{T}{\theta_D} \right)^3 = BT^3$

(b) in metal the specific heat is $C_e = \frac{3}{2} N k_B \left(\frac{k_B T}{E_F} \right) = AT$

(c) In superconductivity $C_{es} = a \exp\left(-\frac{\Delta E}{kT} \right)$

The decrease is exponential which is faster than T & T^3

(d) In magnetic solids, the heat capacity increases near the curie temperature when the magnetic moments become ordered.

Ans. 87: (a) and (d)

Solution: (a) Debye model is a continuum model which is valid only for long wavelength

(b) $\theta_D = \frac{\hbar v_0}{k_B}$, thus θ_D is independent of temperature

(c) The theory does not take into account the actual crystalline nature of the solid. The theory cannot be applied to crystals comprising max than one type of atoms

(d) The total number of vibrational modes are assumed to $3N$

Ans. 88: (b), (c), and (d)

Solution: (a) A crystal consist of atoms which may be regarded as identical and independent harmonic oscillators wrong option

(b) Due to the identical environment of each, all the oscillators vibrate with same frequency. Correct option

(c) Correct option

(d) Correct option

Ans. 89: (a) and (d)

Solution: The resistively of metal increases because of tow reasons

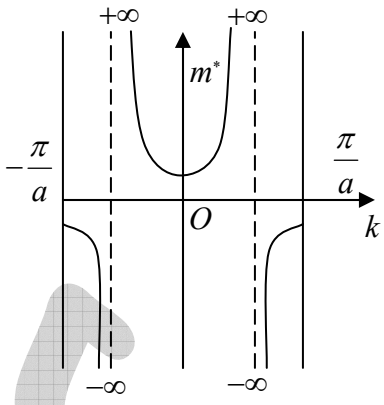
(i) increases in electron-photon scattering. With increase in temperature the electron-photon collision increase and hence resistively

(ii) electron impurity scattering with addition of more number of impurities the electrons collision with impurity increases and hence resistively

Thus correct option are (a) and (d)

Ans. 90: (a) and (b)

Solution: The variation of effective mass is



(a) Near the bottom of the band $\left(F=\frac{\hbar^2k^2}{2m_0}\right)$

$$m^*=\frac{\hbar^2}{\frac{d^2E}{dk^2}}=\frac{\hbar^2}{\left(\frac{\hbar^2}{m_0}\right)}=m_0$$

$m^*=m$ (Rest mass of electron and positive)

(b) Near the top of the band $\left(k=\pm\frac{\pi}{a}\right)$ the m^* is negative i.e. $m^*=-m_0$

Thus correct options are (a) and (b)

Ans. 91: (a), (c) and (d)

Solution: (a) $k_h=-k_e$ (correct)

If an electron is missing from an orbital of wave vector k_e the total wavevector of the system is $-ve$ and is attributed to the hole. $k_h+k_e=0 \Rightarrow k_h=-k_e$

(b) $v_h=-v_e$ (Incorrect)

The velocity of the hole is equal to the velocity of the missing electron. Form the figure we see that $v_h=\frac{\partial E_h}{\partial k_h}=\frac{\partial E_e}{\partial k_e}=v_e \Rightarrow v_h=v_e$

(c) $m_h=-m_e$ (correct)

The effective mass is inversely proportional to the curvature $\frac{\partial^2E}{\partial k^2}$ and for hole band this has the opposite sign to that for an electron in the valence band. Near the top of the valence band m_e is negative, so that m_h is positive

(d) $E_h=-E_e$ (correct)

If the band is symmetric. Then

$$E_e(k_e)=E_e(-k_e)=-E_h(-k_h)=-E_h(k_h)\Rightarrow E_e=-E_h$$

Ans. 92: (a) and (c)

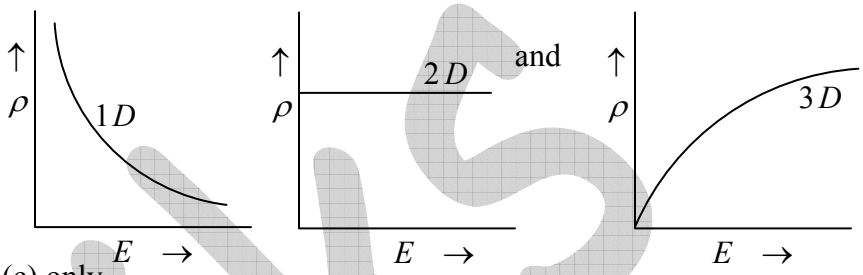
Solution: The density of state is

In $1D$ $\quad \rho(F) \propto E^{\frac{-1}{2}}$

In $2D$ $\quad \rho(F) \propto E^{o}$

In $3D$ $\quad \rho(F) \propto E^{\frac{1}{2}}$

Thus



Correct options are (a) and (c) only

Ans.93: (b) and (d)

Solution: The Fermi energy in one dimension is $E_F = \frac{\hbar^2}{2m}\left(\frac{\pi N}{2L}\right)^2$

The Fermi energy in two dimension is $E_F = \frac{\hbar^2}{2m}\left(\frac{2\pi N}{L^2}\right)$

Ans. 94: (b) and (c)

Solution: The dispersion relation of relativistic electrons is $E = pc = \hbar kc$

$\therefore \quad k = \frac{E}{\hbar c}$ and $dk = \frac{dE}{\hbar c}$

In three dimension, the number k-states in range k and $k + dk$ is

$$g(k)dk = 2\pi\left(\frac{L}{2\pi}\right)^3 .4\pi k^2 dk$$

$$g(E)dE = 8\pi\left(\frac{L}{2\pi}\right)^3 \cdot \frac{E^2}{(\hbar c)^2} \cdot \frac{dE}{\hbar c} = 8\pi\left(\frac{L}{2\pi}\right)^3 \cdot \frac{E^2}{(\hbar c)^3} dE$$

The density of state is $\rho(E) = \frac{g(E)dE}{dE} = 8\pi\left(\frac{L}{2\pi}\right)^3 \frac{E^2}{(\hbar c)^3} \quad \therefore \quad \rho(E) \propto E^2$

In two dimension, the number of k-states in range k and $k+dk$ is

$$g(k)dk = 2\times\left(\frac{L}{2\pi}\right)^2 2\pi k dk$$

$$\therefore \quad g(E)dE = 2\left(\frac{L}{2\pi}\right)^2 2\pi\cdot\frac{E}{\hbar c}\times\frac{dE}{\hbar c} = 4\pi\left(\frac{L}{2\pi}\right)^2\cdot\frac{E}{(\hbar c)^2}dE$$

Density of state is

$$\rho(E) = \frac{g(E)dE}{dE} = 4\pi\left(\frac{L}{2\pi}\right)^2\frac{E}{(\hbar c)^2} \quad \therefore \quad \rho(E)\propto E$$

Ans. 95: (b), (c), and (d)

Solution: The dispersion relation of relativistic electrons is

$$E = pc = \hbar kc \;\Rightarrow\; k = \frac{E}{\hbar c} \Rightarrow dk = \frac{1}{\hbar c}dE$$

(a) The number of k-states in range k and $k+dk$ in 1 dimension is

$$g(k)dk = 2\times\frac{L}{\pi}dk = \frac{2L}{\pi}\times\frac{dE}{\hbar c} = \frac{2L}{\pi}\frac{dE}{\frac{\hbar}{2\pi}c}$$

$$g(E)dE = \frac{4L}{\hbar c}dE \;\therefore \text{ Density of state is } \rho(E) = \frac{g(E)dE}{dE} = \frac{4L}{\hbar c}$$

(b) At $T = 0\,K$, the number of electrons can be calculated as

$$N = \int_0^{E_F} g(E)dE = \frac{4L}{\hbar c}\int_0^{E_F} dE = \frac{4L}{\hbar c}E_F \;\therefore \quad E_F = \frac{\hbar c}{4}\left(\frac{N}{L}\right)$$

(c) Fermi temperature is $T_F = \frac{E_F}{K} = \frac{\hbar c}{4K}\left(\frac{N}{L}\right)$

(d) The Fermi wave vector (k_F) can be calculated as

$$E_F = \frac{\hbar^2 k_F^2}{2m} \quad \Rightarrow\; k_F = \sqrt{\frac{2mE_F}{\hbar^2}}$$

$$k_F = \left(\frac{2m}{\hbar^2}\times\frac{\hbar c}{4}\times\frac{N}{L}\right)^{\frac{1}{2}} = \left(\frac{2m}{\hbar^2}\times 4\pi^2\times\frac{\hbar c}{4}\times\frac{N}{L}\right)^{\frac{1}{2}} = \sqrt{\frac{2\pi^2 mc}{\hbar}9\left(\frac{N}{L}\right)} = \sqrt{\frac{\pi mc}{\hbar}\left(\frac{N}{L}\right)}$$